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E2-80-565

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TENSOR POLARIZATION OF THE FORWARD DEUTERONS IN THE (p, pd) AND (p, πd) REACTIONS

Submitted to ЯФ

1980

INTRODUCTION

Recent experimental investigation of the quadrupole polarization of deuterons in the pd back scattering shows that its value is about zero within the experimental errors throughout a wide range of incident proton energies T_0~400-1000 MeV [1]. This result is in strong disagreement with the predictions of the single nucleon pole model or the nucleon resonance model of Kerman and Kisslinger [2] which predict strongly variable large tensor polarization coefficients [3].

The investigation of the polarization phenomena gives a rigorous test to different models of the pd back scattering at intermediate energies. The differential cross sections of this process, calculated in different models [4-7], can give a good agreement with the experimental data. The difference between different models may be found by the study of vector and tensor polarization phenomena which depend strongly on the dynamics of the process.

The smallness of tensor polarization in the triangle model of Craigie and Wilkin [8] (see fig.1a) was already predicted [9]. The second rank polarization tensor in the triangle model can be constructed only from vector $\vec{q}$ which is the relative momentum of nucleons in the deuteron. Since in the integral on $\vec{q}$ the main contribution is given by the small values of q [4,5], the quantity $\langle q_i q_j \rangle$ must also have a small value.

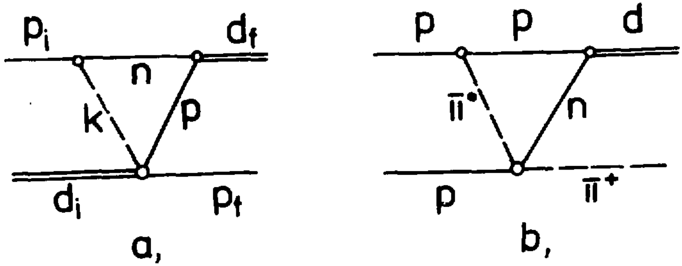

Fig.1. a) The triangle diagram of Craigie and Wilkin for the pd→pd scattering. b) The triangle diagram of Yao for the $pp \to d\pi^+$ reaction.

This argument is not enough as the d-wave component of the deuteron wave function, which gives only a small fraction to the differential cross sections [10], has a very important role in the calculation of quantity $\langle q_i q_j \rangle$. The momentum spectra of the d-wave component is strongly different from the spectra of the s-wave since the d-wave is described by large q values.

But at the incident energy $T_0 = 620$ MeV at which is a threshold energy for the creation of the Δ_{33} resonance in the NN collisions, the Δ_{33} resonance can be created only in the case of $q = 0$ (if the resonance has zero width). So in the models where the main contribution to the amplitude of the scattering is determined by the creation of the Δ_{33} resonance [7]*, in the given energy range the small value of the tensor polarization can be predicted.

The calculations (see further below) show that in the Craigie-Wilkin model of the pd back scattering the final resonance width in the $pp \to d\pi^+$ reaction gives some small contribution.

Basing on the above arguments, we shall show later, that the value of tensor polarization in the $pp \to d\pi^+$ reaction is small. The smallness of tensor polarization of forward deuterons in the quasifree reactions A(p, pd) B and A(p, πd) B is predicted, too.

Preliminary results of these calculations have been published in work [11].

TENSOR POLARIZATION IN THE TRIANGLE MODEL

Using the definition and notation of work [3] for the tensor polarization parameter A we have:

$$A = \frac{N_+ + N_- - 2N_0}{N_+ + N_- + N_0}, \tag{1}$$

where N_+, N_- and N_0 are the numbers of deuterons in the final state in the case of unpolarized beam and target with spin-projection values $M_z = +1, -1$, and 0, respectively. The region of values of A according to (1) is $-2 \le A \le 1$.

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*Calculation of tensor polarization in the pd back scattering in the model [7] has been recently carried out by L.A. Kondratyuk et al.

The triangle model [5] for the pd back scattering ($\theta_{C.M.}$ =180°) gives the following simple expression for A.

$$A = \frac{-|f_2|^2 + 2\sqrt{2}\,\mathrm{Re}(f_0 f_2^*)}{|f_0|^2 + |f_2|^2}. \qquad (2)$$

In the usual peaking approximation, calculating the slowly variable quantities at $\vec{q}=0$, f_ℓ is defined in terms of the deuteron wave function components $\phi_\ell(r)$ with orbital momentum $\ell = 0$ or $\ell = 2$.

$$f_\ell(r) = \int_0^\infty \phi_\ell(r) e^{-\gamma r}\, j_1(\tilde{p} r)(1+\gamma r)\, dr, \qquad (3)$$

$j_m(x)$ is a spherical Bessel function. Numerical factors γ and $\tilde{p}$ are discussed in refs. [5,10], their definition is given in the Appendix, see formulae A2. Their values in the isobar region are $\gamma \sim 0.73$ fm^{-1} and $\tilde{p} \sim 1.65$ fm^{-1}.

In the peaking approximation A, practically, does not depend on the incident energy T_0, its value is $A \sim -0.8$.

The peaking approximation is well established only for the calculation of f_0 as the $d \to pn$ vertex in the case of $\ell = 0$ has its maximum at q=0 and it decreases drastically with the increase of q. As the momentum spectra of the $d \to pn$ vertex in the $\ell = 2$ is found in the region of large q values, the peaking approximation for the calculations of f_2 is rather rough and we have to calculate it with more accurate methods. This is very important as the contribution of the $\ell = 2$ wave, which is quite small for the differential cross sections [10], is comparable to the s-wave contribution see formulae (2).

At first we have converted the nonrelativistic forms of nucleon propagators [5] to relativistic forms. We have assumed for the $pp \to d\pi^+$ amplitude $F_M^{m_1 m_2}(T_0, q_z)$ (m_1, m_2, M are the spin projection quantum numbers of nucleons and deuteron, respectively) a Breit-Wigner form in the resonance region with resonance energy $T_R \approx 620$ MeV:

$$F_M^{m_1 m_2}(T_0, q_z) = F_M^{m_1 m_2}(T_0)\, \frac{T_R - T_0 - i\Gamma/2}{T_R - T(T_0, q_z) - i\Gamma/2}, \qquad (4)$$

$T(T_0, q_z)$ is the relativistically calculated energy of the incident proton in the rest system of proton in the deuteron. Taking into account the experimental total $pp \to d\pi^+$ cross section [12] the width of the resonance has been chosen $\Gamma \approx 300$ MeV.

The dependence of the $N \to N\pi$ vertex on the square of the pion four-momenta k^2 is described by the Ferrari-Selleri factor[18]

$$F(k^2) = [1 + (\mu^2 - k^2)/60\mu^2]^{-1} \quad , \tag{5}$$

where μ is the pion mass.

In order to separate the dependence on q and $\cos\theta = q_z/q_z$ we have expanded the $A(T_0, q_z)$ factor into Legendre series, where $A(T_0, q_z)$ is the product of functions depending on q_z (it contains (4), (5) and other factors defined in Appendix):

$$A(T_0, q_z) = \sum_{\ell=0}^{\infty} A_\ell(T_0, q_z) P_\ell(\cos\theta) \tag{6}$$

The use of expansion (6) in calculations of f_2 will not change formula (2) since the application of (6) does not influence the spin structures. For f_2 we have got the following sum of two-fold integrals.

$$f_2 = \sum_{L\ell\ell_1} B(L\ell\ell_1) \int_0^\infty dr\, e^{-\gamma r} \int_0^\infty dq\, q^2 \phi_2(q)\, j_L(qr)\, A_\ell(T_0, q) \times \\ \times [(1+\gamma r)\, j_{\ell_1}(\tilde{p}r) + (-1)^L\, i^{\ell_1 + \ell + 1}\, \tilde{p}r\, j_L(\tilde{p}r)\, S(q)], \tag{7}$$

where $\phi_2(q)$ is the d-wave component of the deuteron wave function in momentum space. $S(q)$ and $B(L\ell\ell_1)$ are defined in A3 and A13, respectively. The second term in (7) gives a 5-10% contribution to f_2.

TENSOR POLARIZATION IN $pd \to pd$ AND $pp \to d\pi^+$ REACTIONS

The above formalism can be applied equally well for the $pd \to pd$ scattering and for the $pp \to d\pi^+$ reaction as they are different in the triangle model only in the amplitude of the resonance process. Changing the amplitude $F^{pp \to d\pi}$ in fig. 1a into the $F^{\pi N \to N\pi}$ amplitude we get the fig. 1b, the triangle diagram of Yao[14] on the $pp \to d\pi^+$ reaction.

For the deuteron wave function we have used the Reid soft core and Reid hard core wave functions[15]. The results are plotted in fig. 2. The calculated tensor polarization A for the pd back scattering is quite small in absolute value throughout the 400-1000 MeV range which is in agreement with experiment[1]. Authors of ref.[1] did not give the value of A at T=600 MeV as they have no relative normalization between the measure-

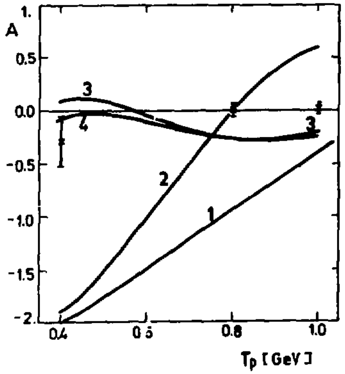

Fig.2. The dependence of tensor polarization A in the pd back scattering on the incident proton energies T_p. Experimental values are from [1]. Theoretical curves are calculated by the single pole model 1 by the isobar model of Kerman and Kisslinger 2, by the triangle model with Reid soft core wave function 3, by the same model with hard core wave function 4.

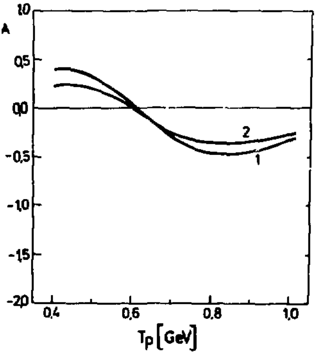

Fig.3. The dependence of tensor polarization A in the $pp \to d\pi^+$ reaction on the incident proton energies at backward pion angle within the triangle model. Curves are calculated by Reid soft core 1 and Reid hard core 2 deuteron wave functions.

ments with full and empty targets. But both results are in agreement with the value A=0. Taking into account the agreement of the calculated and experimental vector polarization in the pd back scattering in the isobar region (500-700 MeV) [13] we can state that the triangular model has a dominant role in the isobar region.

In the case of the $pp \to d\pi^+$ reaction the width of the resonance is ~120 MeV according to the experimental width of the Δ_{33} isobar. The results with Reid soft core and Reid hard core wave functions are plotted in fig.3. It can be seen that the predicted tensor polarizations are small, too, especially in the isobar region. There are no measurements in this region. Our result is in strong contradiction with the prediction of Niskanen's isobar model [17], where the tensor polarization in the $pp \to d\pi^+$ reaction at $\theta_{C.M.}$=180° has its minimal

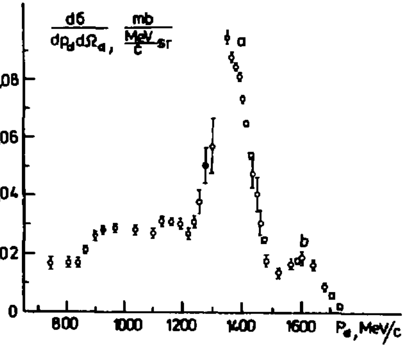

Fig.4. Forward inclusive deuteron spectrum in the $^{12}C(p,Xd)$ reaction at intermediate proton incident energies[20]. The first peak (a) can be related to the $pN \to d\pi$, the second (b) to the $p\langle NN\rangle \to Nd$ quasifree processes.

value, A = −2, independently of the incident proton energy. From the view point of different models the measurement of A is highly desirable.

TENSOR POLARIZATION IN THE A(p, Xd) REACTION

The large number of forward deuterons at intermediate incident proton energies[18] can be explained in terms of the $pp \to d\pi^+$ quasi-free reaction and of the quasi-elastic $pd \to pd$ scattering, see the review[19]. An illustrate example of the forward inclusive deuteron spectra can be seen in fig.4. The diagrams of the quasi-free processes are plotted in fig. 5. These reactions have a strong peripherical character due to the strong absorption of deuteron in the final state.

The quasi-free pd processes are mainly due to the $p\langle pn, t=0\rangle \to pd$ (t is the isospin), elementary processes, where the $\langle pn\rangle$ cluster has the quantum numbers of the deuteron[11]. The cross section ratios are $\sigma(p\langle pn, t=0\rangle \to pd)/\sigma(p\langle pn, t=1\rangle \to pd) = 9$ $\sigma(p\langle pn, t=0\rangle \to pd)/\sigma(p\langle nn\rangle \to nd) = 4.5$ assuming identical space part of the $t=0$ and $t=1$ wave functions[21]. In ref.[10] the dominance of the $\ell=0$ contribution is suggested due to the sensitivity of the amplitude on the short range part of the $\langle pn\rangle$ relative wave function.

In the case of unpolarized target the nucleon and the $\langle pn\rangle$ cluster inside the nucleus are also unpolarized. Using the triangular model for the elementary processes inside the nucleus[10], we can predict the small absolute values of tensor polarization of forward deuterons in the isobar region, independently of the deuteron origin from the $pp \to d\pi^+$ or from the $p\langle pn\rangle \to pd$ elementary processes. The Fermi motion of the nucleon or the $\langle pn\rangle$ cluster inside the nucleus give the effect

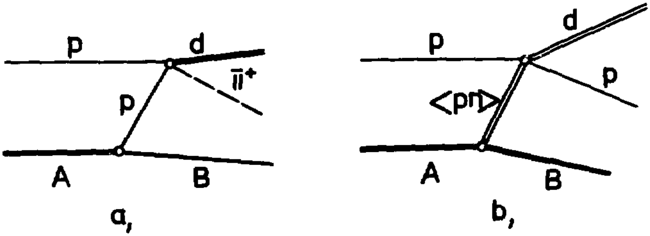

Fig.5. The diagrams of the $A(p,\pi d)B$ (a) and $A(p,Nd)B$ (b) quasi-free processes.

of resonance width increase in the elementary process. The motion of the resonance maximum is possible, too. These effects do not influence our conclusion like for example the increase of the width from 120 to 300 MeV does not change the character of the results (see figs.2,3). The effect of the spin dependent forces may disturbe the polarization of deuterons along their path inside the nucleus, but at these high energies of deuterons (600 MeV from the $pd \to pd$ scattering) the imaginary part of the optical potential is dominant.

The experimental study of tensor polarization of forward deuterons from the reaction $A(p,Xd)B$ at intermediate proton energies in different mass regions is highly desirable. If the measurements give support to the model suggested in /10/, the $A(p,pd)B$ experiments see /21,22/ may give an excellent tool for the study of the pairing effects on the surface of the nuclei.

APPENDIX

The $M^{m_f M_f}_{m_i M_i}$ amplitude of the $pd \to pd$ scattering, where m_a and M_a are the spin projection quantum numbers of the protons and deuterons, according to the diagram 1a, can be written in the following form:

$$M^{m_f M_f}_{m_i M_i} = C \int \frac{D^{M_f}_{\nu_1 \nu_2} \Gamma^{\nu_1}_{m_i} F^{\nu_2 m_f}_{M_i} d\vec{p}\, dE}{(p^2 - m^2 - i\eta)(n^2 - m^2 - i\eta)(k^2 - \mu^2 - i\eta)}, \qquad (A1)$$

where $C = -i\sqrt{2}\,\pi^{-5/2} m\mu$, μ and m are the masses of pion and nucleon, p, n, and k are the four momenta of the virtual proton, neutron and pion, $\vec{p}$ and E are the momentum and energy of the virtual neutron, $D^{M_f}_{\nu_1 \nu_2}$, $\Gamma^{\nu_1}_{m_i}$ are the deuteron vertex and the $p \to n\pi^+$ vertex, respectively. Putting the virtual neutron on the mass shell the integration with the help of the pole in $E=(p^2+n^2)^{1/2}$ can be carried out:

$$M^{m_f M_f}_{m_i M_i} = C' \int \frac{D^{M_f}_{\nu_1\nu_2}\Gamma^{\nu_1}_{m_i} F^{\nu_2 m_f}_{M_i} R(\vec{p})\, d\vec{p}}{(p^2+\chi^2)[(\vec{p}-\vec{\tilde{p}})^2+\gamma^2]}, \tag{A2}$$

where

$$C' = -2^{-1/2}\pi^{-3/2}\,\mu(1+T_i/m)^{-1}, \tag{A2a}$$

$$R(\vec{p}) = \frac{(p^2+\chi^2)[(\vec{p}-\vec{\tilde{p}})^2+\gamma^2]}{[-M_d^2/2+M_d(m^2+p^2)^{1/2}][2m^2-2(m^2+p^2)^{1/2}(m+T_i)+2\vec{p}\vec{p}_i-\mu^2]}, \tag{A2b}$$

$$\vec{\tilde{p}} = \vec{p}_i(1+T_i/m)^{-1}, \tag{A2c}$$

$$\gamma^2 = T_i^2(1+T_i/m)^{-2}+\mu^2(1+T_i/m)^{-1}. \tag{A2d}$$

For the evaluation of the invariant integral we have chosen the rest system of the final deuteron. T_i and $\vec{p}_i$ are the kinetic energy and momentum of the initial proton, in this system, respectively. M_d is the deuteron mass $\chi^2 = m\epsilon_d$, where ϵ_d is the binding energy of the deuteron. With the factor $R(\vec{p})$ we can reserve the relativistic nature of the propagators.

The $\Gamma^{\nu_1}_{m_i}$ vertex can be written in the following form:

$$\Gamma^{\nu_1}_{m_i} = \sqrt{2}\,GF(k^2)\bar{u}(p_i)\gamma_5 u(p) = \sqrt{2}\,GF(k^2)(E_i+m)^{1/2}(E+m)^{-1/2} \times \tag{A3}$$
$$\times (2\mu)^{-1/2}(2m)^{-1}\phi^+_{1/2\,\nu_1}\vec{\sigma}[(\vec{p}-\vec{\tilde{p}})+S(p)\vec{p}_i]\phi_{1/2\,m_i},$$

where

$$S(p) = 1+(1+T_i/m)(m+E)(m+E_i)^{-1} \tag{A3a}$$

with $G^2/4\pi = 14.8$, $\vec{\sigma}$ is the Pauli Matrix, $\phi_{1/2\,m}$ is a two component spinor.

Assuming the energy function described by formula (4) for the $F^{\nu_2 m_f}_{M_i}$ amplitude and collecting the factors from $\Gamma^{\nu_1}_{m_i}$ and $F^{\nu_2 m_f}_{M_i}$ depending on p_z we have the following expression for $A(T_0, p_z)$ in (6):

$$A(T_0,p_z)=R(\vec{p})\,\frac{(2m)^{1/2}}{F(k_1^2)}\cdot\frac{F(k^2)}{(E+m)^{1/2}}\cdot\frac{T_R-T_0-i\Gamma/2}{T_R-T(T_0,p_z)-i\Gamma/2}, \qquad (A4)$$

where $k_1^2=k^2(\vec{p}=0)$. The definition of the expansion coefficients in formulae (6) is as follows

$$A_\ell(T_0,p)=\frac{2}{2\ell+1}\int_{-1}^{1}A(T_0,p_z)P_\ell(x)\,dx \qquad x=\cos\theta. \qquad (A5)$$

Taking into account the following integral representations:

$$(p^2+\gamma^2)^{-1}=(4\pi)^{-1}\int e^{-\gamma r}/r\cdot e^{i\vec{p}\vec{r}}\,d\vec{r} \qquad (A6a)$$

and

$$\frac{\vec{p}}{p^2+\gamma^2}=\frac{-i}{4\pi}\int\frac{\vec{r}}{r}\left(\frac{\gamma}{r}+\frac{1}{r^2}\right)e^{-\gamma r}\,e^{i\vec{p}\vec{r}}\,d\vec{r} \qquad (A6b)$$

and using the formulas (A3) and (6) the (A2) amplitude obtains the following form

$$M^{m_f M_f}_{m_i M_i}=C''\phi^{+}_{1/2\,\nu_1}\vec{\sigma}\phi_{1/2\,m_1}F^{\nu_2 m_f}_{M_i}(T_0)\int\frac{D^{M_f}_{\nu_1\nu_2}}{p^2+\chi^2}\sum_\ell A_\ell(T_0,p)\,P_\ell(\cos\theta)\times \qquad (A7)$$

$$\times\int\left[\frac{\vec{r}}{r}\left(\frac{\gamma}{r}+\frac{1}{r^2}\right)e^{-\gamma r}-i\vec{p}_1\,S(p)\,\frac{e^{-\gamma r}}{r}\right]e^{i(\vec{p}-\vec{\tilde{p}})\vec{r}}\,d\vec{r}\,d\vec{p},$$

where

$$C''=\frac{-i}{4\pi}\sqrt{2}\,G\left(\frac{E_1+m}{2\mu}\right)^{1/2}\frac{1}{2m}\,\frac{F(k_1^2)}{(2m)^{1/2}}C'. \qquad (A7a)$$

We express the $D^{M_f}_{\nu_1\nu_2}$ vertex in terms of the $\phi_\ell(p)$ deuteron wave function components as follows

$$\frac{D^{M_f}_{\nu_1\nu_2}}{p^2+\chi^2}=\sum_{\ell_0}\sum_{\substack{m_s\\ m}}(1/2\,\nu_1\,1/2\,\nu_2\,|\,1m_s)(1m_s\,\ell_0 m|\,1M_f)\,\phi_{\ell_0}(p)Y_{\ell_0 m}(\hat{p})\,\phi_{1/2\,\nu_2}\phi_{1/2\,\nu_1}.$$

Expanding the $e^{i(\vec{p}-\vec{\tilde{p}})\vec{r}}$ factor in (A7) by spherical harmonics and using the expression, for the spherical harmonics products we have

$$M^{m_f M_f}_{m_i M_i} = G E_i^{-1} F(k_1^2)\cdot(2m)^{-1/2} [3\mu(E_i+m)]^{1/2} \times$$
$$\times \sum_{\ell_0 \ell L \ell_1} \frac{[(2\ell_f+1)(2\ell_0+1)]^{1/2}}{2L+1} (\ell_1 010|L0) \times \tag{A9}$$
$$\times (\ell_0 0 \ell 0|L0) H^{\ell_0}_{\ell L \ell_1} \sum_{\substack{\nu_1 \nu_2 m \\ m_s m_1 m_2}} (\tfrac{1}{2}\nu_1 \tfrac{1}{2}\nu_2|1m_s)(1m_s \ell_0 m|1M_f) \times$$
$$\times (\tfrac{1}{2}\nu_1 1m_2|\tfrac{1}{2}m_1)(\ell_1 m_1 1m_2|\ell m) Y_{\ell_1 m_1}(\hat{p}_i) F^{\nu_2 m_f}_{M_i}(T_0) \phi_{\frac{1}{2}\nu_1} \phi_{\frac{1}{2}\nu_2}$$

with

$$H^{\ell_0}_{\ell L \ell_1} = \int_0^\infty dr\, e^{-\gamma r} \int_0^\infty dq\, q^2 \phi_{\ell_0}(q) j_L(qr) A_\ell(T_0, q) \times$$
$$\times [(1+\gamma r) j_{\ell_1}(\tilde{p} r) + (-1)^L i^{\ell_1+\ell+1} j_L(\tilde{p} r) \tilde{p} r S(q)]. \tag{A10}$$

In the case of the $\ell_0 = 0$ amplitude $\phi_{\ell_0}(q)$ decreases drastically with the increase of q. In the region of $q \approx 0$ $A_\ell(T_0, q) \sim \delta_{\ell 0}$ and by taking into account that $S(0) \ll 1$ we get the formula of ref. /5/.

Using the formulae (A9) for the amplitude $M^{m_f M_f}_{m_i M_i}$ we have calculated the tensor polarization A defined in (1), where

$$N_+ = \sum_{m_i M_i m_f} |M^{m_f 1}_{m_i M_i}|^2 , \tag{A11}$$

and for N_- and N_0 we have the same formulae with the change of index $M_f = 1$ to -1 and 0 respectively. We have got the formulae (2), where f_2 has the following definition:

$$f_2 = \sum_{L \ell \ell_1} B(L \ell \ell_1) H^2_{L \ell \ell_1} \tag{A12}$$

with

$$B(L\ell\ell_1) = -(5/\pi)^{1/2} (-1)^{\ell_1} i^{\ell_1+L+1} (\ell_1 010|L0)(20\ell 0|L0) \times$$
$$\times \sum_\mu (-1)^\mu (1-\mu 2\mu|10)(2\mu \ell 0|L\mu)(\ell_1 01\mu|2\mu). \tag{A13}$$

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Received by Publishing Department
on August 14 1980.