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QCD CORRECTIONS TO THE WEAK HAMILTONIAN AND PARITY VIOLATION IN THE N-N REACTIONS

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At the present time there is no doubt in the importance of the effect of quantum chromodynamics (QCD) at the intermediate distances $x > m_c^{-1}$ while considering weak nonleptonic interactions. First of all this concerns $\Delta S = 1$ decays [1]. As for weak N-N interactions, the region $x > m_c^{-1}$ was taken into account in the framework of QCD for parity-violating $NN\pi$ vertex, and this yielded great enhancement of the corresponding coupling constant f_π [2]. However, this great enhancement was mainly due to the appearance of masses of current u, d -quarks in the denominator of relevant expression rather than to the logarithmic factor characteristic of the asymptotically free field theories.

Here we consider the gluon corrections to parity - violating NNV vertices for different x (also for $x > m_c^{-1}$). The obtained results are used to estimate the values of circular γ -quanta polarization P_γ in the reaction of radiative capture of thermal neutrons by protons, $np \to d\gamma$ and of asymmetry parameter A_{pp} in elastic scattering of polarized protons on protons, $\vec{p}p \to pp$.

In the standard model of electro-weak interactions with four flavours $\bar{q} = (\bar{c}\,\bar{u}\,\bar{d}\,\bar{s})$ [3], a nonleptonic part of the effective weak Hamiltonian conserving strangeness can be written in the form (we do not consider Higgs scalar exchange [4])

$$H^W_{\Delta S=0} = \frac{G}{\sqrt{2}}[M_W^2 \int d^4x D(x,M_W)\, T(J^+_\mu(x), J^{-\mu}(0)) + + M_Z^2 \int d^4x D(x,M_Z)\, T(J^0_\mu(x), J^{0\mu}(0))]_{\Delta S=0} . \quad (1)$$

Here $D(x,M_{W(Z)})$ is the propagator of $W(Z)$ boson,

$$J^+_\mu = :\bar{q}_i \gamma_\mu (1+\gamma_5) C_+ q_i: , \qquad J^-_\mu = (J^+_\mu)^+ , \quad (2)$$

$$J^0_\mu = :\bar{q}_i \{[C_3(1-2s_W^2) - \frac{1}{3}s_W^2]\gamma_\mu + C_3\gamma_\mu\gamma_5\} q_i : ,$$

$$C_+ = C_1 + iC_2 = \begin{pmatrix} 0 & 0 & -s & c \\ 0 & 0 & c & s \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad C_3 = \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad (2)$$

$s_W^2 = \sin^2\theta_W = 0.23$, $s = \sin\theta_C = \sqrt{1-c^2} = 0.23$, and we mean summation over colour index $i = 1,2,3$.

Let us consider the parity-violating part of Eq.(1). T - product in Eq.(1) can be written with the help of Wilson's expansion, then

$$H^{P.V.}_{\Delta S=0}(M_W) = \sqrt{2}\, G \sum_r b^r(M_W)\, O^r(M_W) \,. \qquad (3)$$

We confine ourselves to the four-quark operators $O^r(m)$ [4], which are linear combinations of the operators $O(M, N) = {:}\bar{q}\gamma_\mu\gamma_5 M q \bar{q}\gamma^\mu N q{:}$, q being quarks with masses $m_q \leq m$ and M, N being matrices in flavour and colour spaces.

The introduction of the quark-gluon interaction leaves Eq.(3) practically unchanged in the region $x \lesssim M_W^{-1}$ (region I), whereas in the region $x > M_W^{-1}$ leads to modification of the Hamiltonian. The gluon corrections in the one-loop approximation in the region $M_W^{-1} < m_1^{-1} < x \leq m_2^{-1}$ modify $H(m_1)$ in the following way [1,5]

$$H(m_1) = \sqrt{2}\, G \sum_r b^r(m_1)\, O^r(m_1) \to H(m_2) =$$

$$= \sqrt{2}\, G \sum_r b^r(m_1) \left[\frac{\alpha_s(m_2)}{\alpha_s(m_1)}\right]^{d_r} O^r(m_2) \equiv \sqrt{2}\, G \sum_r b^r(m_2)\, O^r(m_2) \,. \qquad (4)$$

Here d_r are (matrices of) anomalous dimensions of the operators $O^r(m_2)$; $\alpha_s(m) = 6\pi[(33 - 2n(m)) \ln \frac{m}{\Lambda}]^{-1}$ is the effective coupling constant of the quark-gluon interaction, $n(m)$ is the number of flavours with $m_q \lesssim m$, Λ is the mass scale QCD parameter, and minimal m_2 is defined by $\alpha_s(m_2) = 1$ (more detailed discussion of the choice of the minimal m_2 can be found in [5]).

In the region $m_1^{-1} < x \leq m_2^{-1}$ it is convenient to choose as O^r in Eq.(4) the operators transforming along irreducible representations (IR) of the group of flavours SU(n): $O^S(m_2)$, $O^A(m_2)$, $O^J(m_2)$ and $O^1(m_2)$ with $n = n(m)$ and $S = \frac{1}{4}n^2(n+1)^2 - n^2$, , $A = \frac{1}{4}n^2(n-1)^2 - n^2$, , $J = n^2 - 1$ and 1 being dimensions of the corresponding IR . In [4] (matrices of) anomalous dimensions of such operators were found

$$d_S = -\frac{6}{33-2n}, \quad d_A = -2d_S, \tag{5}$$

$$d_J = -\frac{1}{33-2n}\begin{pmatrix} 0 & 1 & 0 & \frac{9}{2} \\ 0 & \frac{12n-85}{6} & 16 & \frac{15}{2} \\ 0 & \frac{11}{2} & 0 & 0 \\ 16 & \frac{41}{6} & 0 & -\frac{27}{2} \end{pmatrix}, O^J = \begin{pmatrix} O(P,1) \\ O(Pt^a, t^a) \\ O(1,P) \\ O(t^a, Pt^a) \end{pmatrix}, \tag{6}$$

$$d_1 = -\frac{1}{33-2n} \cdot \begin{pmatrix} 0 & \frac{11}{2} \\ 16 & \frac{6n-20}{3} \end{pmatrix}, \qquad O^1 = \begin{pmatrix} O(1,1) \\ O(t^a,t^a) \end{pmatrix}, \tag{7}$$

t^a being Gell-mann's matrices in the colour space, and we mean summation over $a = 1,...,8$, P being diagonal matrix in the flavour space, e.g., $P = C_3$ for $n = 4$ and $P = \lambda_3, \lambda_8$ for $n = 3$, where $\lambda_{3,8}$ are Gell-Mann's matrices.

In earlier calculations of gluon corrections to the weak NNV-vertex, a region $M_W^{-1} < x \leq m_c^{-1}$ was considered (region II)[6,7] . Here $n(m_c) = 4$ and the index r runs the values S = 84, A = 20, J = 15 and 1. The corresponding operators are[4]:

$$O^{84(20)}(m_c) = O^{84(20)}(C_1,C_1) + O^{84(20)}(C_2,C_2) + (1-2s_W^2)\, O^{84(20)}(C_3,C_3),$$

$$O^{84}(C_i,C_i) = \frac{2}{3}[O(C_i,C_i) - \frac{1}{20}O(1,1)] + \tag{8}$$

$$+\frac{1}{4}[O(C_i t^a, C_i t^a) - \frac{1}{20}O(t^a,t^a)],$$

$$O^{20}(C_i, C_i) = \frac{1}{3}[O(C_i, C_i) + \frac{1}{12}O(1,1)] -$$
$$- \frac{1}{4}[O(C_i t^a, C_i t^a) + \frac{1}{12}O(t^a, t^a)], \tag{8}$$

and $O^{15}(m_c)$, $O^{1}(m_c)$ as well as d_r are defined by Eqs.(5)-(7) at $n = 4$. The coefficients $b^r(M_W)$ are

$$b^{84}(M_W) = b^{20}(M_W) = 1, \quad b^{15}(M_W) = -\frac{s_W^2}{3}(1\ \ 0\ \ 0\ \ 0),$$
$$b^{1}(M_W) = \frac{3-2s_W^2}{180}(1\ \ 6). \tag{9}$$

At $M_W = 88.6$ GeV, $m_c = 1.2$ Gev, $\Lambda = 0.1$ GeV the ratio $\frac{\alpha_s(m_c)}{\alpha_s(M_W)}$ is equal to 2.73 and the gluon corrections transform $b^r(M_W)$ into $b^r(m_c)$ according to Eq.(4):

$$b^{84}(m_c) = 0.79, \quad b^{20}(m_c) = 1.62,$$
$$b^{15}(m_c) = -\frac{s_W^2}{3}(1.07\quad -0.015\quad 0.008\quad -0.24),$$
$$b^{1}(m_c) = \frac{3-2s_W^2}{180}(-2.78\quad 5.88). \tag{10}$$

At $\frac{\alpha_s(m_c)}{\alpha_s(M_W)} = 4$ and 10 we arrive at $b^r(m_c)$ derived in [7].

Let us consider now the region $m_c^{-1} < x \le \mu^{-1}$ (region III). With our choice of Λ $\mu = 0.2$ GeV and $n(\mu) = 3$, index r in (4) runs the values S = 27, J = A = 8 and 1. The operator $O^{27}(\mu)$ is defined by the formula

$$O^{27}(\mu) = \sum_i (\alpha_i + \beta_i)\, O(\lambda_i, \lambda_i) + \gamma[O(\lambda_3, \lambda_8) + O(\lambda_8, \lambda_3)], \tag{11}$$
$$\alpha_{1,2,3} = -\frac{1}{3}\alpha_{4,5,6,7} = \frac{1}{9}\alpha_8 = \frac{1}{60}(-\frac{1}{4} + c^2 - s_W^2),$$

$$\beta_{1,2} = -\frac{1}{2}\beta_3 = -\frac{1}{12}(s^2 - 2s_W^2),$$

$$\beta_{4,5} = -\beta_{6,7} = -\frac{1}{\sqrt{3}}\gamma = -\frac{1}{20}(c^2 - 2s_W^2), \quad \beta_8 = 0, \tag{11}$$

and O_I^8, O^1 are defined by Eqs.(6-7) at $n = 3$, $P = \lambda_3$ being for isospin value $I = 1$, while $P = \lambda_8$ for $I = 0$. Omitting c quarks from the Hamiltonian $H^{P.V.}_{\Delta S=0}(m_c)$, one can rewrite it through new operators with the following coefficients:

$$b^{27}(m_c) = 0.79, \quad b_0^8(m_c) = (-0.19 \quad 0.15 \quad -0.17 \quad 0.15),$$

$$b_1^8(m_c) = (-0.13 \quad 0.056 \quad -0.09 \quad 0.065), \tag{12}$$

$$b^1(m_c) = (-0.025 \quad 0.08).$$

The quark-gluon interactions in the region $m_c^{-1} < x \leq \mu^{-1}$ modify the Hamiltonian according to Eq.(4), and for our choice of parameters $\frac{\alpha_s(\mu)}{\alpha_s(m_c)} = \frac{1}{\alpha_s(m_c)} = 3.56$ and $b^r(\mu)$ are

$$b^{27}(\mu) = 0.59, \quad b_0^8(\mu) = (-0.34 \quad 0.23 \quad -0.30 \quad 0.27),$$

$$b_1^8(\mu) = (-0.21 \quad 0.076 \quad -0.14 \quad 0.15), \tag{13}$$

$$b^1(\mu) = (-0.09 \quad 0.098).$$

Note, that the gluon corrections suppress the contributions of symmetric operators O^S and enhance those of antisymmetric ones O^A. The same is valid for the (anti)symmetric parts of O_I^8 and O^1.

Taking into account gluon corrections let us estimate now γ-polarization P_γ in the reaction $np \to d\gamma$ and asymmetry parameter A_{pp} in the reaction $\vec{p}p \to pp$. The effective parity-violating Hamiltonian of the weak NNV-interaction reads

$$H^{P.V.}_{ef.} = \frac{Gm_\rho^2 g_A}{\sqrt{2} f_\rho}\bar{N}[\sqrt{2}\,a\,(\rho_\mu^- \tau^+ + \rho_\mu^+ \tau^-) + \rho_\mu^\circ(\frac{1}{2}b\tau^3 + \frac{1}{2}\zeta c) + \tag{14}$$

$$+\omega_\mu \left(\frac{1}{2} c' \tau^3 + \frac{1}{2}\zeta d)\right] \gamma^\mu \gamma_5 N . \tag{14}$$

τ^i being the Pauli matrices, $\tau^\pm = \frac{1}{2}(\tau^1 \pm i\tau^2)$ and $\zeta = 2\sqrt{3}\xi =$ $= 0.4$ at $\frac{D}{D+F} = 0.65$. The coefficients a, b, c, c', d are defined by the coefficients b^r and by matrix elements $\langle N'V|O^r|N\rangle$. Using for estimations of the latter, the method proposed in [8], we obtain

$$\begin{pmatrix} a \\ b \\ d \end{pmatrix} = \begin{pmatrix} \frac{2}{3} & \frac{32}{9} & \frac{1}{15}(1-8s^2+12s_W^2) \\ \frac{4}{3} & \frac{64}{9} & \frac{2}{15}(1+12s^2-28s_W^2) \\ \frac{28}{3} & \frac{64}{9} & \frac{2}{15}(3-4s^2-4s_W^2) \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{3}}(b^8_{I,1}+b^8_{I,3})+b^1_1 \\ \frac{1}{\sqrt{3}}(b^8_{I,2}+b^8_{I,4})+b^1_2 \\ b^{27} \end{pmatrix}_{I=0}, \tag{15}$$

$$c = c' = \frac{16}{3}\left[b^8_{I,1} + b^8_{I,3} + \frac{4}{3}(b^8_{I,2} + b^8_{I,4}) + \frac{1}{10}(c^2 - 2s_W^2)\, b^{27}\right]_{I=1} .$$

In the region I, $x \lesssim M_W^{-1}$ (no quark gluon interactions), one has

$$a = c^2 - \frac{1}{6}(1-2s_W^2), \quad b = -\frac{2}{3}c^2 + \frac{7}{3}(1-2s_W^2),$$
$$c = c' = -\frac{8}{9}s_W^2, \qquad d = \frac{2}{3}c^2 + \frac{1}{3}(1-2s_W^2). \tag{16}$$

In the region $x > M_W^{-1}$ a, b, c, c', d can be found from Eqs. (10), (13) and are given in Table 1. To estimate now P_γ and A_{pp} let us use parametrizations of refs. [9] and [10]. In our notation for the Kishi-Sawada-Watari potential

$$P_\gamma = (1.38a - 1.86b + 1.68\zeta d) \times 10^{-8},$$
$$A_{pp}\,(50\ \text{MeV}) = -[4.28(b+\zeta c) + 4.47(c'+\zeta d)] \times 10^{-8}, \tag{17}$$

while for that of Hamada-Jonhston

$$P_\gamma = (3.18a - 1.29b + 1.29\zeta d) \times 10^{-8},$$
$$A_{pp}\,(50\ \text{MeV}) = -[4.63(b+\zeta c) + 4.68(c'+\zeta d)] \times 10^{-8}. \tag{18}$$

Table 1

x	a	b	c=c′	d
I	0.86	0.63	-0.20	0.81
II	0.92	0.99	-0.15	-0.16
III	1.19	1.73	-0.13	-1.46

Our results are set into Table 2 for various regions of x and are conformed to those of Ref. /6/ and Ref. /7/, if we correct Eq.(38) of the latter. We see that gluon corrections give large negative contributions either to P_γ or to A_{pp} (P_γ^{K-S-W} even changes a sign), changing uncorrected results several times for P_γ and by a factor of 1.5 for A_{pp}. Reasonable variations of the ratio $\frac{\alpha_s(m_2)}{\alpha_s(m_1)}$ do not influence strongly numerical values of P_γ and A_{pp}. It is important however to note, that our calculations are performed in the one-loop approximation and we perhaps meet the same problem as say in the calculations of charmonium decays where gluon corrections are also of the same order of magnitude as the first approximation.

Table 2

x	$P_\gamma^{K-S-W} \times 10^8$	$P_\gamma^{H-J} \times 10^8$	$A_{pp}^{K-S-W} \times 10^8$	$A_{pp}^{H-J} \times 10^8$
I	0.56	2.34	-2.85	-3.13
II	-0.68	1.57	-2.93	-3.30
III	-2.56	0.80	-3.82	-4.43

Nevertheless the values obtained for P_γ^{th} remain 1.5 - 2 order of magnitude as small as $P_\gamma^{exp} = (-130 \pm 45) \times 10^{-8}$ /11/, while the values obtained for A_{pp}^{th} are smaller than A_{pp}^{exp} (45 MeV) = $(-3.2 \pm 1.1) \times 10^{-7}$ /12/ by a factor of $8 \div 10$.

The reason of these discrepancies, in our opinion, is either in the difficulties in calculating the matrix elements $\langle N'V|O^r|N\rangle$ (see, e.g., ref. /13/) or/and in our unsatisfactory knowledge of strong potentials at short distances.

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