объединенный
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исследований
дубна

E2-80-547

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STATISTICAL MECHANICS OF THE INTERACTING YANG-MILLS INSTANTON GAS

Submitted to "Nuclear Physics B"

1980

1. Introduction

A broad program of semiclassical calculations in QCD has been pioneered by Callan, Dashen and Gross (CDG) /1a,b,c/, based on the instanton solutions /2/ of Euclidean Yang-Mills theory. But role and relevance of instantons are highly controversial, in QCD as well as in the two-dimensional CP^{N-1} model /3/ (often considered as a toy model for the former). This concerns both the question of which field configurations are the important ones within the functional integral, as well as the relation of the semiclassical approximation as such to other approximation schemes enjoying good reputation as the $1/N$ expansion.

The apparent conflict /4/ between instantons and $1/N$ seems to be overcome: they do not necessarily exclude each other /5/, and instanton effects are not necessarily $O(e^{-N})$ /6/. The impressive beauty of the exact, selfdual solutions with multiple topological charge /7/ and the progress in doing the semiclassical integration around them /8,9,10/ have led to the hope, that "dense gases" described by the corresponding multiinstanton partition function could be treated some day to give the "true vacuum" amplitude and to overcome the notorious infrared troubles of the "dilute gas approximation" /1a/ (DGA). But the latter is so far the only way in QCD to deal with field configurations of mixed duality, being approximate stationary points, so far ignored in the "dense gas" partition function. Only in the CP^1 model there were recently attempts /11/ to include multiinstanton-multiantiinstanton configurations.

Almost all calculations of physical instanton effects have been done up to now within the DGA. In essence, they show the effects of vacuum field fluctuations being near to superpositions of individual single instantons and antiinstantons, taking their classical and quantum interactions into account or not. But where does this kind of fluctuations show up physically with noticeable effects, calculable free of ambiguities? Corrections to high momentum transfer processes accessible to perturbative QCD have been found to be small by large inverse powers in Q^2 /12/. Qualitative effects of localised topological charge responsible for chiral symmetry breaking /13/ and a solution of the $U(1)$ problem /14/ must be expected. However, an essential source of dissatisfaction with all those DGA calculations, where no external size cut-off is provided by a large momentum, has been the inavoidable ad hoc infrared cut-off ρ_c introduced by hand, and the arbitrariness of the supplemented diluteness criteria used to fix ρ_c. It seems to be worth while to look for a more justified diluteness criterion for the DGA, or better to find within the DGA a natural cut-off for the instanton density, in order to describe (at least this part of) the vacuum fluctuations selfconsistently.

As long as we are considering the four-dimensional dilute gas of instantons and antiinstantons being essentially dipole field configurations, one is led to some kind of magnetostatics (in four "spatial" dimensions) of an analog magnetisable medium. This point of view has been invented by CDG /1b/ and has led them to conjecture two bulk effects of the instanton medium: a first order phase transition between the "bag" /15/ and "vacuum" phases /1b/ (expected to occur at some critical strength of the color electric field) and the renormalization of the coupling constant /1c/ subsumming nonperturbative fluctuations below a given length scale. Both phenomena are related to the response of the instanton medium to external fields, i.e., the permeability of the instanton gas.

How dense could this dilute gas be? The usual criterion, involving the average occupied space-time fraction ($f < 1$), guarantees neither the validity of the semiclassical approximation for the quantum weight of each many-instanton configuration nor that the dipole interaction is really the dominating one. In fact, the partition function of this gas should be restricted by constraints of the hard-core type, $\rho_1^2 \rho_2^2 / |z_1 - z_2|^4 < 1/a'$. Having realized this, one has to study the statistical mechanics of a gas with short range hard-core and long range dipole interactions instead of resorting to more intuitive considerations of a "continuous" instanton medium.

An attempt in this direction has been made by Jevicki /16/, who has studied the interplay of the two interactions. However, not much of immediately useful results has been presented there. We have attacked the microscopic treatment of the dipole interaction in Ref. /17/ , aiming at a numerical test of the two above-mentioned conjecturable instanton mechanisms, but we have accounted for the hard-core repulsion only in a very poor fashion. In the present paper we will do better and extract "phenomenologically" relevant quantities from the hard-core dipole gas. The first step will be disentangling the long range dipole interaction by some variant of the random field functional trick. The remaining hard-core problem with arbitrary external field will be treated using the generating functional of the hard-core gas, which leads to a simple, analytically easy collective suppression of large instantons, that can be specified further by a selfconsistency condition. All this provides a new scheme for DGA instanton calculations, which substitutes the uncomfortable ρ_c cut-off and allows better to control the validity of the dilute gas picture as such.

In Section 2 we write down the Euclidean path integral in the form of the grand partition function of the interacting instanton-antiinstanton gas, fix our notation, and decouple the dipole interaction by the functional trick. In Section 3 we explain how to obtain from the hard-core instanton gas the co-operative suppression of large instantons, and define the quantities describing the state of this gas. Section 4 contains the results of the functional averaging, that defines the final partition function, as far as the permeability is concerned. In Section 5 the renormalization of the coupling constant is considered. The interpolation between the weak and strong coupling behaviour of the Gell-Mann-Low β-function is studied under the aspect of the influence of instanton interactions, the applicability of the dilute gas picture and the relation between the Λ parameters of continuum and lattice QCD /10,18/, respectively. By comparison with recent lattice calculations /19/ we draw our conclusions.

2. The Partition Function of the Interacting Instanton Gas

We are going to consider the vacuum-to-vacuum transition amplitude /1a/ $Z = \langle vac|e^{-HT}|vac\rangle$ for the pure SU(N)- Yang-Mills theory. Calculating the path integral within the quasi-classical approximation around superpositions

$$A_\mu^a(x) = \sum_{i=1}^{n_+} \overset{+}{A}{}_\mu^a(x-\overset{+}{z}_i,\overset{+}{\rho}_i,\overset{+}{R}_i) + \sum_{j=1}^{n_-} \bar{A}_\mu^a(x-\bar{z}_j,\bar{\rho}_j,\bar{R}_j) \qquad (2.1)$$

of individual instantons ($\varepsilon = +$) and antiinstantons ($\varepsilon = -$)

$$\overset{\varepsilon}{A}{}_\mu^a(x,\rho,R) = \overset{\varepsilon}{D}{}_{\mu\nu}^a \frac{2x_\nu}{x^2(x^2+\rho^2)}, \qquad (2.2)$$

where D denotes the dipole moment of an (anti)instanton

$$\overset{\varepsilon}{D}{}_{\mu\nu}^a = \frac{\rho^2}{g} R^a{}_\alpha \overset{\varepsilon}{\eta}{}_{\mu\nu}^\alpha \qquad (2.3)$$

(we use standard notations for location, size and group orientation /1a/ ; $\overset{+}{\eta}{}^\alpha_{\mu\nu} = \bar{\eta}_{\alpha\mu\nu}$, $\bar{\eta}^\alpha_{\mu\nu} = \eta_{\alpha\mu\nu}$ in terms of 't Hooft's symbols /20/), one is led to write down Z as the grand partition function of an interacting instanton-antiinstanton gas

$$Z(V) = \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \sum_{\varepsilon_i} \int \frac{d\rho_i}{\rho_i} \int_V d^4z_i\, d_0(\rho_i) \int dR_i\, e^{2\pi^2 D(i)^a_{\mu\nu} \tilde{H}^a_{\mu\nu}} e^{-V_{int}}. \quad (2.4)$$

Here $d_0(\rho)$ is the single-instanton amplitude calculated by 't Hooft /20/ and others (see, e.g., Ref. /21/ for arbitrary N)

$$d_0(\rho) = C_N^R \frac{1}{\rho^4} x_0^{2N} e^{-x(\rho)} \qquad (2.5)$$

with

$$x_0 = \frac{8\pi^2}{g_0^2}, \qquad x(\rho) = \frac{8\pi^2}{g^2(\rho)} = \frac{11N}{3} \log\frac{1}{\rho\Lambda},$$

$$C_N^R = \frac{4.6}{\pi^2} \frac{1}{(N-1)!(N-2)!} \left(\Lambda^{PV}/\Lambda^R\right)^{\frac{11N}{3}} e^{-1.68N} \qquad (2.6)$$

(R refers to a particular regularization scheme, PV to Pauli-Villars).

In the partition function (2.4) the interaction of the (anti) instanton with an external field $\tilde{H}^a_{\mu\nu} = F^{ext\,a}_{\mu\nu}$ is included. The interaction term V_{int} among the (anti)instantons has to collect all nonfactorizing contributions, arising from the classical action and from quantum effects, i.e., from the multiscattering expansion of the fluctuation determinants as well as from the expansion of the collective coordinate Jacobian /22/ . The leading part of V_{int} , behaving like a sum of two-body interactions of $\mathcal{O}(\rho_i^2\rho_j^2/|z_i-z_j|^4)$, involves the classical instanton-antiinstanton interaction of the dipole-dipole form

$$-V_{int} \propto -S_{int} = 8\pi^2 \sum_{ij} \overset{+}{D}(i)^a_{\mu\nu}\, \overset{-}{D}(j)^a_{\mu\nu'}\left(\delta_{\nu\nu'} - 4\hat{\Delta}_\nu\hat{\Delta}_{\nu'}\right)/\Delta^4 ,$$
$$\Delta = \overset{+}{z}_i - \overset{-}{z}_j \quad , \qquad \Delta_\nu = |\Delta|\,\hat{\Delta}_\nu , \tag{2.7}$$

Our task, as it has been in Ref. /17/ , will be the treatment of the partition function with pair interactions of just this form.

It has been pointed out by Levine and Yaffe /22/ , that there must be furthermore constraints of hard-core type, restricting distances between instantons to be $\Delta^4 = |z_1-z_2|^4 > a'\rho_1^2\rho_2^2$. They will ensure, that in the limit $g \to 0$ the superpositions (2.1) tend towards really stationary points of the action. The variation of a' with the coupling constant will become clear later. For a given instanton-antiinstanton pair a' determines the ratio between S_{int} and the action $S_0 = \times(\rho_1) + \times(\rho_2)$ of the noninteracting pair. Moreover, it allows to estimate to what extent other interactions, falling with higher powers in ρ/Δ , contribute. We add therefore to V_{int} a sum of two-body potentials

$$\mathcal{U}(i,j) = \begin{cases} 0 \\ \infty \end{cases} , \quad |z_i - z_j|^4 \gtrless a'\rho_i^2\rho_j^2 \tag{2.8}$$

representing a strong repulsion at small distances.

In most of all instanton calculations the zero-mode factor $\times_0^{2N}$ in the instanton amplitude (2.5) used to be expressed through the running coupling constant as $\times(\rho)^{2N}$ without compelling reasons. For instance, inclusion of the two-loop running coupling in the exponentiated action, $exp(-\times_{2\,loop}(\rho))$, would almost cancel the assumed ρ dependence of the zero-mode factor $\times_{1\,loop}(\rho)^{2N}$, taken in the one-loop approximation. We will take in our explicit integrals over the instanton density the zero-mode factor $\times_0^{2N}$ fixed, to be determined afterwards selfconsistently.

We prefer to linearize the dipole-dipole interaction (2.7) as in Ref. /17/ by the following functional representation

$$e^{-S_{int}} = e^{-\frac{1}{2}\sum_{ij}\int d^4x\, \overset{+}{A}{}^a_{i\mu}(x-\overset{+}{z}_i)\,\Box_x\,\bar{A}^a_{j\mu}(x-\bar{z}_j)}$$
$$= (\det\hat{\Box})^{-\frac{1}{2}}\int \mathcal{D}\ell\; e^{-\frac{1}{2}\int\hat{\ell}\,\hat{\Box}^{-1}\hat{\ell}}\; e^{-\frac{i}{\sqrt{2}}\int \hat{A}\hat{\ell}}\,, \qquad (2.9)$$

where we have used the matrix notation

$$\hat{A}^a_\mu(x) = \begin{pmatrix} \sum_i \overset{+}{A}{}^a_{i\mu}(x-\overset{+}{z}_i) \\ \sum_j \bar{A}^a_{j\mu}(x-\bar{z}_j) \end{pmatrix}, \quad \hat{\ell}^a_\mu(x) = \begin{pmatrix} \overset{+}{\ell}{}^a_\mu(x) \\ \bar{\ell}^a_\mu(x) \end{pmatrix}$$

and

$$\hat{\Box} = \begin{pmatrix} 0 & \Box_x \\ \Box_x & 0 \end{pmatrix}\delta(x-x')\,\delta^{aa'}\delta_{\mu\mu'}\,.$$

Then the partition function will be obtained as the integral

$$Z(V) = \left(\det\hat{\Box}\right)^{-\frac{1}{2}}\int\mathcal{D}\ell\; e^{-\frac{1}{2}\int\hat{\ell}\,\hat{\Box}^{-1}\hat{\ell}}\; Z_{\ell_c}(\phi,V)\,, \qquad (2.10)$$

where $Z_{\ell_c}(\phi,V)$ is the partition functional for a pure hard-core instanton-antiinstanton gas, to be determined in Section 3, and where the test function ϕ is set equal to

$$\phi(x,\rho;\epsilon) = \int dR\; e^{2\pi^2\overset{\epsilon}{D}\tilde{H}}\; e^{-\frac{i}{\sqrt{2}}\int\overset{\epsilon}{A}(y-x)\,\overset{\epsilon}{\ell}(y)\,d^4y}. \qquad (2.11)$$

We postpone the evaluation of (2.10) to Section 4 and add here what we want to extract from Z .

The magnetization tensor, i.e.,the average dipole density

$$\tilde{M}^a_{\mu\nu} = \langle m^a_{\mu\nu}\rangle,$$
$$m^a_{\mu\nu}(x) = \sum_{i=1}^{n_+}\overset{+}{D}(i)^a_{\mu\nu}\,\delta(x-\overset{+}{z}_i) + \sum_{j=1}^{n_-}\bar{D}(j)^a_{\mu\nu}\,\delta(x-\bar{z}_j) \qquad (2.12)$$

will be obtained from the partition function

$$4\pi^2\tilde{M}^a_{\mu\nu} = \frac{\partial}{\partial\tilde{H}^a_{\mu\nu}}\,\frac{1}{V}\,\log Z(V) \sim 4\pi^2\chi\,\tilde{H}^a_{\mu\nu}\,, \qquad (2.13)$$

χ is called the susceptibility. The magnetization is related to the macroscopic, averaged field [23]

$$\tilde{B}_{\mu\nu} = \langle\tilde{F}^a_{\mu\nu}\rangle = \tilde{H}^a_{\mu\nu} + 2\pi^2\tilde{M}^a_{\mu\nu} \sim \mu\,\tilde{H}^a_{\mu\nu} \qquad (2.14)$$

and there is defined a permeability $\mu = 1 + 2\pi^2\chi$. In particular, we are interested to find the permeability of the interacting instanton gas.

3. The Hard-Core Instanton Gas

In this section we have to find the partition functional $Z_{hc}(\phi, V)$ in (2.10) of the hard-core instanton-antiinstanton gas under the influence of an arbitrary external field, but without any other interactions. Jevicki /16/ has dealt with the hard-core gas, but of instantons having fixed size. We will consider here constraints of the type $|z_i - z_j|^4 > a^4 \varrho_i^2 \varrho_j^2$ involving both locations *and* sizes. The bonus will be to obtain a cooperative suppression of large instantons.

In the following we will denote the single-instanton amplitude $d_0(\varrho_i)$ for short as $z(i)$ with $i = (z_i, \varrho_i, \varepsilon_i)$ as its arguments, and $\int di = \sum_{\varepsilon_i} \int_V d^4 z_i \int d\varrho_i/\varrho_i$. With $\phi(i)$ as test function we define a hard-core partition functional

$$Z_{hc}(\phi, V) = \sum_n \frac{1}{n!} \int \prod_{i=1}^{n} di \; z(i)\, \phi(i)\, e^{-\sum_{i<j} U(i,j)} , \qquad (3.1)$$

where $\mathcal{U}(i,j)$ is defined in (2.8). Z_{hc} fulfills obviously the following relations

$$\begin{aligned} \delta/\delta\phi(1)\; Z_{hc}(\phi, V) &= z(1)\; Z_{hc}(\phi(\cdot|1), V) , \\ &\vdots \\ \delta/\delta\phi(1) \ldots \delta/\delta\phi(n)\, Z_{hc}(\phi, V) &= z(1) \ldots z(n)\, c(1, \ldots, n)\; Z_{hc}(\phi(\cdot|1 \ldots n) V) \end{aligned} \qquad (3.2)$$

with

$$\begin{aligned} c(1 \ldots n) &= e^{-\sum_{i<j}^{n} U(i,j)} , \\ \phi(\ell|1 \ldots n) &= \phi(\ell)\, e^{-\sum_{j=1}^{n} U(\ell, j)} . \end{aligned} \qquad (3.3)$$

Derivatives at $\phi = 1$ are just the many-instanton densities, apart from the normalisation factor $Z_{hc}(\phi = 1, V)$, e.g.,

$$n(1) = \frac{1}{Z_{hc}(1,V)} \frac{\delta}{\delta\phi(1)} Z_{hc}(\phi, V)\Big|_{\phi=1} = z(1)\, \frac{Z_{hc}(\phi(\cdot|1), V)\big|_{\phi=1}}{Z_{hc}(1, V)} , \qquad (3.4)$$

where

$$\phi(\ell|1)\Big|_{\phi=1} = \begin{cases} 0 \\ 1 \end{cases}, \qquad |z_\ell - z_1|^4 \lessgtr a' \varrho_\ell^2 \varrho_1^2$$

cuts out an excluded volume $v_1 = \frac{\pi^2}{2} a' \varrho_\ell^2 \varrho_1^2 / 2$ around the considered instanton 1. We assume now a behaviour

$$Z_{\ell_c}(1, V) \sim \exp PV \tag{3.5}$$

with the hard-core instanton gas pressure P , and anticipate, that ϱ integrals will be cut off such that we may define a r.m.s. radius $\bar{\varrho}$. Then

$$Z_{\ell_c}(\phi(\cdot|1), V)\Big|_{\phi=1} \simeq Z_{\ell_c}(\phi=1, V - v_1 \tag{3.6}$$

with

$$v_1 = \frac{\pi^2}{2} a' \bar{\varrho}^2 \varrho_1^2 . \tag{3.7}$$

Thus we obtain a cooperative instanton suppression as follows

$$n(1) = z(1)\, e^{-Pv_1} = d_0(\varrho_1)\, e^{-A(P)\varrho_1^2} \tag{3.8}$$

governed by the pressure P, the average radius $\bar{\varrho}$ and the parameter a' . These parameters will turn out intimately related. Similarly the many-instanton densities will be

$$n(1,2) = z(1)\, z(2)\; C(1,2)\; e^{-P(v_1+v_2)} , \tag{3.9}$$

etc., i.e.,they factorize into single-instanton densities or vanish, if any two of the (anti)instantons would approach each other too much. Employing the general relation between multiple densities and correlation functions we can write

$$Z_{\ell_c}(\phi, V) = Z_{\ell_c}(\phi=1, V) \sum_{n=0}^{\infty} \frac{1}{n!} \int n(1,\dots,n) \prod_{i=1}^{n} (\phi(i) - 1)\, di , \tag{3.10a}$$

$$= Z_{\ell_c}(\phi=1, V) \exp \sum_{n=1}^{\infty} \frac{1}{n!} \int \tilde{c}(1,\dots,n) \prod_{i=1}^{n} (\phi(i) - 1)\, di , \tag{3.10b}$$

where

$$\tilde{c}(1) = n(1) = z(1)\, e^{-Pv_1} ,$$

$$\tilde{c}(1,2) = n(1,2) - n(1)\, n(2) = z(1)\, z(2)\, e^{-P(v_1+v_2)} \left(e^{-U(1,2)} - 1\right) , \tag{3.11}$$

etc. The pressure

$$P = \frac{1}{V} \log Z_{\ell\ell}(\phi = 1, V) \tag{3.12}$$

can be found by scaling down $z(i) \longrightarrow \lambda z(i)$, $0 \leq \lambda \leq 1$, upon which all quantities become dependent on λ : $P(\lambda)$, $\tilde{\varepsilon}(1|\lambda)$, $\tilde{\varepsilon}(1,2|\lambda)$, ... Employing now (3.10b) in order to find the derivative $dZ_{\ell\ell}/d\lambda$, we obtain a differential equation for $P(\lambda)$ that involves the single-instanton density,

$$\frac{dP(\lambda)}{d\lambda} = \sum_{\varepsilon} \int d_0(\rho)\, e^{-A(P(\lambda))\rho^2} \frac{d\rho}{\rho} . \tag{3.13}$$

This equation must be solved for $P = P(\lambda = 1)$ with the condition $P(\lambda = 0) = 0$.

As announced in the preceding section, for all size integrations we adopt the following way to proceed. In the instanton amplitude d_0 we consider $C_N' = C_N x_0^{2N}$ as a constant to be determined afterwards. The r.m.s. radius $\bar{\rho}$ is completely fixed in terms of P and a' as

$$\frac{\pi^2}{2} a' \bar{\rho}^4 = \left(\frac{11N}{6} - 2\right)\frac{1}{P} \tag{3.14}$$

and may be eliminated in (3.8) in favour of $A(P)$. The differential equation (3.13) can then be solved and gives

$$\frac{P}{\Lambda^4} = \left[\frac{11N}{12} C_N' \,\Gamma\!\left(\frac{11N}{6} - 1\right) \frac{\pi^2 a'}{2}\right]^{\frac{12}{11N}} \left[\frac{\pi^2 a'}{2}\left(\frac{11N}{6} - 2\right)\right]^{-1} . \tag{3.15}$$

Calculating the occupied space-time fraction

$$f_0 = \pi^2 \int \frac{d\rho}{\rho} C_N' (\rho\Lambda)^{\frac{11N}{3}} e^{-P_V} \tag{3.16}$$

we obtain, independently of C_N'

$$f_0 = \left(1 - \frac{6}{11N}\right) \frac{2}{a'} \tag{3.17}$$

such that nothing else than a' determines the fraction f_0.

We have already mentioned that a' gives much better control over the validity of the semiclassical approximation and the relative importance of various interactions falling with powers of ρ/Λ not less than fourth. For a given instanton-antiinstanton pair, the condition $|S_{int}| \simeq S_0$, requiring $a' \sim O(10)$, certainly violates the semiclassical approximation, anyhow, and does not allow to separate different powers in ϱ/Λ . A still extreme, but yet tolerable order of magnitude might be $a' \sim O(10^3)$. This reduces reasonable fractions f_0 to much smaller numbers than had been allowed usually. In our numerical study below we shall let a' vary, thus considering more or less dilute gases, each having its typical instanton size.

We calculate the ideal gas susceptibility

$$\chi_0 = \frac{1}{N^2-1} \int \frac{d\varrho}{\varrho} \, C_N' \, (\varrho\Lambda)^{\frac{11N}{3}} e^{-P_V} x(\varrho) \tag{3.18}$$

and find

$$\pi^2 \chi_0 \simeq f_0 \, \frac{x(\bar\varrho)}{N^2-1} \quad . \tag{3.19}$$

For given a' the coupling constant $x(\bar\varrho)$ is formally defined

$$x(\bar\varrho) = \log\left\{ C_N' \, \Gamma\left(\frac{11N}{6}-1\right) \frac{11N}{6} \, \frac{\bar\pi^2 a'}{4} \left(\frac{11N}{6}-2\right)^{-\frac{11N}{6}} \right\} \tag{3.20}$$

which does not help unless we relate the zero-mode factor x_0^{2N} in C_N' to $x(\bar\varrho)$. For definiteness and most naturally, we will identify them, $x_0 \equiv x(\bar\varrho)$, and obtain a' and hence all other quantities characterizing the instanton gas parametrically dependent on a typical coupling constant x_0 (in other words, on a typical size $\bar\varrho$):

$$\frac{1}{a'} = \Gamma\left(\frac{11N}{6}-1\right) \frac{11N}{6} \left(\frac{11N}{6}-2\right)^{-\frac{11N}{6}} \frac{\bar\pi^2}{4} \, C_N \, e^{-x_0} x_0^{2N} , \tag{3.21}$$

$$f_0 = \Gamma\left(\frac{11N}{6}\right) \left(\frac{11N}{6}-2\right)^{-\frac{11N}{6}} \frac{\bar\pi^2}{2} \, C_N \, e^{-x_0} x_0^{2N} , \tag{3.22}$$

$$\pi^2 \chi_0 = \Gamma\left(\frac{11N}{6}\right) \left(\frac{11N}{6}-2\right)^{-\frac{11N}{6}} \frac{1}{N^2-1} \, \frac{\bar\pi^2}{2} \, C_N \, e^{-x_0} x_0^{2N+1} , \tag{3.23}$$

$$\frac{P}{\Lambda^4} = \Gamma\left(\frac{11N}{6}-2\right) \frac{11N}{12} \left(\frac{11N}{6}-2\right)^{2-\frac{11N}{6}} C_N \, e^{-\left(1-\frac{12}{11N}\right) x_0} x_0^{2N} . \tag{3.24}$$

We notice the appearance of the factors $C_N e^{-x_0} x_0^{2N}$ or $C_N e^{-x_0} x_0^{2N+1}$, resembling the defining integrals for f_0 and $\pi^2 \chi_0$, respectively, while the pressure is immediately proportional to the instanton density at $\bar{\varrho}$. We observe also, that the hard-core parameter a' , now in turn expressed through the coupling constant $x_0 = 8\pi^2/g_0^2$, must tend to infinity exponentially with $g_0 \to 0$ ensuring that we single out infinitely dilute configurations in this limit /22/ .

In this section we have dealt so far with the hard-core instanton gas without any external field and seen how it cuts off itself, i.e., we considered its partition functional at $\phi = 1$. Excluding for the time being the dipole interaction, we should here in passing look at the response of this gas to an external field $\tilde{H}$ by taking

$$\phi(x,\varrho;\varepsilon) = \int dR \exp 2\pi^2 \overset{\varepsilon}{D} \tilde{H} \simeq 1 + \pi^2 x(\varrho) \frac{\varrho^4}{N^2-1} \left(\tilde{H}_{-\varepsilon}\right)^2_{\mu\nu} \tag{3.25}$$

($\tilde{H}_{+(-)}$ denotes the selfdual (antiselfdual) part of $\tilde{H}$) into the lowest terms of the correlation expansion in (3.10b)

$$Z_{hc}(\phi,V) = e^{PV} \exp\left\{ V \sum_{\varepsilon} \int \frac{d\varrho}{\varrho} d_0(\varrho) e^{-P v} \pi^2 \frac{\varrho^4}{N^2-1} x(\varrho) \left(H_{-\varepsilon}\right)^2 - \right.$$
$$\left. - \frac{\pi^2}{4} a' V \left[\sum_{\varepsilon}' \int \frac{d\varrho}{\varrho} d_0(\varrho) e^{-P v} \pi^2 \frac{\varrho^6}{N^2-1} x(\varrho) \left(H_{-\varepsilon}\right)^2 \right]^2 + \ldots \right\} . \tag{3.26}$$

For sufficiently small external field, $\tilde{H}^2 \bar{\varrho}^4 \ll \frac{2(N^2-1)}{\pi^2 x_0}$, the correlation terms can be neglected. The only effect of the hard core onto the susceptibility of the instanton gas (without dipole interactions!) is that it is expressed, according to (3.18), in terms of the damped instanton density. For larger external field, reaching the order of magnitude $\tilde{H}^2 \bar{\varrho}^4 \sim \frac{N^2-1}{\pi^2 x_0}$ higher orders in (3.25) as well as in the correlation expansion (3.26) become essential. In the following section we attempt to find the permeability taking the dipole interactions among instantons and antiinstantons into account.

4. Interplay of Dipole and Hard-Core Interaction

In order to perform the functional averaging indicated in (2.10) we apply the formalism of the preceding section, including this time the external field into

$$z(x,\varrho;\varepsilon) = d_0(\varrho) \int dR \exp 2\pi^2 \overset{\varepsilon}{D} \tilde{H} = d(\varrho) \tag{4.1}$$

and the random field h into

$$\phi(x,\varrho;\varepsilon) = \Big\langle \exp\Big(-\frac{i}{\sqrt{2}}\int d^4y\ \overset{\varepsilon}{H}(y-x)\ \overset{\varepsilon}{h}(y)\Big)\Big\rangle . \tag{4.2}$$

Here the average is understood with respect to the "angular" distribution

$$W_\varepsilon(R) = \exp 2\pi^2 D^\varepsilon \tilde{H} \Big/ \int dR' \exp 2\pi^2 D^{\varepsilon\,\prime} \tilde{H} . \tag{4.3}$$

Then the correlation expansion (3.10b) becomes a convenient expansion in powers of h , whereas

$$Z_{h_c}(\phi=1,V) = e^{P(\tilde{H})V} \tag{4.4}$$

depends on the external field, in particular

$$P(\tilde{H}) = P + \pi^2 \varkappa_0 \tilde{H}^2 + \dots \tag{4.5}$$

is seen from (3.26) and (3.18), giving the (ideal gas) susceptibility without dipole interactions, corrected for the hard-core effect. We want to do the functional average (2.10) within Gaussian approximation and expand $\phi(x,\varrho;\varepsilon)-1$ up to second order in h ,

$$\phi(x,\varrho;\varepsilon)-1 = -\frac{i}{\sqrt{2}}\int \langle \overset{\varepsilon}{H}{}^a_\mu(y-x)\rangle \overset{\varepsilon}{h}{}^a_\mu(y)\, d^4y - \tag{4.6}$$
$$-\frac{1}{4}\int \langle \overset{\varepsilon}{H}{}^a_\mu(y-x)\, \overset{\varepsilon}{H}{}^{a'}_{\mu'}(y'-x)\rangle \overset{\varepsilon}{h}{}^a_\mu(y)\, \overset{\varepsilon}{h}{}^{a'}_{\mu'}(y')\, d^4y\, d^4y' ,$$

where the averages are understood depending on $\tilde{H}$. Thus we have to average over h (pressure and exponential cut-off are defined via $d(\varrho)$!) the following expression

$$Z_{h_c}(\phi,V) = e^{PV} \exp\Big\{\int d^4x \sum_\varepsilon{}' \int\frac{d\varrho}{\varrho}\, d(\varrho)\, e^{-PV} \times \tag{4.7}$$
$$\times\Big[-\frac{i}{\sqrt{2}}\int\langle \overset{\varepsilon}{H}(y-x)\rangle \overset{\varepsilon}{h}(y)\, d^4y - \frac{1}{4}\int \overset{\varepsilon}{h}(y)\langle \overset{\varepsilon}{H}(y-x)\,\overset{\varepsilon}{H}(y'-x)\rangle \overset{\varepsilon}{h}(y')\, d^4y\, d^4y'\Big]$$
$$+\frac{\pi^2}{4} a' \int d^4x \Big[\sum_\varepsilon \int\frac{d\varrho}{\varrho}\, d(\varrho)\, e^{-PV} \varrho^2 \int\langle \overset{\varepsilon}{H}(y-x)\rangle \overset{\varepsilon}{h}(y)\, d^4y\Big]^2\Big\} .$$

The last factor comes from the hard-core two-instanton correlation and can be written

$$\Big(\det\frac{2}{\pi^2 a'}\Big)^{\frac{1}{2}} \int D\lambda\, e^{-\frac{1}{\pi^2 a'}\int \lambda^2(x)\, d^4x}\, e^{\int\lambda(x)\sum_\varepsilon\int\frac{d\varrho}{\varrho} d(\varrho)\, e^{-PV} \varrho^2 \int\langle \overset{\varepsilon}{H}(y-x)\rangle \overset{\varepsilon}{h}(y)\, d^4y\, d^4x} . \tag{4.8}$$

Thus, apart from the final λ integration, we have to integrate

$$(\det \hat{\square})^{-\frac{1}{2}} \int D\ell \, \exp\left(-\tfrac{1}{2}\int \hat{\ell}\,\hat{\square}^{-1}\hat{\ell}\right)\exp PV \times$$

$$\times \exp\left\{\int d^4x \sum_{\varepsilon} \int \frac{d\rho}{\rho}\, d(\rho)\, e^{-Pv}\left[-\tfrac{i}{\sqrt{2}} + \rho^2\lambda(x)\right] \int \langle \overset{\varepsilon}{A}(y-x)\rangle \overset{\varepsilon}{\ell}(y)\, d^4y \, - \right. \tag{4.9}$$

$$\left. -\tfrac{1}{4}\sum_{\varepsilon} \int \overset{\varepsilon}{\ell}(y) \int \frac{d\rho}{\rho}\, d(\rho)\, e^{-Pv} \langle \overset{\varepsilon}{A}(y-x)\overset{\varepsilon}{A}(z-x)\rangle \overset{\varepsilon}{\ell}(z)\, d^4x\, d^4y\, d^4z \right\}$$

$$= \det(1+\hat{\square}\hat{m})^{-\frac{1}{2}}\, e^{PV} \exp\left(-\tfrac{1}{2}\int \hat{j}\,(\hat{\square}^{-1}+\hat{m})^{-1}\hat{j}\right),$$

where the matrix abbreviations

$$\hat{m} = \begin{pmatrix} \overset{+}{m} & 0 \\ 0 & \bar{m} \end{pmatrix}, \tag{4.10}$$

$$\overset{\varepsilon}{m}{}^{aa'}_{\mu\mu'}(y,z) = \tfrac{1}{2}\int d^4x \int \frac{d\rho}{\rho}\, d(\rho)\, e^{-Pv} \langle \overset{\varepsilon}{A}{}^{a}_{\mu}(y-x)\overset{\varepsilon}{A}{}^{a'}_{\mu'}(z-x)\rangle$$

and

$$\hat{j} = \begin{pmatrix} \overset{+}{j} \\ \bar{j} \end{pmatrix} = \hat{j}_0 + \hat{j}_1$$

$$\overset{\varepsilon}{j}{}^{a}_{\mu}(y) = \frac{1}{\sqrt{2}}\int d^4x \int \frac{d\rho}{\rho}\, d(\rho)\, e^{-Pv}\left[1 + i\sqrt{2}\,\rho^2\lambda(x)\right] \langle \overset{\varepsilon}{A}{}^{a}_{\mu}(y-x)\rangle \tag{4.11}$$

have been used; j_0 denotes $j(\lambda=0)$. Letting formally $a' \to 0$ in (4.8) would make it a δ-measure in λ , and we had reobtained the partition function with dipole interaction but without hard-core repulsion /17/ . We do the λ-integration, which is strictly Gaussian, noticing that both $\hat{j}_0$ and $\hat{j}_1$ are $O(\tilde{H})$. Therefore only the functional determinant does yield terms of $O(\tilde{H}^2)$:

$$Z(V) = e^{PV} \det(1+\hat{\square}\hat{m})^{-\frac{1}{2}} \exp\left(-\tfrac{1}{2}\int \hat{j}_0(\hat{\square}^{-1}+\hat{m})^{-1}\hat{j}_0\right) \times$$

$$\times \exp\left(\frac{\pi^2}{4} a' \int d^4x \sum_{\varepsilon\varepsilon'} \int \frac{d\rho}{\rho}\, d(\rho)\, e^{-Pv} \rho^2 \langle \overset{\varepsilon}{A}(y-x)\rangle (\hat{\square}^{-1}+\hat{m})^{-1\,\varepsilon\varepsilon'}(y,z) \times \right. \tag{4.12}$$

$$\left. \times \int \frac{d\rho'}{\rho'}\, d(\rho')\, e^{-Pv'} \rho'^2 \langle \overset{\varepsilon'}{A}(z-x)\rangle \, d^4y\, d^4z \right).$$

This is correct up to $O(\tilde{H}^2)$, sufficient to determine the susceptibility of the interacting gas, to be obtained according to (2.13) from the free energy density $\frac{1}{V}\log Z(V)$. The pressure P (eq. (4.5)) yields the susceptibility χ_0 (eq. (3.18)), corrected for the hard-core effect. The contribution of the determinant in (4.12) to the free energy density can be visualized as rings of interacting instantons and antiinstantons in alternating sequence. It gives a correction to the susceptibility, $\Delta\chi_{rings}$, owing to the dependence of $\hat{\mathcal{M}}$ defined in (4.10) on $\tilde{H}$, since it is a group average over the distribution (4.3). The quadratic term in $\hat{\mathfrak{Z}}$, to be imagined as chains of alternating instantons and antiinstantons in interaction, gives a contribution

$$\Delta\chi_{chains} = \chi_0 \frac{(\pi^2\chi_0)^2}{1-(\pi^2\chi_0)^2} \tag{4.13}$$

to the susceptibility, whereas the last correction due to the hard-core two-instanton correlations gives a negative contribution $\Delta\chi_{corr}$ to the susceptibility. The explicit expressions for the corrections $\Delta\mu_{rings}$ and $\Delta\mu_{corr}$ are given in the Appendix (eq. (A.1) and (A.2)). The permeability of the hard-core instanton gas with dipole interaction will be

$$\mu = 1 + 2\pi^2\chi = \mu_0 + \Delta\mu_{rings} + \Delta\mu_{chains} + \Delta\mu_{corr}\,. \tag{4.14}$$

In the next section the dependence of μ on the diluteness is studied numerically in connection with the intermediate coupling behaviour of the β-function. In the Table the different contributions to μ are separately shown. Within the tolerably dense instanton gas the permeability does not essentially exceed unity (weak paramagnetism), and the most important correction to μ is coming from the ring diagrams.

5. The Gell-Mann-Low β-Function in the Intermediate Coupling Region and the Degree of Diluteness

It has been suggested that instantons show up their influence in a restricted, intermediate range of scale. CDG have checked this, e.g., by proposing an instanton induced interpolation between the weak and strong coupling behaviour of the β-function /1o/ . If the QCD vacuum fluctuations - at least in a certain range of scale- can be reasonably well described in terms of magnetizable media, the coupling constant should be renormalized by the "vacuum permeability" μ (a), defined at a corresponding length scale a, as follows

$$g^2(a) = \mu(a)\ g^2_{RF}(a). \tag{5.1}$$

The β-function can then be found as

$$-\frac{\beta}{g} = \frac{\partial \log g(a)}{\partial \log(a\Lambda)}\Bigg|_{a=a(g)} . \tag{5.2}$$

Deviating from CDG, who identified the scale parameter a approximately with the infrared cut-off ς_c, it seems natural in our scheme to study g^2 as a function of the length scale $a \approx \bar{\varsigma}$ characteristic for the gas of fluctuations. The r.m.s. radius $\bar{\varsigma}$ has been shown to be a function of the diluteness parameter a' only. Therefore we are able to check point by point the validity of the dilute gas approximation and the influence of the dipole interaction. Numerically we have computed the β-function for the case of SU(3) applying formula (4.14) and using a parameterization by $x_0 = -11 \log \bar{\varsigma}\, \Lambda^{Latt}_{SU(3)}$,

$$g^2 = \mu(x_0)\frac{8\pi^2}{x_0}, \tag{5.3}$$

$$-\frac{\beta}{g} = \frac{11}{2x_0}\left(1 - \frac{x_0}{\mu}\frac{\partial\mu}{\partial x_0}\right). \tag{5.4}$$

For comparison with the strong coupling result from Euclidean lattice QCD [19] (cf. curve II in the Figure) we must adopt a coupling constant definition according to the lattice regularization scheme. We decided to take the estimate of A. and P. Hasenfratz [18] which provides, in the end, the Λ parameter and hence the corresponding factor C_N in the instanton density in relation to the corresponding ones in the Pauli-Villars scheme: $\Lambda^{PV}_{SU(3)} / \Lambda^{Latt}_{SU(3)} \approx 39.$, $C_3^{Latt} / C_3^{PV} \approx 3.7 \cdot 10^{17}$.

Our main result is given by curve III in the Figure. If we had applied the rough estimate for the Λ parameter corresponding to the lattice given in Ref./10/ $\left(\Lambda^{PV}_{SU(3)} / \Lambda^{Latt}_{SU(3)} \approx 6.6\right)$, we would end up with a similar curve, shifted however to somewhat bigger g $(\Delta g \simeq .3)$, with a slightly slower rise. The β-functions computed from large order calculations (in the inverse coupling constant) of the string tension within the Euclidean as well as in the Hamiltonian formulation seem to bend over to the asymptotic freedom curve near $g \simeq 1$, occasionally

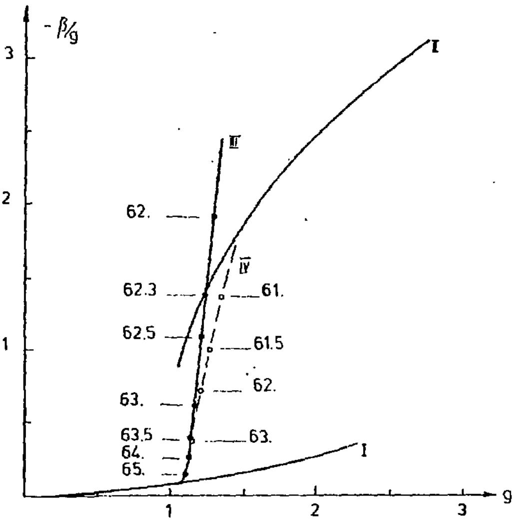

Figure: β-function in the transition region between weak and strong coupling for SU(3);

Curve I : Perturbative result in one-loop approximation;

II : Strong coupling result from Euclidean lattice QCD /19/;

III : instanton-mediated transition with account of dipole-dipole and repulsive interactions acc. to (4.14);

IV : same as III, but without interactions.

Some points are labeled by their corresponding x_0 values (compare the Table).

Table: Numerical values of different quantities defined in this paper, corresponding to the points marked on the curves III and IV in the Figure (calculated in accordance to the lattice regularization scheme of Ref. /18/ for SU(3)).

range	x_0	a'	f_0	$\pi^2\chi_0$	μ_0	$\Delta\mu_{chains}$ equ. (4.13)	$\Delta\mu_{rings}$ equ. (A.1)	$\Delta\mu_{corr}$ equ. (A.2)	μ
III	62.0	170	$9.5\cdot10^{-3}$	.074	1.15	$8\cdot10^{-4}$	.20	-.03	1.32
II	62.3	230	$7.3\cdot10^{-3}$	.057	1.11	$4\cdot10^{-4}$	.12	-.02	1.21
	62.5	270	$6.3\cdot10^{-3}$	.047	1.09	$2\cdot10^{-4}$	.08	-.01	1.17
	63.0	420	$3.9\cdot10^{-3}$	.030	1.06	$6\cdot10^{-5}$	.03	-.004	1.09
I	63.5	670	$2.5\cdot10^{-3}$	.019	1.04	$1\cdot10^{-5}$	.01	-.002	1.05
	64.0	1050	$1.6\cdot10^{-3}$	.013	1.03	$4\cdot10^{-6}$	.006	-.0008	1.03
	65.0	2600	$6.3\cdot10^{-4}$	.005	1.01	$3\cdot10^{-7}$	.001	-.0001	1.01

extrapolated by Padé approximation /19/. In this sense the instanton calculation given here serves to check the relation between the Λ parameters of continuum and lattice QCD. We are inclined to expect an even somewhat smaller $\Lambda^{Latt}_{SU(3)}$ than that obtained in Ref. /18/ . In any case, the sudden and steep rise of the β -function is happening at small coupling, which indicates, that it is a weak coupling albeit nonperturbative phenomenon. Instantons are one language to describe this.

How far can the instanton mechanism be trusted? On our curves have been marked several points corresponding to different values of x_0 . These values, the corresponding diluteness parameter a' and f_0 , respectively, the ideal gas susceptibility $\pi^2\chi_0$ together with the different corrections to the permeability of the interacting gas are exhibited in the Table. Obviously we can distinguish three ranges of scale, or diluteness:

I : instanton induced rise of the β- function, no influence of instanton interactions ($5000 \gtrsim a' \gtrsim 600$);

II : instanton interactions give a nonnegligible contribution ($600 \gtrsim a' \gtrsim 200$);

III : for $a' \lesssim 200$ we expect the dipole-dipole approximation to become a bad one; just this range coincides with the cross-over of curve III with the strong coupling curve II.

These are the diluteness parameter values a' (or, correspondingly, the space-time fractions f_o , which are in good agreement with recent calculations by Neuberger /24/), which set the limits for dilute gas instanton calculations. Since the magnitude of a' takes care for the smallness of the classical dipole-dipole interaction (as well as of all corrections to it) this parameter plays its role independently of the particular regularization scheme. In any case, the usual diluteness criterion, $f_o < 1$, is too unrestrictive as to guarantee consistent calculations within the dilute gas approximation taking no more than dipole interactions into account. It should be replaced by something like $f_o < .01$. Once the instanton gas is taken so dilute, there is no chance to save the proposed first order phase transition, signalled by an instability in the D vs. E plot ($D^a_k = iH^a_{k4}$, $E^a_k = iB^a_{k4}$, cf. equation (2.14)) /25/ , that was proposed /1b/ to justify the bag model from first principles. We have indeed checked the equation of state $D = D(E)$ in the simpler case of SU(2) taking into account the two-body repulsive correlation contribution (cf. eq. (3.26)) to the free energy density at arbitrary field strength. Even for denser gases than allowed by $f_o < .01$ we have not found any instability.

The comparison of the β-function provided by instanton effects with the strong coupling one /17/ had made us suspicious of whether instanton calculations must be restricted to much smaller length scales than usually assumed. The actual reason for this has been pointed out here.

Acknowledgements

We are indebted to Prof. D.V.Shirkov for continuous support and acknowledge gratefully a useful discussion with M.A.Shifman.

Appendix

In ref. /17/ we have found the contributions of the ring and chain diagrams, respectively, to the free energy density and the susceptibility by straightforward but tedious spelling out the matrix and tensor structure of the corresponding expressions, given here in (4.12). Similar manipulations with the new correlation term in (4.12) need not be repeated here. For completeness, however, the final expressions should be given here:

$$\Delta\mu_{rings} = \frac{3\pi^2}{8}\int_0^\infty dk\, k^3\, \tilde{K}(k)\,\frac{\pi^2\tilde{\chi}(k)}{1-(\pi^2\tilde{\chi}(k))^2} + \frac{\pi^2}{4N}\int_0^\infty dk\, k^3\,\tilde{\Omega}(k)^2\left(\frac{\pi^2\tilde{\chi}(k)}{1-(\pi^2\tilde{\chi}(k))^2}\right)^2, \tag{A.1}$$

$$\Delta\mu_{corr} = -\frac{a'}{4}\int_0^\infty dk\, k^3\,(\pi^2\tilde{\psi}(k))^2\,\frac{\pi^2\tilde{\chi}(k)}{1-(\pi^2\tilde{\chi}(k))^2} \tag{A.2}$$

with

$$\tilde{\chi}(k) = \frac{1}{N^2-1}\int\frac{d\rho}{\rho}\, d_0(\rho)\, e^{-Pv} x(\rho)\,\rho^4\, F^2(k\rho), \tag{A.3}$$

$$\tilde{K}(k) = \frac{1}{N^2-1}\int\frac{d\rho}{\rho}\, d_0(\rho)\, e^{-Pv} x^2(\rho)\,\rho^8\, F^2(k\rho), \tag{A.4}$$

$$\tilde{\Omega}(k) = \frac{1}{N^2-1}\int\frac{d\rho}{\rho}\, d_0(\rho)\, e^{-Pv} x^{3/2}(\rho)\,\rho^6\, F^2(k\rho), \tag{A.5}$$

$$\tilde{\psi}(k) = \frac{1}{N^2-1}\int\frac{d\rho}{\rho}\, d_0(\rho)\, e^{-Pv} x(\rho)\,\rho^6\, F(k\rho) \tag{A.6}$$

and

$$F(x) = \frac{4}{x^2}\left(1-\frac{x^2}{2}K_2(x)\right). \tag{A.7}$$

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Received by Publishing Department
on August 4 1980.