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BAG MODEL WITH LINEAR CONFINING POTENTIAL

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The bag model may describe a great deal of data on the hadron properties[1-5]. In these models the quark confinement is ensured by the conditions $M_q \to \infty$, where M_q is the free quark mass. Such a mechanism of quark confinement of the bag model is identical with the choice of potential of quark interaction with the gluon field in a form of the infinite square-well potential. The difference between works[1] and[2] is that in[1] the bag radius is fixed but in ref.[2] the value of $B = -\frac{\partial E_h}{\partial V_h}$ (the pressure of physical vacuum on bag) is fixed (E_h - the hadron energy, V_h - the bag volume).

In this work we require that quarks inside a hadron should produce the same constant fixed pressure B on the quark potential wall as gluons on the bag, that is, one takes $B_h \equiv -\frac{\partial E_h}{\partial V_h} = B_q \equiv -\frac{\partial E_q}{\partial V_q}$ (E_q - the quark energy, V_q - the volume of space from the bag center to quark potential surface). We shall study the case when the potential of quark interaction with the gluon fields is an infinite potential well and also the potential which is power-increasing with distance from the bag center.

Physically the requirement $B_q = B_h$ means that in transition from one excited hadron state to another the rearrangement of action of gluon fields on quarks causes. This may be taken into account, for instance, by changing the potential parameters which characterise the strength of interaction like in the case of quantum chromodynamics.

It is essential to emphasize that the phenomenon of rearrangement of quark interaction with gluons we have considered does not occur if no conditions on pressure are imposed.

Following[2] we choose the mass of u, d light quarks inside the bag equal zero. It is supposed the gluon field is concentrated inside the bag and has the energy of $E_G = BV_h$.

A rather successfull method of the bag model was the treatment of relativistic motion of quarks as quasi-independent particles[1]. This allowed the explanation of a great number of experimental data on static properties of hadrons (mass spectrum, baryon magnetic moments, and r.m.s. charge radius, axial-vector coupling constant, etc.). We shall follow this approach. In this model the hadron wave function is represented as a product of the wave functions of individual quark satisfying the Dirac equation:

$$[\vec{\alpha}\vec{p} + \beta(m_q + V_n(r))]\Psi(\vec{r}) = \epsilon_n \Psi(\vec{r}), \tag{1}$$

where ϵ_n is the dimensionless energy of an n-th quark mode, m_q is the mass of effective quark.

Let us define the n-th hadron mode as a state in which both quarks are in the n-th mode. The meson masses, as in the bag model, is a sum of the quark kinetic energy and the energy of the gluon field:

$$M_n = \frac{2\epsilon_n}{\tilde{R}_n} + \frac{4}{3}\pi R_n^3 B, \tag{2}$$

where R_n is the radius of the gluon bag inside which quarks are in the n-th mode, n is the principal quantum number, $\tilde{R}_n$ is the characteristic scale of quark orbit.

The stability condition of the system is identical to defining the minimum of (2) with respect to $\tilde{R}$ and R. If those radii are regarded as independent variables, then this condition provides that the gluon bag will contract to a point and the quarks will fly away. That is why it is necessary to impose a coupling $f(\tilde{R}, R) = 0$ so that the bag be stable. The dimensional considerations lead to the following form of this coupling

$$R_n = a r_n(\tilde{R}_n), \tag{3}$$

where r_n is the radius of "classical" region of the quark motion.

Let the confining potential for quarks produced by gluons be an infinite potential well. The pressure of quark which is in the n-th mode is calculated by the following expression:

$$B_n \equiv -\frac{\partial E_n}{\partial V_n} = -\frac{1}{4\pi r_n^2}\,\frac{\partial(\epsilon_n/r_n)}{\partial r_n} = \frac{\epsilon_n}{4\pi r_n^4}, \tag{4}$$

where E_n and ϵ_n are the quark energies written in the dimensional and dimensionless manner, respectively. The requirement that the quark field pressure should be constant

$$B_0 = B_1 = B_2 = \dots \tag{5}$$

provides the recurrence relation for the potential

$$r_n = r_0 \left(\frac{\epsilon_n}{\epsilon_0}\right)^{1/4}. \tag{6}$$

Substituting (3) in (2) we have:

$$M_n = \frac{2\epsilon_n}{r_n} + \frac{4}{3}\pi a^3 r_n^3 B. \tag{7}$$

Minimum of this expression with respect to r_n provides the stability of the bag that is realized for $a^3 = 2$. Substituting this value in (7), we have:

$$M_n = M_0 \left(\frac{\epsilon_n}{\epsilon_0}\right)^{3/4}, \tag{8}$$

where M_0 is the mass of the meson ground state.

Generally speaking, one may use an arbitrary form of the confining potential. This is also true for the models [1,2]. The linear potential is different from other confining potentials within relativistic quark models because it provides linear Regge trajectories at large n : $E_n^2 \sim n + \mathrm{const}$ [7]. For the sake of generality we shall consider the problem with an arbitrary power potential $V(r) = gr^m$ $(m \neq -1)$. In this case the energy will become dimensionless by the following expression $\epsilon_n = E_n \tilde{R}_n$ $(\tilde{R}_n = g_n^{-1/(m+1)})$ and the radius r_n is calculated using: $r_n = \epsilon_n^{1/m} \tilde{R}_n$. Then the pressure will be equal to:

$$B_n = \frac{\epsilon_n^{1-3/m}}{4\pi \tilde{R}_n^4}. \tag{9}$$

The condition that B_n be constant leads to the recurrence relation for the potential parameters:

$$\tilde{R}_n = \tilde{R}_0 \left(\frac{\epsilon_n}{\epsilon_0}\right)^{(m-3)/4m}. \tag{10}$$

By using relation (3): $R_n = a\epsilon_n^{1/m} \tilde{R}_n$ (2) is reduced to:

$$M_n = \frac{2\epsilon_n}{\tilde{R}_n} + \frac{4\pi}{3} a^3 \epsilon_n^{3/m} \tilde{R}_n^3 B \ . \qquad (11)$$

As in the above-mentioned case $a^3 = 2$ ensures the stability condition of the bag. Then the mass formula for meson excited states reads:

$$M_n = M_0 \left(\frac{\epsilon_n^{(m)}}{\epsilon_0^{(m)}}\right)^{3(m+1)/4m} . \qquad (12)$$

At last consider the interaction between quark and gluons on the base of this model. The scalar Coulomb potential for small distances from the center of the bag and the linear potential for large distances corresponds to this interaction, that is we consider a potential $V(r) = -\frac{a}{r} + br$.

For this problem we have two parameters: dimensional b and dimensionless a and the only condition on these parameters, the condition of stability of the system. So, in order not to complicate the treatment, we take into account the contribution of pressure of the Coulomb field to the total pressure of the quark field and then fix a , the varying parameter being b . Now the dimensionless energy depends on parameter a : $\epsilon_n(a) = E_n / \sqrt{b_n}$. Calculating the pressure by (4) we get:

$$B_n = \frac{1}{4\pi} \frac{b_n^2}{\tilde{r}_n^2} f_n^* ; \qquad f_n^*(a) = 1 + f_n(a) - f_n(0) , \qquad (13)$$

where

$$f_n(a) = \frac{2[\epsilon_n(\epsilon_n - \epsilon_n') \sqrt{\epsilon_n^2 + 4a_n} - \epsilon_n'(\epsilon_n^2 + 4a_n) + 4\epsilon_n]}{(4+\epsilon_n)^2 \sqrt{\epsilon_n^2 + 4a_n} + \epsilon_n(4 + \epsilon_n^2 + 4a_n)}$$

$$r_n = \frac{\tilde{r}_n}{\sqrt{b_n}} = \frac{\epsilon_n + \sqrt{\epsilon_n^2 + 4a_n}}{2\sqrt{b_n}} , \qquad \epsilon_n' = \frac{\partial \epsilon_n}{\partial a_n} .$$

The value $f_n(a = 0) \neq 0$ is derived in calculating B by (4) owing to changing the potential form, but not bag volume and consequently has nothing to do with pressure. Therefore it is needed to subtract $f(0)$ from $f(a)$.

Using (5) we obtain

$$b_n = b_0 \frac{\tilde{r}_n}{\tilde{r}_0} \left(\frac{f_0^*}{f_n^*}\right)^{1/2}. \tag{14}$$

We assume that the parameter α_n does not depend on n . Minimizing this expression for the mass of the bag:

$$M_n = \frac{2\epsilon_n}{\sqrt{b_n}} + \frac{4\pi}{3} a^3 r_n^3 B \tag{15}$$

With respect to b_n , we find that $a^3(\alpha) = \frac{2\epsilon_n}{\tilde{r}_n f_n}$. Substitution of this relation into (15) yields the formula for the meson mass:

$$M_n = M_0 \frac{\epsilon_n}{\epsilon_0} \left(\frac{\tilde{r}_n}{\tilde{r}_0}\right)^{1/2} \left(\frac{f_0^*}{f_n^*}\right)^{1/4}. \tag{16}$$

Using formulas (8),(12),(15) let us calculate the mass spectrum of radial excitations of ρ-meson. Let the meson with mass 773 MeV corresponds to the lowest state of the spectrum. The calculated masses, bag radius, and quantity $B^{1/4}$ are shown in the table. The eigenvalues ϵ_n for the potentials are found by numerical solving of the Dirac equation.

Table

Potential type	$B^{1/4}$(MeV)	R_0(fm)	n = 1	n = 2	n = 3
Experiment /6/			0.773	1.600	
$m = \infty$	90	1.75	0.773	1.601	2.267
$m = 1$	75	2.25	0.773	1.574	2.238
$m = 2$	85	1.90	0.773	1.648	2.347
$m = 1;\ \alpha = 0.05$	74	1.92	0.773	1.641	2.364
$m = 1;\ \alpha = 0.2$	71	1.75	0.773	1.744	2.548

From the data in the table it is seen that qualitative correspondence with the experimental masses is achieved both for the infinite potential well and for a power potential with small admixture of the Coulomb term. The obtained values of pressure B are of an order of the values $B^{1/4} \sim 150$ MeV from[2] and $B^{1/4} \sim 240$ MeV[8].

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