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A.V.Efremov, A.V.Radyushkin

ON PERTURBATIVE QCD OF HARD AND SOFT PROCESSES

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1. INTRODUCTION

It is now safe to assert that quantum chromodynamics (QCD) agrees qualitatively with all the experimental data related to strong interactions phenomena. At the same time, the QCD predictions are usually too flexible for precisive quantitative tests. This is caused mainly by the fact that all the calculations in QCD are based on perturbation theory (PT), i.e. on the expansion over the coupling "constant" $\alpha_s(k)$ that depends really on the momentum scale k related to the process investigated. Asymptotic freedom [1] enables one to use PT at short distances (or large momenta). However, any physical process involves also long distances, i.e. each process involves small momentum scales p^2 (e.g., quark and hadron masses), and as a rule, this results in the appearance of the logarithmic contributions $\ln(Q^2/p^2)$, that are singular for $p^2=0$ (mass singularities [2-4]). In such a situation p^2 cannot be neglected. However, within PT it is possible to show that for inclusive [2-6] and some hadronic exclusive hard processes [6-8] the Q^2-dependence of the corresponding amplitude $T(Q^2,p^2)$ can be factorized from the p^2-dependence (see fig.1):

$$T(Q^2,p^2)=Q^N\{E(Q^2/\mu^2,\alpha_s(\mu))\otimes f(\mu^2,p^2)+R(Q,p)\}, \qquad (1.1)$$

where N is the dimension of T in mass units and R is the sum of contributions which are power suppressed with respect to $E\otimes f$. . The parameter μ is a boundary between large and small momenta, and $E\otimes f$ does not depend on a particular choice of μ. .

The functions f describe long-distance interactions. This means they cannot be reliably calculated in perturbative QCD and must be treated phenomenologically. As a result, the QCD predictions are more ambiguous.

The functions E describe short-distance interactions. In principle, they are given by a perturbative series expansion over $\alpha_s(\mu)$. . In practice, only a few terms are known, and one is forced to make some plausible hypotheses about the magnitude of the noncalculated higher-order corrections. According to most of recent estimates, $\alpha_s(\mu)/\pi$ is of the order 0.1 for $\mu^2 \lesssim 10$ GeV^2. This means that taking into account only a few first terms of the series is a good approximation only if the coefficients of the expansion of E over $\alpha_s \pi$ are not too large.

An analogous uncertainty exists also for power corrections absorbed by $R(Q, p)$. It is known that the hadron-and quark-mass corrections for the most simple inclusive processes can be calculated exactly with the help of the ξ-scaling formalism [9,10]. For light quarks u,d,s, their masses ($m_u \simeq 4$ MeV $m_d \simeq 7$ MeV, $m_s \simeq 120$ MeV) usually may be neglected at all. The main uncertainty is due to power corrections caused by the finite size of the hadrons, by the Fermi-motion of quarks inside the hadrons, etc. All these effects have a nonperturbative origin. The magnitude of the corresponding corrections $(M^2/Q^2)^n$ is determined by a characteristic scale $M = 1/R_{conf} \simeq 300 \div 500$ MeV, but in some cases they play a very important rol up to very high momentum transfers.For example, in high $-p_T$ hadron production, the effects of the primordial transverse momentum of partons dominate the cross section up to $p_T^2 \simeq 30$-40 GeV^2 [11]. It should be emphasized, however, that all the phenomenological methods of taking into account the power suppressed terms have no reliable theoretical basis. So, it is highly important to develop methods of field-theoretical analysis of power corrections.

2. HIGHER-ORDER CORRECTIONS

In general, the functions $E(Q^2/\mu^2, \alpha_s(\mu))$ in eq. (1.1) depend on the calculation scheme, namely, on the chosen recipe of the R-operation for the ultraviolet divergences and the recipe of separating the contributions related to short and long distances (i.e. on the R-operation for composite operators). In particular, E is μ-dependent. The functions $f(\mu, p)$ also depend on the chosen scheme and only the product $E \otimes f$ is scheme-independent. If we take $\mu = Q$, then the resultant expression would not have an explicit dependence on μ. Note, that this procedure removes from $E(Q^2/\mu^2)$ the logarithms $\ln(Q^2/\mu^2)$ which, for $Q \gg \mu$, lead to growth of the coefficients in the expansion of E over α_s. The meaning of the choice $\mu = Q$ is clear: one must take μ equal to a scale characterizing the off-shellness of the particles taking part in the short-distance subprocess and the latter is proportional to Q^2: $\langle k^2 \rangle \simeq a^2 Q^2$.

If, however, the parameter a is very large (or very small) compared to 1, then the choice $\mu \simeq aQ$ should be preferred. It is implicit here that we use a "physical" renormalization scheme, i.e. $\bar{g}(k)$ corresponds to a vertex with external momenta k^2 . However, for direct calculations within QCD it is more convenient to use various "unphysical" schemes based on dimensional regularization. In this case the meaning of $\bar{g}(k)$ is less transparent. So, for the time being, we will adhere to the choice $\mu = Q$. .

Recently, in a series of papers /12-15/ it has been established that the scheme-dependence of the results obtained may be reduced by expanding the coupling constant $\alpha_s(Q)$ over $(\ln Q^2/\Lambda^2)^{-1}$:

$$\frac{\alpha_s(Q)}{4\pi} = \frac{4}{b_0 \ln Q^2/\Lambda^2} \{1 - \frac{b_1}{b_0^2} \frac{\ln\ln Q^2/\Lambda^2}{\ln Q^2/\Lambda^2} + \dots \}, \tag{2.1}$$

where $b_0, b_1, \dots$ are the coefficients of the expansion of the β-function over g. . After this change we have the following representation for $E \circledast f$

$$E \circledast f = \{ (\ln Q^2/\Lambda^2)^{-\gamma_0/b_0} \sum_{k=0}^{\infty} \sum_{\ell=0}^{\infty} a_{\ell k} \frac{(\ln\ln Q^2/\Lambda^2)^k}{(b_0 \ln Q^2/\Lambda^2)^\ell} \} \circledast \tilde{f} =$$

$$= \{ (\ln Q^2/\Lambda^2)^{-\gamma_0/b_0} [a_{00} + \frac{a_{10}}{b_0 \ln Q^2/\Lambda^2} + \frac{a_{20}}{b_0^2 \ln^2 (Q^2/\Lambda^2)} + \tag{2.2}$$

$$+ a_{21} \frac{\ln\ln Q^2/\Lambda^2}{b_0^2 \ln^2 (Q^2/\Lambda^2)} + \dots \} \circledast \tilde{f}.$$

All information about the long-distance dynamics is accumulated in $\tilde{f}$, whereas the coefficients $a_{\ell k}$ can be calculated in PT. . Moreover, the coefficients $a^{(1)}_{\ell k}$, $a^{(2)}_{\ell k}$ related to two different schemes can be obtained from one another by the change $\Lambda_1 = \kappa_{12} \Lambda_2$ for the appropriately chosen parameter κ_{12}. Thus, if one uses the expansion (2.2), then various schemes differ only in magnitude of the parameter Λ .

Let us assume that, in a scheme S , some first coefficients $a_{\ell k}$ are numbers of the order 1. Then in another scheme S', which has $\Lambda' = 100\Lambda$ (or $\Lambda' = 0.01\Lambda$), the coefficients $a'_{\ell k}$ are numbers of the order $b_0 \ln 100 \simeq 40$. It is easy to notice the analogy with our previous discussion about the optimal choice of the parameter μ and to conclude that the scheme S is very close to a "physical" scheme, since the choice $\mu = Q$ (assumed in eq. (2.2)) minimizes for this scheme the higher-order corrections. Note also that the choice $\mu = aQ$ is equivalent to an expansion over $(\ln(a^2Q^2/\Lambda^2))^{-1}$ rather than over $(\ln Q^2/\Lambda^2)^{-1}$, i.e. to the change $\Lambda \to \Lambda/a$.

So, let us assume that if one uses a physical scheme with a properly chosen subtraction point (i.e. $\mu^2 = \langle k^2 \rangle$), then the resultant expansion over (α_s/π) has coefficients of the order 1 (this is just the situation usually encountered in QED, where one has no problems with the momentum-dependence of the coupling constant $\alpha_{QED} \simeq 1/137$). If this assumption is valid, then the higher-order corrections may be calculated using the following rules:

1) One may calculate in an arbitrary scheme. The most convenient, in our view, is the $\overline{MS}$-scheme [16], which is free from spurious terms $\ln(4\pi)$ and γ_E present in the minimal-subtraction (MS)-scheme. The parameter $\Lambda_{\overline{MS}}$ may be chosen to be a fundamental scale of QCD. Note, that $\Lambda_{\overline{MS}}$ is close to Λ_{PH} related to a physical scheme: $\Lambda_{PH} = \kappa \Lambda_{\overline{MS}}$, where $\kappa \simeq 2$ is almost independent of the vertex chosen to define $\bar{g}(k)$ [15].

2) In general, however, there are no a priori reasons to expect that the $\overline{MS}$-scheme minimizes the coefficients $a_{\ell k}$ of in eq. (2.2). If it is known that the average off-shellness of lines related to a short-distance subprocess is a^2Q^2, and $a \gg 1$ (or $a \ll 1$), then $E \otimes f$ must be expanded over $(\ln(a^2Q^2/\kappa^2\Lambda^2_{\overline{MS}}))^{-1}$ rather than over $(\ln Q^2/\Lambda^2_{\overline{MS}})^{-1}$.

3) Usually the value of a is not known. However, this value may be estimated by requiring that the coefficient a_{10} (or a_{20}, if $\gamma_0 = 0$) vanish after the change $\Lambda_{\overline{MS}} \to 2\Lambda_{\overline{MS}}/a$.

In this approach all results of the calculations are expressed in terms of the only parameter $\Lambda_0 \equiv 2\Lambda_{\overline{MS}}$. However, in the expansion (2.2) for different processes one may use different $\Lambda^{(i)}_{eff} = \Lambda_0/a_i$ (with known a_i's).

3. POWER CORRECTIONS

Our derivation of eq. (1.1), given in refs. [5-7], is based on the analysis of Feynman diagrams in the α-representation [17] (see also [18,19]), i.e. on the formula

$$\frac{1}{m_\sigma^2 - k^2} = i \int_0^\infty d\alpha_\sigma \exp[i\alpha_\sigma(k^2 - m_\sigma^2)] \qquad (3.1)$$

applied to propagators of all lines σ of the diagram. After integrations over all k_l this gives the representation

$$T(Q,p) \sim \int_0^\infty \prod_\sigma d\alpha_\sigma D^{-2}(\alpha) G(Q,p,\alpha) \times$$
$$\times \exp[iQ^2 A(\alpha) + ip^2 I(\alpha) - i\sum_\sigma \alpha_\sigma m_\sigma^2] , \tag{3.2}$$

which has many advantages for analysis of the large Q^2 behaviour of T. In particular, from eq. (3.2) it follows that integration over a region where $A(\alpha) > \rho$ gives for $Q^2 \to \infty$ an exponentially damped contribution $O(\exp(-Q^2\rho))$. Hence all contributions having a power $(O(Q^{-N}))$ behaviour for $Q^2 \to \infty$ are due to integration over regions where $A(\alpha)$ vanishes. There exist three main possibilities to get $A(\alpha) = 0$:

1) short-distance (or small-α) regime, when $\alpha_{\sigma_1} = \alpha_{\sigma_2} = \dots = \alpha_{\sigma_n} = 0$ for some lines $\sigma_1, \sigma_2, \dots \sigma_n$;

2) infrared (or $\alpha \to \infty$) regime, when $\alpha_{\sigma_1} = \alpha_{\sigma_2} = \dots = \alpha_{\sigma_n} = \infty$ for a set of lines $\{\sigma_1, \dots, \sigma_n\}$;

3) pinch regime, when $A = 0$ for nonzero finite α's. This regime works when $A(\alpha)$ may be represented as a difference of two positive terms.

It is possible also to get $A(\alpha)$ making up a combination of the three basic regimes. In the momentum representation the first regime corresponds to integration over a region $k \sim Q$, the second one over $k \sim p^2/Q$, and the third over $k \sim p$. This means that perturbative QCD is applicable only when the regimes 2,3 and the combined regimes either do not contribute at all or give a power suppressed contribution compared to that of the pure SD-regime. There exists a wide class of processes for which the pinch regime does not work (see ref. /5/), and it is sufficient to analyse only the SD- and IR-regimes. In this case it is very useful to visualize a diagram as an electric circuit and to treat the parameters α_σ as the resistances of the corresponding lines σ. Note, that according to eq. (3.2), for $A = 0$ the amplitude T lacks its Q^2-dependence. Hence, one must find the subgraphs that should possess the following topological properties: when lines of these subgraphs are contracted into point $(\alpha_\sigma = 0)$ and/or removed from the diagram $(\alpha_\sigma = \infty)$ then the resulting diagram does not depend on Q^2. Each configuration of this type corresponds to some power-behaved $O(Q^{-N})$ -contribution. The power N may be easily estimated with the help of the rules $k_{SD} \sim Q$, $k_{IR} \sim p^2/Q$:

$$t_V^{SD} \lesssim Q^{4-\Sigma t_i} ;$$

$$t_S^{IR} \lesssim Q^{-\Sigma t_j} ; \qquad (3.3)$$

$$t_{V;S}^{SD;IR} \lesssim Q^{(4-\Sigma t_i - \Sigma t_j)} ;$$

where t_i (t_j) is twist [20] of the i-th (j-th) external line of the subgraph V(S) corresponding to the SD-(IR-) integration. Recall that $t_{i,j} = 1$ for ψ, $\bar{\psi}$ -fields and the curl $G_{\mu\nu}$, whereas $t_{i,j} = 0$ for the vector field A_μ. That is why in QCD (in covariant gauges) it is necessary to sum up over external gluon lines of the subgraphs V, S . However, for the forward amplitudes (corresponding to inclusive cross sections) and for amplitudes of exclusive processes involving colour singlet particles, after such a summation the field A_μ either disappears (and the gluon lines correspond to the curl $G_{\mu\nu}$ that has a nonzero twist) or enters into covariant derivatives $D_\mu = \partial_\mu - igA_\mu$ present in composite operators that naturally arise when the contribution of the corresponding configuration is written in the coordinate representation (cf. [5-7]).

Consider, e.g. the forward amplitude $T(r, Q^2)$ corresponding to the total cross section of the Drell-Yan process $AB \to \mu^+\mu^- X$. In this case all the configurations responsible for the leading contribution $T^{lead}(r, Q^2) = O(Q^\circ)$ have the structure shown in fig.2a. Here, the subgraph V corresponds to the E-function, whereas subgraphs resulting after contraction of V into point correspond to the function $f = f_A \otimes f_B$ . . The configurations shown in fig.2b,c give power suppressed contributions. Note also that eq. (3.3) gives only an upper estimate. This means the contribution 2a itself apart from the leading contribution, contains also power corrections. These corrections appear in the following cases:

1) If we project the spinor structure of the subgraph V onto 1 rather than onto γ_μ . In the latter case we obtain a composite operator having twist equal to 2 and in the former one the resulting operator has twist 3. Note that γ_5, $\gamma_5\gamma_\mu$ - and $\sigma_{\mu\nu}$ - projections have vanishing matrix elements for spin-averaged amplitudes.

2) If we expand a bilocal operator $O^\mu(\xi, \eta)$ over the local ones, then there appear operators

$$\{\bar{\psi}(\gamma^\mu D^{\mu_1} \dots D^{\mu_n})_{\text{symmetrized}} \overbrace{g_{\mu.\mu.} \dots g_{\mu.\mu.}}^{j \text{ times}} \psi\} \qquad (3.4)$$

having twist $2+2j$. Each factor $g_{\mu.\mu.}$ adds $(\xi-\eta)^2$ to the corresponding function $E(\lambda,\eta;\xi',\eta')$ and this leads to suppression of the resulting contribution by an additional factor $1/Q^2$.

The expansion of the function $E(\xi,\eta,...)(\xi-\eta)^{\mu_1}...(\xi-\eta)^{\mu_n}$ over symmetric-traceless structures $E(\xi,\eta,...)[(\xi-\eta)^2]^j\{(\xi-\eta)^{\mu_1}...(\xi-\eta)^{\mu_\ell}\}$ corresponds in the momentum representation to an expansion of the amplitude $\tilde{E}(k,Q,...)$ over k^2:

$$\tilde{E}(k,...)=\tilde{E}(k,...)|_{k^2=0}+\sum_{j=1}^{\infty}(k^2)^j\,\tilde{E}_j(k,...)|_{k^2=0} \tag{3.5}$$

i.e. over the off-shellness of the particle corresponding to the external line of the SD-subgraph V (quark masses are assumed to be zero). In the α-representation this corresponds to an expansion of the integrand in eq. (3.2) into a power series over $\lambda_V=\sum_{\sigma\in V}\alpha_\sigma$. Thus, the coefficient function E for the leading contribution corresponds to an on-shell amplitude. This means that E is formally gauge-invariant in each order of PT. However, because of logarithms $\ln Q^2/k^2$ present in $\tilde{E}(k,Q,...)$, taking the limit $k^2=0$ is a rather delicate procedure. Note, that $E(Q^2/\mu^2,...)$ in eq. (1.1) corresponds to integration over small λ_V , i.e. the contribution of the region $\lambda_V>1/\mu^2$ must be subtracted off. As a result, one has $\ln Q^2/\mu^2$ in place of $\ln Q^2/k^2$, , and it is then safe to take $k^2=0$. To maintain gauge invariance, one may introduce the cut-off at $\lambda_V>1/\mu^2$ using, e.g., the dimensional regularization $d^4k\to d^{4+2\epsilon}k(\mu^2)^{-\epsilon}$ combined with subtraction of poles $1/\epsilon$. These poles formally correspond to $\ln(\mu^2/k^2)|_{k^2=0}$ (cf. ref. [23]).

Thus in the leading power approximation quarks corresponding to external lines of the parton subprocess should be treated as the on-shell particles. Real quarks are, of course, off-shell. But according to eqs. (3.4), (3.5) this phenomenon leads to power corrections only. They may be analysed just in the same way as the leading term, although the analysis is more involved. In particular, for each new set of operators one must introduce a new function. However some of these functions are linearly dependent due to equations of motion. As it was emphasized in a classic paper [24] using the equation of motion $D_\mu\gamma^\mu\psi=0$, one may get rid of the operators containing $D_\mu\gamma^\mu$ and $D_\mu D^\mu$. The resulting operators are built of the fields ψ , $\bar{\psi}$, $G_{\mu\nu}$ and covariant derivatives

D_μ . The reduced matrix elements of such operators may be identified with the moments of functions which are generalizations of the parton distribution functions, e.g.

$$\langle P || \bar{\psi}_{(a)} \{ (\sigma_{\mu\nu} G^{\mu\nu}_{;\mu_1 \ldots \mu_k}) \psi^{(a)}_{;\mu_{k+1} \ldots \mu_{n+k}} \} || P \rangle =$$

$$= \frac{1+(-1)^k}{2} [\tilde{f}_{a,g} (n,k) + (-1)^n \tilde{f}_{\bar{a},g} (n,k)] =$$

$$= \frac{1}{2} [\tilde{f}_{ag,a} (n,k) + (-1)^k \tilde{f}_{a,ga} (n,k) + (-1)^n \tilde{f}_{\bar{a}g,\bar{a}} (n,k) + \tag{36}$$

$$+ (-1)^{n+k} \tilde{f}_{\bar{a},g\bar{a}} (n,k)] = \int_0^1 dx_1 dx_2 dx_3 [f_{ag,a} (x_1, x_2; x_3) \delta(\quad + x_2 - x_3) +$$

$$+ (-1)^k f_{a,ga} (x_1; x_2, x_3) \delta(x_1 - x_2 - x_3) +$$

$$+ (-1)^n \ldots + (-1)^{n+k} \ldots] x_1^{n-1} x_2^{k-1} ,$$

where ; denotes covariant differentiation, a denotes the quark flavour and $\bar{a}$ that of antiquark. The function $f_{ag,a}$ ($f_{\bar{a}g,\bar{a}}$) describes a quark (antiquark) with momentum $x_1 P$ and a gluon with momentum $x_2 P$ in the initial state and quark (antiquark) with momentum $x_3 P = (x_1 + x_2) P$ in the final state. The functions $f_{a,ga}$, $f_{\bar{a},g\bar{a}}$ have analogous meaning. Such a construction was introduced first in ref. [25] for operators $\bar{\psi} \{ \gamma_\mu (\partial_{\mu_1} \ldots \partial_{\mu_k} \psi) \times \times (\partial_{\mu_{k+1}} \ldots \partial_{\mu_{n+k}} A_{\mu_{n+k+1}}) \}$ used in the analysis of factorization in the Feynman gauge.

Operators of eq. (3.6) appear also in configurations 2a if the subgraph V has external lines corresponding to the curl $G_{\mu\nu}$. Apart from matrix elements $\langle P | O | P \rangle$, these configurations contain also matrix elements $\langle 0 | G \ldots G | 0 \rangle$, $\langle 0 | (\bar{\psi}\psi) \ldots (\bar{\psi}\psi) | 0 \rangle$, $\langle 0 | G \ldots G (\bar{\psi}\psi) | 0 \rangle$ etc. In each order of PT these matrix elements vanish, but in QCD, due to nonperturbative effects, these vacuum matrix elements may be nonzero. As it was demonstrated in ref. [26], these contributions are very important for understanding the dynamics of hadrons. The main problem is to generalize the methods developed in ref. [26] to more complicated amplitudes.

All the configurations considered above correspond to the SD-regime $\lambda(V) \sim 0$. . One must take into account also the configurations 2c corresponding to the combined SD-IR regime $\lambda(V) \sim 0$, $\lambda(S) \to \infty$. Physically this regime corresponds to a short-distance subprocess accompanied by the exchange of soft quanta between the hadrons A and B. According to eq.(3.3), these contributions also have a power behaviour $O(Q^{-N})$, where N is the number of external lines of subgraph S . If all N lines are gluonic, then the corresponding diagrams describe a multipole interaction of hadrons. However, if the quarks are massless, then the subgraph may possess quark lines also. The main contribution for the IR-regime is given by the region $k^2 \sim (p^2/Q)^2$, where p^2 may be treated as hadronic mass. For massive fields (e.g., quarks) the contribution of the IR-regime is damped by the mass present in the propagator $(k^2+m^2)^{-1}$ if $k^2 \sim m_H^4/Q^2 \ll m^2$, , i.e. for $Q^2 \gtrsim m_H^4/m^2$ the IR-regime does not work. The mass m in this case works as an infrared cut-off.

Since the gluons are massless, the IR-regime in PT always works for gluons, and there are power corrections due to integration over $a \sim \infty$. However, the contributions corresponding to the configuration 2c do not factorize in the usual sense. This suggests that complete analysis even of the lowest power corrections in perturbative QCD is impossible.

However, if the exchanged system is coloured, then the corresponding contribution should be damped by confinement (non-perturbative) effects, i.e. in this case even for $m=0$ there exists an IR cut-off $M \equiv 1/R_{conf} \simeq 300 \div 500$ MeV. Thus, if we add to PT a confinement hypothesis, then for a coloured system S the contribution of configuration 2c is damped for $Q^2 \gtrsim m_H^4/M^2 \sim M^2$ i.e. for all hard processes. On the other hand, if the exchanged system is colour-singlet (e.g., colour-singlet glue-ball, π-meson, ρ-meson, pomeron etc.), then there are no a priori grounds to neglect the configuration 2c. We feel that the methods of the "old" hadronic theory, such as the Reggeon calculus and potential models (e.g. the quasipotential approach [27]), will be much more suitable for analysis of these contributions than the perturbative QCD methods. Highly instructive in this connection is the result of ref.[28], where it is shown that if one describes the soft exchange by an exponentially vanishing quasipotential, then the soft interactions in initial and final states give power ($1/p_T$ and $1/p_T^2$) rather than exponential ($\exp(-ap_T)$) corrections to the amplitude of high-energy wide-angle elastic πp-and pp-scattering. What is more, in the available energy range these corrections give an essential contribution.

Thus, a consistent analysis of power corrections in QCD seems to be a highly nontrivial but maybe not a hopeless task.

After this paper was essentially completed, we received a preprint by Politzer /29/, where the power corrections are analysed using the methods similar to ours. There is no surprise that the analyses are similar, because both are based on the classic work /24/. However, there exists also a conceptual difference between the two approaches. Our approach is based directly on the analysis of the corresponding amplitude $T(Q^2,p^2)$ in the large-Q^2 region whereas Politzer's approach is based on the analysis of small-p^2 behaviour of $T(Q^2,p^2)$ (i.e. on the analysis of mass singularities). Both the approaches are (almost) equivalent if one analyses logarithmic $\ln(Q^2/p^2)$-corrections. However, the power corrections $O(Q^{-2N})$ in mass-singularity analysis correspond to contributions $(p^2)^N(\ln p^2)^k$, that simply vanish for $p^2 \to 0$. So, we are very sceptic about the main idea of ref. /29/ that a complete analysis of power corrections may be performed within the mass-singularity approach. Incompleteness of this approach reveals itself in the fact that the soft exchanges (configuration 2c) are completely ignored in ref. /29/. Another disadvantage of the mass-singularity approach is that it incorporates perturbation theory just in the region $k^2 \to 0$, where one should expect in QCD large nonperturbative effects. In particular, within the mass-singularity approach, it seems impossible to understand the origin and structure of power corrections to the "$e^+e^- \to$ hadrons" process, since the related amplitude $T(Q^2)$ does not depend (in a massless theory) on small momentum variables like p^2. It should be emphasized that the analysis of these effects given in ref. /26/ is the only serious analysis of power corrections in QCD and it is based (not by chance) on the operator product expansion, i.e. just on the analysis of the large-Q^2 behaviour of the relevant amplitude.

4. PION FORM FACTOR AT MODERATELY LARGE Q^2

As an example illustrating the importance of a detailed study of higher order and higher twist effects, let us consider the behaviour of pion electromagnetic form factor $F_\pi(Q)$ for moderately large Q^2.

During the last 3 years a definite progress has been made in understanding of the asymptotical $Q^2 \to \infty$ behaviour of $F_\pi(Q)$ /6-8,30-33/ in the QCD framework. The main result here is the proof that in a region where the power corrections may be neglected the form factor may be written in a factorized form [6-8]:

$$F_\pi(Q) = \frac{1}{Q^2}\int_0^1 dx \int_0^1 dy\, \phi^*(y, \mu^2, \mu_R^2, \alpha_S(\mu_R)) \times$$
$$\times E(Q^2/\mu^2, \mu_R^2/\mu^2, x, y, \alpha_S(\mu_R))\,\phi(x, \mu^2, \mu_R^2, \alpha_S(\mu_R)), \tag{4.1}$$

where $\phi(x)$ is the wave function describing the splitting of the pion into a $q\bar{q}$ -state and E/Q^2 is the amplitude of the short-distance subprocess $\gamma^* q\bar{q} \to q'\bar{q}'$. Note, that, in general, the normalization parameter μ_R of the R-operation for ordinary UV -divergences may differ from the splitting parameter μ that separates small and large momenta. The latter may be treated also as the normalization parameter for composite operators. The moments of the function $\phi(x)$ are equal to the reduced matrix elements of the twist-2 operators $\bar{\psi}\gamma_5\gamma_\mu D^n\psi$. . The SD-amplitude, as usual, is a series expansion over $\alpha_S(\mu_R)$ (see fig.3):

$$E(Q^2/\mu^2, \mu_R^2/\mu^2, x, y, \alpha_S(\mu_R)) =$$
$$= \frac{2\pi\alpha_S(\mu_R)C_F}{N_c} \cdot \frac{xQ^2}{(xQ^2)(xyQ^2)}\{1 + O(\alpha_s)\} \tag{4.2}$$

where $C_F = 4/3$, $N_c = 3$. . The factor xQ^2 in the numerator of of eq. (4.2) is due to the trace over Dirac γ-matrices. Note that $E(x, y)$ is rather singular for $x, y \sim 0$. Hence, the main contribution is given by integration over small x, y. In the next order the most singular terms are $\frac{1}{xy}\ln(xyQ^2/\mu_R^2)$ and $\frac{1}{xy}(\ln x)(\ln y)$, , $(\ln Q^2/\mu^2)\cdot(\ln xy)/(xy)$. The first term is given by divergent parts (notice μ_R) and the others by convergent ones. Thus, to minimize the α_s -corrections we must take $\mu^2 \simeq \bar{x}Q^2 = \bar{y}Q^2$ and $\mu_R^2 \sim \bar{x}\bar{y}Q^2$, where $\bar{x}(\bar{y})$ is the average value of x (or y):

$$\ln\bar{x} = \langle \ln x \rangle = \left(\int_0^1 \ln x \frac{\phi(x)}{x}dx\right)\left(\int_0^1 \frac{\phi(x)}{x}dx\right)^{-1} . \tag{4.3}$$

If $\phi(x) \sim \delta(x-\frac{1}{2})$ (noninteracting quarks), then $\bar{x} = \frac{1}{2}$. However, for very broad functions, e.g. for $\phi(x) \sim [x(1-x)]^R$ with $R \ll 1$ we have a very small value $\bar{x} \sim \exp(-1/R)$.

In the problem investigated we encounter just the same mass scales (m_π, m_q and $M \equiv 1/R_{conf}$) as in deep inelastic scattering. So, it seems natural to expect that eqs. (4.1), (4.2), for the appropriately chosen wave function $\phi(x)$ must provide a good approximation for F_π in the region $Q \gtrsim 1$ GeV. The wave functions $\phi(x,\mu^2)$ are in general unknown. In perturbative QCD one may calculate only their evolution with growing μ^2. In particular $\phi(x,\mu^2) \to 6f_\pi x(1-x)$ as $\mu^2 \to \infty$ /34/, where f_π = 133 MeV. Presence of the f_π-factor is due to the normalization condition

$$\int_0^1 \phi(x,\mu^2)\,dx = f_\pi . \qquad (4.4)$$

It is clear, however, that for $\mu \lesssim 1$ GeV the wave function $\phi(x,\mu^2)$ may strongly differ from its limiting form. For noninteracting particles $\phi(x) \sim \delta(x-1/2)$. . When the interactions are switched on, the wave function broadens. The width Γ of $\phi(x)$ may be estimated as $\Gamma \sim (E_{int}/m_q)^2$. Hence, for heavy mesons (e.g. for J/ψ or Y-particles) $\phi(x)$ is rather narrow, since $E_{int} \simeq M \simeq 300 \div 500$ MeV and $m_q \gtrsim 1$ GeV. On the other hand, for pions the wave function must be very broad, because $m_u \simeq 4$ MeV, $m_d \simeq 7$ MeV /26/, i.e. pion must be treated as an ultrarelativistic system. To obtain a more accurate estimate of the width Γ for such a system we assume that the (soft) Bethe-Salpeter wave function $\chi_P(k_1,k_2)$ is exponentially damped for moderately large, spacelike k_i^2 $(i=1,2)$:

$$\chi_P(k_1,k_2) \sim \frac{1}{k_1^2 k_2^2}\exp(k_i^2/M^2); \quad -k_i^2 \gtrsim M^2 . \qquad (4.5)$$

The exponential damping is suggested by the observed spectra of particles produced in high-energy hadronic reactions. For our wave function $\phi(x)$ (which may be obtained from $\chi_P(k_1,k_2)$ by integration over $k_- \equiv k_0 - k_3$ and k_+ /35,36/) the coice (4.5) gives

$$\phi(x,\mu^2 \simeq M^2) \simeq f_\pi \cdot \begin{cases} \exp[-m_u^2/xM^2]; & x \ll 1 \\ \exp\{-m_d^2/((1-x)M^2)\}; & (1-x) \ll 1 . \end{cases} \qquad (4.6)$$

Thus, $\phi(x)$ is very close to f_π everywhere outside the regions $0 \le x \le m_u^2/M^2 \sim 10^{-4}$; $0 \le 1-x \le m_d^2/M^2 \sim 10^{-3}$. In these regions $\phi(x)$ vanishes rapidly. Note that for such a wave function $\bar{x} \sim 10^{-3} \div 10^{-4}$, i.e. the main contribution into F_π is given by the region where the gluon has a catastrophically small off-shellness $\bar{x}^2 Q^2$, which for $Q^2 \lesssim 100$ GeV^2 is much smaller that the value $|k^2| \sim 0.1 \div 0.3$ GeV^2 where the confinement effects must be taken into account. Thus, for a broad wave function short distances do not contribute, in fact, and eq. (4.2) is unreliable. In particular, it is not justifiable to neglect the power corrections that may really have a form $(M^2/\langle k^2\rangle) \sim (M^2/\bar{x}Q^2)$ rather than simply M^2/Q^2. We assume that the confinement effects eventually remove the infrared singularity from the "hard" quark and gluon propagators $1/xQ^2$ and $1/xyQ^2$. . So, we change $1/xyQ^2 \to 1/(xyQ^2+2M^2) \sim 1/(xyQ^2 + \langle (k_\perp - k_\perp')^2\rangle)$ and $1/xQ^2 \to 1/(xQ^2+M^2) \sim 1/(xQ^2 + \langle k_\perp^2\rangle)$. The connection between M^2 and $\langle k_\perp^2\rangle$ is a pure mnemonics and must not be understood too literally. However, as an order-of-magnitude estimate this connection must be true. So, we should expect that $M^2 \simeq 0.1 \div 0.3$ GeV^2.

As a result, we have in place of eqs. (4.1), (4.2)

$$F_\pi^{AA}(Q) = \frac{2\pi C_F}{N_c} \int_0^1 dx\, dy\, \phi(x)\,\phi(y) \frac{\alpha_s(\mu_R^2)(xQ^2)}{(xQ^2+M^2)(xyQ^2+2M^2)} \{1+O(\alpha_s)\}, \quad (4.7)$$

where AA stands for projection onto the "axial" operators $\bar{\psi}\gamma_5\gamma_\mu D^n \psi$. .

From eq. (4.7) it is clear that for not too large Q^2 the contribution of the soft region $x \sim 0$ is damped at $x \sim M^2/Q^2$, whereas the wave function damps only the region $x \le m_q^2/M^2$. Thus, up to $Q^2 \sim (M^2/m_q)^2 \gtrsim 10^3$ GeV^2 the magnitude of the pion form factor is determined by the value of M^2, i.e. by the confinement radius.

The main contribution into the integral in eq. (4.7) is given by the region $xyQ^2 \sim 2M^2$. . To minimize the α_s-correction we should take μ_R^2 equal to the average off-shellness of the gluon: $\mu = \bar{x}\bar{y}Q^2 + 2M^2 \simeq 4M^2$.

$$\alpha_s(\mu_R^2) \to \alpha_s(4M^2) = \frac{4\pi}{9\ln(4M^2/\Lambda^2_{PH})}. \quad (4.8)$$

It should be realized that eqs. (4.7), (4.8) are meaningful only if $\alpha_s/\pi \ll 1$, i.e. for $4M^2/\Lambda^2 \geq 50$ (in this case $\alpha_s(4M^2)/\pi \simeq 0.1$). If $M^2 \sim 0.2$ GeV^2, then eqs. (4.7), (4.8) may work only for $\Lambda_{PH} \lesssim 100$ MeV. Note that this is just the value preferred in ref. /26/. We emphasize that the authors of ref. /26/ just have taken into account power corrections. In standard analyses of deep inelastic data (neglecting higher-twist effects) larger values of Λ are usually obtained. It is known, however, that if one includes in a phenomenological analysis the effects of higher twists, then it is possible to describe the data using an arbitrarily small Λ /37/.

Apart from power corrections related to the primordial transverse momentum of quarks (which correspond to operators involving the curl $G_{\mu\nu}$), there exist also power corrections due to twist-3 operators $\bar\psi\gamma_5 D^n\psi$ and $\bar\psi\gamma_5\sigma_{\mu\nu} D^n\psi$. In the large-$Q^2$ limit their contribution has an additional factor λ^2/Q^2 compared to the contribution of the twist-2 operators $\bar\psi\gamma_5\gamma_\mu D^n\psi$. Note, however, that λ is anomalously large

$$\langle 0|\bar d\gamma_5 u|P\rangle = if_\pi\lambda = if_\pi \frac{m_\pi^2}{m_u+m_d} \simeq if_\pi\cdot(1.8\ \mathrm{GeV}) \qquad (4.9)$$

i.e. for $Q^2 \lesssim 6$ GeV^2 these operators cannot be neglected. For the pseudoscalar $\bar\psi\gamma_5 D^n\psi$ -operator we have

$$F_\pi^{(PP)}(Q) = \frac{4\pi\alpha_s(4M^2)}{N_c} C_F \int_0^1 \phi_P(x)\,\phi_P(y)\,dx\,dy\ \times$$
$$\times \frac{1-x}{(xQ^2+M^2)(xyQ^2+2M^2)}\{1+O(\alpha_s)\}, \qquad (4.10)$$

where $\phi_P(x) = \lambda\phi(x) \simeq \lambda f_\pi$.

It should be noted that for $M^2=0$ the amplitude $E^{(PP)}(x,y)$ is as singular at $x \sim 0$ as $1/x^2$. . As a result, integration over x gives an additional factor Q^2/M^2 that compensates the absence of the Q^2-factor in the numerator of eq. (4.10). In other words, in the region $Q^2 < (M^2/m_q)^2$ the contribution $F_\pi^{(PP)}(Q)$ has $1/Q^2$ -behaviour rather than $1/Q^4$. . Moreover, $F_\pi^{(PP)}$ has an additional large factor $(\lambda/M)^2 \geq 10$ compared to $F_\pi^{(AA)}$. The same factor has also the $F_\pi^{(TP)}$-contribution ((TP) stands for the $\bar\psi\gamma_5\sigma_{\mu\nu}\psi \otimes \bar\psi\gamma_5\psi$ -projection):

$$F_\pi^{(TP)} = \frac{2\pi\alpha_s(4M^2)C_F}{N_c}\int_0^1 \frac{\phi_T(x)\,\phi_P(y)\,dx\,dy}{(xyQ^2+2M^2)(xQ^2+M^2)}\times$$

$$\times\left(3-\frac{(1+x)Q^2}{xQ^2+M^2}-\frac{y(1+x)Q^2}{xyQ^2+2M^2}\right)\{1+O(\alpha_s)\}. \qquad (4.11)$$

Note that for $M^2=0$ the amplitude E^{TP} has the $1/x^3$-singularity. However, using the equations of motion it is possible to show that the function $\phi_T(x)$ has an additional x-factor for $x\sim 0$. In particular, if $\phi_P(x) \simeq \lambda f_\pi$, , then $\phi_T(x) \simeq \lambda f_\pi x(1-x)$. As a result, $F_\pi^{(TP)} \sim 1/Q^2$ in the region $Q^2 \lesssim (M^2/m_q)^2$. This contribution is negative and small for $Q^2 \geq 4$ GeV^2.

The curves for F_π given by the sum of eqs. (4.7),(4.10), (4.11) have a right form (fig.4) and for $M^2 \simeq 0.1$-0.2 GeV^2, $\Lambda=100$ MeV they are close to existing experimental data.

It is easy to realize that since the passive quark in our case has a very small fraction of the pion momentum $x\sim M^2/Q^2$ ("wee" parton), we deal really with the mechanism proposed by Feynman /38/ to explain the power-law fall-off of hadronic form factors. Thus, we might have considered the diagram shown in fig.5 and write $F_\pi(Q)$ in the standard bound state formalism /35,36/

$$F_\pi(Q) \sim \int_0^1 \frac{dx}{x(1-x)}\int d^2k_\perp\,\phi(1-x,k_\perp)\times$$

$$\times\,\phi(1-x,\ k_\perp+xq). \qquad (4.12)$$

Note that according to the Bethe-Salpeter equation the diagrams shown in figs.3 and 5 are equivalent up to $O(\alpha_s)$-corrections.

If the function $\phi(x,k_\perp)$ is that given by eq. (4.6) then, performing $k_\perp$-integration in eq. (4.12) we obtain

$$F_\pi(Q) \sim \int_0^1 dx\,\exp\left(-\frac{xQ^2}{2M^2(1-x)} - 2\left(\frac{m_1^2}{xM^2}+\frac{m_2^2}{(1-x)M^2}\right)\right) \qquad (4.13)$$

whence it follows that for $Q^2 \lesssim M^4/m_q^2$ the main contribution is given by the region $x\sim M^2/Q^2$. If the function $\phi(x)$ (i.e. $\phi(x,k_\perp)$ integrated over $k_\perp$) behaves like x^R for $x\sim 0$, then $F_\pi(Q)\sim (Q^2)^{-1-R}$. Our choice (4.5) corresponds to $R=0$ and as a result $F_\pi(Q)\sim 1/Q^2$. . If one assumes that

$$\chi_P(k_1,\dots,k_n) \sim \exp\{\Sigma k_i^2/M^2\} \tag{4.14}$$

for a system composed by n valence quarks, then $\phi(x) \sim x^{n-2}$ for $x \sim 0$ and thus

$$F_{(n)}(Q) \sim (Q^2)^{1-n} \quad . \tag{4.15}$$

This relation corresponds formally to the well-known quark counting rule (QCR) /39,40/. In our case, however, this rule has nothing in common with short distances and scale invariance. The short-distance mechanism proposed in ref./40/ to explain QCR, according to our estimate, works only for $Q^2 \gtrsim 10^3$ GeV^2. In an intermediate region one must take into account the fact that the contribution of the Feynman mechanism is damped by the Sudakov form factor of the active quark. Thus, one must multiply the curves shown in fig.4 by the Sudakov QCD form factor /41/

$$S(Q^2,M^2) = \exp\{-\frac{2C_F}{b_0}[(\ln\frac{Q^2}{\Lambda^2}-\frac{3}{2})(\ln\frac{\ln Q^2/\Lambda^2}{\ln M^2/\Lambda^2}) - \ln\frac{Q^2}{M^2}]\}. \tag{4.16}$$

For Λ =100 MeV, M^2= 0.22 GeV^2 this gives the curve shown in fig.6. In the region $Q^2 = 1 \div 4$ GeV^2 there is good agreement with experimental data /42/. Decrease of $Q^2F_\pi(Q)$ for $Q^2 \gtrsim 10$ GeV^2 is due to the Sudakov form factor. In the region $Q^2 \gtrsim 100$ GeV^2 the short-distance regime begins to work. In this region the average off-shellness of the gluon increases, μ^2 grows, and as a result the wave function becomes narrower:

$$\phi(x,\mu^2) \simeq (x(1-x))^{\frac{2C_F}{b_0}[\ln\ln\frac{\mu^2}{\Lambda^2} - \ln\ln\frac{2M^2}{\Lambda^2}]} \tag{4.17}$$

and this, in turn, damps the contribution of the Feynman regime. For $Q^2 \gtrsim 10^3$ GeV^2 one may neglect the $F_\pi^{(PP)}$ contribution and use eqs. (4.1), (4.2) with the wave functions $\phi(x) \sim (x(1-x))^{0.2\pm0.3}$. The asymptotic formula $\phi(x) \simeq 6f_\pi x(1-x)$ may be used only for $Q^2 > 10^{20}$ GeV.

Thus, in the region $Q^2 \gtrsim 10^3$GeV^2 begins the asymptopia, and $F_\pi(Q)$ is again given by the quark counting rules. In this case they are due to the short-distance scale invariance, as expected in the pioneering works by Matveev, Muradyan, and Tavkhelidze /39/ and Brodsky and Farrar /40/.

Quark counting rules for ultrarelativistic systems were considered first by Terentyev [43]. However, he used constituent quark masses, $m_q \sim M$. In this case the range of applicability of our analysis ($M^2 \ll Q^2 \ll M^4/m_q^2$) is zero. We insist on using the current quark masses ($m_q \simeq$ 4-7 MeV) in the wave function (4.6). Note that the eventual IR cut-off in our analysis is of an order of M (i.e. of an order of constituent quark mass) in accordance with the common wisdom.

The last but not least observation is that the magnetic proton form factor in QCD is negative for narrow wave functions like $\phi(x_1,x_2,x_3) \sim \Pi\delta(x_i - 1/3)$ [44] and positive for the broad ones, e.g. for that given by eq. (4.14).

Summarizing this section, we may conclude that although our analysis is semi-phenomenological and some assumptions are very crude, it is clear, nevertheless, that a consistent treatment of power "corrections" (in fact, they give the main effect) is the main problem for perturbative QCD of hard elastic processes in the now (and, perhaps, forever) available energy range.

5. SOFT PROCESSES AND PERTURBATIVE QCD

The main fraction of the total cross-section at high energies is due to the processes with small transverse momentum (soft processes). These are the elastic and quasielastic processes in the diffraction region $|t| \ll 1$ (GeV/c)^2 and multiple production processes with low p_T: $p_T \lesssim 1$ GeV/c. The conventional phenomenology of processes in this region is the Regge-Mueller picture. Till now, however, it was not clear whether perturbative QCD can give any information about these processes. Below we discuss this problem analyzing a process $12 \to 1'2'$ in the region $s \gg |t|, m^2_{hadr}$. We will assume also that the t-channel is flavour nonsinglet. This assumption simplifies the analysis.

For scalar gluons, i.e. in a Yukawa-type field theory soft processes have been studied 10 years ago [45]. It was shown that summation of all logarithmic terms $(\log s)^N$ coming from the short-distance integration (regime 1), see sect.2) gives the following representation

$$f^{\pm}(j,t) = C^{\pm}(j,t)[1 - B^{\pm}(j,t)\, v^{\pm}(j)]^{-1} v^{\pm}(j)\, C^{\pm}(j,t) \tag{5.1.}$$

for the Mellin transform of the scattering amplitude $F^{\pm}(s,t)$:

$$F^{\pm}(s,t) = \frac{1}{2i}\int_{-i\infty}^{i\infty} \frac{s^{j}(e^{i\pi j} \pm 1)}{\Gamma(j+1)\sin\pi j} f^{\pm}(j,t)\,dj \,, \tag{5.2}$$

where ± stands for signature, C, v and B are some matrices (to be discussed below); e.g., $B = B_{ab}$, and $a, b = S,V,T,A,P$ are structures appearing in the Fierz identity applied to factorize the spinor structure of the relevant contributions.

According to this representation (eq. (5.1)) the Mellin transform $f^{\pm}(j,t)$ possesses moving (t-dependent) Regge poles due to zeros of $\mathrm{Det}[1-B(j,t)\,v(j)]$. . It has also fixed (t-independent) singularities in the complex j-plane accumulated in the function v. The type of fixed singularities depends on the ultraviolet asympltotics of the effective coupling constant. In particular, in a fixed point theory (where $\bar{g}(\mu) \to g_0$ as $\mu \to \infty$) the function $v(j)$ has square-root branch points, the position of which depends on g_0, i.e. on the asymptotical value of $\bar{g}(\mu)$. On the other hand, in an asymptotically free field theory $v(j)$ has the infinite number of poles condensing to $j = 0$.

Let us now discuss briefly the derivation of eq. (5.1). Consider a particular diagram of a binary process $12 \to 1'2'$ in the region $S \gg |t|, m^2_{hadr}$. The Mellin transform of its contribution has the following structure in the α-representation (eq. (3.2)):

$$f^{\pm}(j,t) \sim \int \prod_{\sigma} d\alpha_{\sigma} D^{-2}(\alpha)\, d(j,t,\alpha_{\sigma})\, |A(\alpha)|^{j} \times$$
$$\times [\,\theta(A) \pm \theta(-A)]\, \exp[\,iJ(\alpha,t,m^2)], \tag{5.3}$$

where $g(j,t,\alpha)$ is a polynomial in j (it corresponds to the function G in eq. (3.2)) and A is the coefficient in front of the large variable $S = s - u$. . As is well-known, the asymptotical behaviour of $F(S,t)$ for large S is determined by the rightmost singularities of its Mellin transform $f(j,t)$. These are poles j^{-N} generated by integrations corresponding to the regimes 1)-3) discussed in sect.3. However, using eq. (3.3) it can be shown that in Yukawa theory the IR-regime gives only nonleading poles at $j = -1, -2,$. Furthermore, the pinch regime contributes only to the negative-signature amplitude $F^-(S,t)$. . Thus, for $F^+(S,t)$ it is sufficient to consider only the poles due to the short-distance regime.

According to eq. (3.3), the leading poles (at $j = 0$) are due to the subgraphs V_i with 4 external lines. We recall that V_j should possess the property that if it is contracted into point, the diagram becomes S-independent (i.e. V_j must be an S-subgraph). The most general configuration is shown in fig.7. Note that in general the SD-subgraphs V_j may be 2-particle-reducible, i.e. they may contain smaller S-subgraphs with 4 external lines and the total singularity due to the SD-regime of V_j may be a multiple pole j^{-N_j}. It makes sense to treat a particular diagram as a ladder composed by 2-particle irreducible blocks k_j. Then the maximal value of N_j is determined by the number of k_j's inside V_j (and also by the number of the UV-divergent subgraphs inside V_j). The contribution $f_V(j)$ of each S-subgraph V may be represented as a sum of two terms $f_V = f_V^{pole} + f_V^{reg}$. The first term (f_V^{pole}) is due to integration over the region $\Sigma \alpha_\sigma = \lambda_V < 1/\mu^2$ and the second one is due to that over the region $\lambda_V > 1/\mu^2$. This procedure corresponds to a subtraction of the pole due to the small - λ_V integration. However, if V is composed by two or more k_j's then f_V^{reg} may also possess the poles at $j = 0$ due to the SD-integration for a smaller subgraph $V_1 \subset V$. Thus, one must represent f_V^{reg} as $f_{V\,V_1}^{reg\,pole} + f_{V\,V_1}^{reg\,reg}$, and so on. An example of such a decomposition is shown in fig.8, where the pole part are circled by the broken line and the regular ones by the slashed lines. Note that fig.7 is really a decomposition of the whole diagram. Summing over all diagrams we obtain (in the coordinate representation):

$$(2\pi)^4 \delta^4(p_1 + p_2 - p_1' - p_2')\, F(S,t) =$$

$$= \sum_{n=1}^{\infty} \int C(p_1, p_1'; x_1, y_1) \prod_{i=1}^{n-1} [d\chi]_i \; v(x_i, y_i; z_i, w_i) \times \tag{5.4}$$

$$\times B(z_i, w_i; x_{i+1}, y_{i+1}) \, v(x_n, y_n; z_n, w_n)\, C(p_2, p_2'; z_n, w_n)\, [d\chi]_n ,$$

where $[d\chi]_i = dx_i dy_i dz_i dw_i$ (see fig.7), and the functions C, v, B are given by the following matrix elements

$$C_{(a)}(p_1, p_1'; x_1, y_1) = R_{\mu^2} \langle p_1' | S^+ T(:\bar\psi(x_1)\Gamma_a \psi(y_1):S) | p_1 \rangle , \tag{5.5}$$

$$B_{(ab)}(z_i, w_i; x_{i+1}, y_{i+1}) = R_{\mu^2} \langle 0 | S^+ T(:\bar\psi(z_i)\Gamma_a \psi(w_i): \times \tag{5.6}$$

$$\times :\bar\psi(x_{i+1})\Gamma_b \psi(y_{i+1}) :S) | 0 \rangle ,$$

$$v(x_i, y_i ; z_i, w_i) = P_{\mu^2} \langle 0 | S^+ T(:\bar\eta(x_i)\Gamma_a \eta(y_i): \times$$
$$\times :\bar\eta(z_i)\Gamma_b \eta(w_i):S)|0\rangle , \tag{5.7}$$

$$\Gamma_a , \Gamma_b = 1, \gamma_\mu, \sigma_{\mu\nu}, \gamma_5, \gamma_5\gamma_\mu . \tag{5.8}$$

Here ,:: denotes the usual normal product; η , $\bar\eta$ are the spinor currents (e.g. $\bar\eta = S^+(\delta S/\delta\psi)$); S is the S-matrix; P_{μ^2} means that $\lambda_V < 1/\mu^2$ for each diagram V contributing to v, and R_{μ^2} means that $\lambda_V > 1/\mu^2$ for all leading S-subgraphs having lines related to B or C .

If we expand B or C into the Taylor series over $\xi = x - y$, $\zeta = z - w$,

$$C_{(a)}(p_1, p_1' ; x_1, y_1) = \tag{5.9}$$
$$= \exp(irX_1) R_{\mu^2} \sum_{j=0}^{\infty} (1/j!) \langle p_1' | S^+ T(O_{a\mu_1 \ldots \mu_j} S)|p_1\rangle \xi_1^{\mu_1} \ldots \xi_1^{\mu_j},$$

$$B_{(ab)}(z_i, w_i ; x_{i+1}, y_{i+1}) = \tag{5.10}$$
$$= R_{\mu^2} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} (1/j!k!) \xi_i^{\nu_1} \ldots \xi_i^{\nu_j} \zeta_{i+1}^{\mu_1} \ldots \zeta_{i+1}^{\mu_k} \times$$
$$\times \langle 0 | S^+ T[O_{a\nu_1 \ldots \nu_j}(Z_i) O_{b\mu_1 \ldots \mu_k}(X_{i+1}) S]|0\rangle,$$

(where $r = p_1 - p_1' = p_2' - p_2$, $X_i = \frac{x_i + y_i}{2}$, $Z_i = \frac{z_i + w_i}{2}$), then

R_{μ^2} provides the renormalization recipe for the resulting composite operators $O_{a\nu_1 \ldots \nu_j} = \bar\psi \Gamma_a \partial_{\nu_1} \ldots \partial_{\nu_j} \psi$. It should be emphasized, however, that R_{μ^2} in addition, subtracts from B , C also the contributions due to integration over small λ_v -parameters for S-subgraphs that do not contain the vertices $\bar\psi\Gamma\partial^n\psi$ corresponding to composite operators.

Just like for hard processes, only the lowest-twist operators (i.e. the traceless-symmetric part $O_{\{a\mu_1 \ldots \mu_j\}}$ of O) give the leading contributions. Using the translation invariance of the functions (5.5)-(5.7), integrating over ξ_i, ζ_i, X_i and Z_i and summing over n (n is the number of SD-integrations), we obtain

$$F(S,t) = \sum_{j=0}^{\infty} \frac{|S|^j}{j!} C(j,t) [1 - v(j) B(j,t)]^{-1} v(j) \cdot C(j,t), \tag{5.11}$$

where $t = r^2$, $S = 2(PQ)$, $P = p_1 + p_1'$, $Q = p_2 + p_2'$ . . The functions $B(j,t)$, , $C(j,t)$, $v(j)$ are given by

$$R_{\mu^2} \langle p_1' | S^+ T(O_{\{a\mu_1 \dots \mu_j\}}(0)\, S) | p_1 \rangle = C(j,t) \{ P_a P_{\mu_1} \dots P_{\mu_j} \} + O(r_\mu),$$

$$\int dX\, e^{irX} R_{\mu^2} \langle 0 | S^+ T(O_{\{a\nu_1 \dots \nu_j\}}(X)\, O^{\{b\mu_1 \dots \mu_k\}}(0) S) | 0 \rangle = \quad (5.12)$$

$$= j!\, \delta_{jk}\, \delta^{\{b}_{\{a}\, \delta^{\mu_1}_{\nu_1} \dots \delta^{\mu_k\}}_{\nu_j\}} B(j,t) + O(r_\nu), \quad (5.13)$$

$$\int \tilde{v}(X-Z, \xi, \zeta) \{ \xi^{\nu_1} \dots \xi^{\nu_j} \} \{ \zeta_{\mu_1} \dots \zeta_{\mu_k} \} d(X-Z)\, d\xi\, d\zeta\, \exp[ir(X-Z)] \Big|_{r^2=0} =$$

$$= j!\, v(j)\, \delta_{jk}\, \delta^{\{\nu_1}_{\{\mu_1} \dots \delta^{\nu_j\}}_{\mu_k\}} + O(r_\nu), \quad (5.14)$$

where $\tilde{v}(X-Z, \xi, \zeta)$ is defined by

$$v(x,y;z,w) = v(X + \frac{\xi}{2}, X - \frac{\xi}{2}; Z + \frac{\zeta}{2}, Z - \frac{\zeta}{2}) = \tilde{v}(X-Z, \xi, \zeta) \quad (5.15)$$

and $O(r_\nu)$ denotes terms containing r_ν . These give zero contribution into eq. (5.11) because $(Pr) = (Qr) = 0$. Note that eqs. (5.1), (5.2) give just a Mellin transformed version of eq. (5.11). To construct the functions B, C, v one must apply first the R-operation for ordinary divergent subgraphs (this procedure is characterized by the renormalization parameter μ_R) and then the operations P_{μ^2} and $R_{\mu^2} = 1 - P_{\mu^2}$ that separate small and large λ-parameters (this corresponds to splitting of the mass-logarithms $\ln S/p^2$ into "short-distance" ($\ln S/\mu^2$) and "long-distance" ($\ln \mu^2/p^2$) parts). The whole amplitude $F(S,t)$, , of course, must be independent both of μ and μ_R . . The μ_R-independence of F leads to a standard renormalization-group equation

$$(\mu_R\, \partial/\partial\mu_R + \beta(g)\, \partial/\partial g - 4\gamma_\psi)\, v(j, \mu_R, g, \mu) = 0. \quad (5.16)$$

The subtraction procedure R_{μ^2} for the problem considered is more complicated than that for hard processes. This is mainly due to the fact that, for soft processes, we deal in general with the configuration (fig.7) that has several nonoverlapping SD-subgraphs $v_1, \ldots, v_n$. We recall that for hard processes we always have a configuration with the only SD-subgraph (see, e.g., fig. 2a). Straighforward analysis gives the following equation for C :

$$\mu \frac{d}{d\mu} C \equiv C' = \gamma(j)\, C + (1 - Bv)^{-1} (1 - Bv)', \tag{5.17}$$

where all functions entering into eq. (5.17): B, C and v depend on μ. . The second term in eq. (5.17) is just due to the additional subtraction discussed above. The function $\gamma(j)$ is the ordinary anomalous dimension of the composite operator $O_{a\nu_1 \ldots \nu_j}$ . . In our case it is convenient to single out from γ the terms singular at $j = 0$. . It can be shown that these terms are proportional to v, i.e. that $\gamma(j) = c(j) + b(j) v(j)$ where $c(j)$ is regular at $j = 0$.

The equation similar to eq. (5.17) can be obtained also for $(1 - Bv)$:

$$-(1 - Bv)' = [2\gamma(j) + bv(1 - Bv) + Bv'][1 - Bv]. \tag{5.18}$$

Requiring that $\Phi^{pole}(j)$ - the sum of the leading poles, does not depend on μ, we obtain that $(\Phi^{pole}(j))'$ must - gular at $j = 0$:

$$\mu \frac{d}{d\mu} \Phi^{pole}(j) = r(j), \qquad 9)$$

where $r(j)$ is some function regular at $j = 0$. . Using eqs. (5.17)-(5.19) we obtain the equation for v :

$$v' + 2\gamma v + bv^2 = -r. \tag{5.20}$$

The meaning of $r(j)$ becomes clear, it is just the residue of $v(j)$ at $j = 0$, because $v' \sim jv$.

It should be remarked that eq. (5.20) differs from its analogue given in ref. [45] because of another choice of the R_{μ^2} -operation.

Using eq. (5.20) one can sum up all the poles at $j = 0$ due to the SD-regime of all possible S-subgraphs (i.e. to sum $\log^N(S/p^2)$ contribution). The solution of eq. (5.19) has square root branch points in the complex j-plane [45] (see also [46]).

However, $v(j)$ has also poles V_j due to divergent subgraphs. These poles (i.e. $\log^N(S/\mu_R^2)$ -contributions) are summed by eq. (5.16). If we take $\mu = \mu_R$ and combine eqs. (5.16), (5.20), we obtain

$$\beta(g)\,\frac{\partial v}{\partial g} = (j - 2\gamma - 4\gamma_\psi)v - bv^2 - r\,, \tag{5.21}$$

where v, γ and r depend on j, and $(-j)$ is the canonical dimension of v. In the lowest order of PT $b=1$, $r \sim \gamma \sim \gamma_\psi \sim g^2$, $\beta(g) \sim g^3$ and the solution of eq. (5.21) has condensing poles at $j=0$ /45,46/.

Summarizing the preceding discussion, we conclude that if one assumes that the asymptotical behaviour of $F(S,t)$ is given by the sum of the leading terms of all contributing Feynman diagrams, then $F(S,t)$ has for large S a Regge-type behaviour $F(S,t) \sim C^2(t)\,S^{\alpha(t)}$ since its Mellin transform has just a t-dependent singularity at $j=\alpha(t)$. . To find the function $\alpha(t)$ explicitly, one must solve the equation

$$\mathrm{Det}[1 - B(j,t,\mu_R,g,\mu,m_q)\,v(j,\mu,\mu_R,g)] = 0. \tag{5.22}$$

It can be shown that eq. (5.18) guarantees that $\alpha(t)$ does not depend on μ and μ_R:

$$\alpha(t) = \phi\left(\frac{m_q^2}{t}, \frac{t}{\mu^2}, \bar g(\mu)\right) = \phi\left(\frac{m_q^2}{t}, 1, \bar g(t)\right)\,. \tag{5.23}$$

Hence, one may try to calculate the Regge trajectories in the region where $\bar g(t)$ is small, e.g. in QED, where $\alpha = 1/137$, or in QCD for sufficiently large t. However, there arises a question whether eq. (5.11) is valid in vector theories.

In QCD one encounters the complication discussed in sect.3. First a SD-subgraph V_i may have an arbitrary number of external lines. But if the t-channel is colour singlet, then the only change is

$$\bar\psi(x)\,\Gamma\,\psi(y) \to \bar\psi(x)\,\Gamma\,P\exp\left(ig\int_y^x \hat A_\mu(z)\,dz^\mu\right)\psi(y) \tag{5.24}$$

for all bilocal operators entering into B- and C-functions. For local operators this corresponds to the change $\partial_\mu \to D_\mu = \partial_\mu - ig\hat A_\mu$. The second complication is due to the IR-regime (soft exchanges, see fig.9). However, just like for hard processes, if the j-channel is colour-singlet, then the sum of all soft exchanges give only power corrections in each order of perturbation theory. Thus, all terms responsible for the leading power contribution have the structure of fig.7 and as a result, we get eq. (5.11). In other words, if we sum the leading j-singularities of all relevant Feynman diagrams in QCD, we obtain a Regge-type picture for binary processes and, hence, a multiperipheral picture for the multiple production at low p_T.

We recall, however, that we have discussed above only the flavour-nonsinglet, positive-signature amplitude F^{+}_{NS}. For F^{-}_{NS} the pinch regime (see sect.3) also gives leading j-poles for nonplanar diagrams. It is known, however, that the nonplanar diagrams have an additional colour factor $(1/N_c)^2 = (1/3)^2$. This suggests that the pinch contributions in QCD must be suppressed. There exists also an experimental evidence in favour of this suppression: the well-known signature degeneracy of the Regge trajectories.

For flavour-singlet amplitudes F_S ("vacuum" exchange) the poles generated by the pinch regime are at $j=1$ rather than at $j=0$ due to the 2-gluon intermediate states and after summation one obtains for F^{+}_{S} a square-root branch point at $j = 1 + O(g^2)$. . This suggests that the pinch regime plays a highly important role in formation of the Pomeron singularity.

The most intriguing possibility is to utilize the asymptotic freedom of QCD for a calculation of the Regge-trajectories and of the resonance masses in the region of large t (see eq. (5.23)). Note, however, that the function $B(t)$ describes the long-distance dynamics, i.e. by its construction, B has an UV cut-off but there is no IR cut-off. This means that if the IR region of integration gives a sizeable contribution, one must (in some way) take into account nonperturbative effects. It seems that the most effective tool here is the method proposed in ref. /26/. This and related problems are under investigation just now.

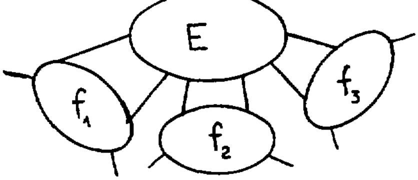

Fig.1

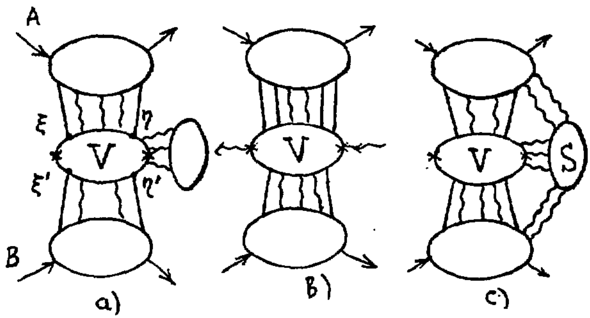

Fig.2

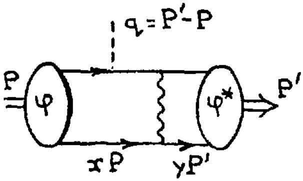

Fig.3

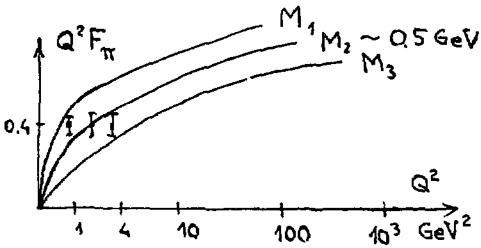

Fig.4

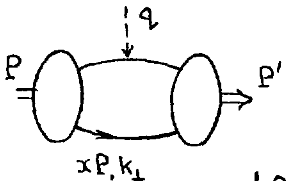

Fig.5

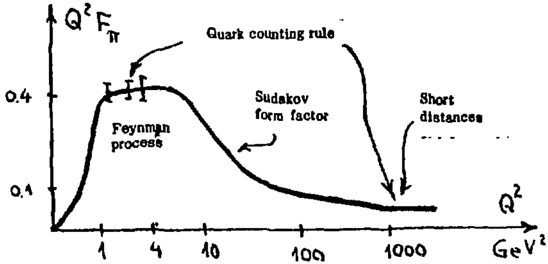

Fig.6

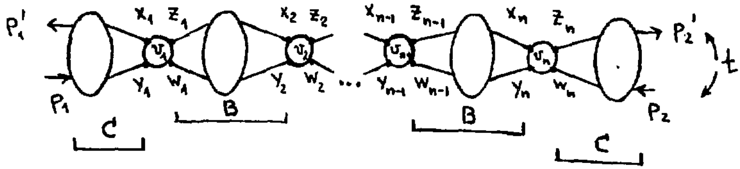

Fig.7

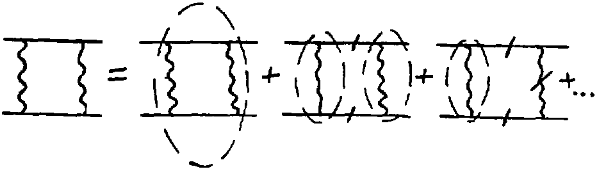

Fig.8

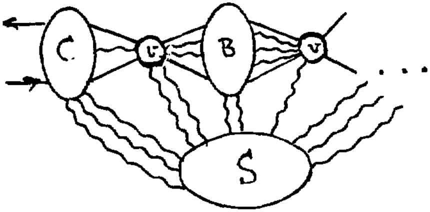

Fig.9

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