сообщения
объединенного
института
ядерных
исследований
дубна

E2-80-483

O.V.Tarasov, A.A.Vladimirov

THREE-LOOP CALCULATIONS IN NON-ABELIAN GAUGE THEORIES

1980

Тарасов О.В., Владимиров А.А. **E2-80-483**

Трехпетлевые вычисления в неабелевых калибровочных теориях

Детально описан метод аналитического вычисления вкладов фейнмановских диаграмм в ренормгрупповые функции, основанный на размерной регуляризации и позволяющий вычислить все трехпетлевые диаграммы любой ренормируемой теории. С помощью этого метода в трехпетлевом приближении найдены функция ренормировки заряда и аномальные размерности полей неабелевой калибровочной теории с фермионами. Приведено выражение для эффективного заряда квантовой хромодинамики с учетом трех петель. Продемонстрировано отсутствие ренормировки заряда в SU(4) -суперсимметричной калибровочной модели на трехпетлевом уровне. В приложении дана полная сводка формул, необходимых для трехпетлевых вычислений в неабелевых калибровочных теориях.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1980

Tarasov O.V., Vladimirov A.A. **E2-80-483**

Three-Loop Calculations in Non-Abelian

1. INTRODUCTION

The renormalization group method when applied to asymptotically free models results in an "improved" perturbation theory. Its expansion parameter, an effective charge $\bar{g}^2(Q^2/\Lambda^2, g^2)$, decreases logarithmically with the increase in the momentum transfer Q^2. The existent QCD calculations of various deep inelastic processes in the first two orders in $\bar{g}^2$ appear to be consistent with the present experimental data [1]. However, the next-to-leading corrections (i.e., those $\sim \bar{g}^4$) are fairly large. It leaves open the possibility that the higher-order contributions will be important.

The calculations in higher orders are also of interest from another standpoint. They might serve us a starting point for summing the perturbation theory expansions of QCD, as it is done, for instance, in the ϕ^4 model [2]. Moreover, these calculations can shed light on some peculiar aspects of certain field theory models. For example, in the SU(4) -supersymmetric non-Abelian gauge model derived in [3,4] the charge renormalization effects are shown to vanish to the two-loop order [5]. The corresponding three-loop calculations presented below give the same answer: The charge renormalization function $\beta(g^2)$ is equal to zero. Apparently, the vanishing of $\beta(g^2)$ at the three-loop level is not a sheer coincidence, but an indication that this effect holds to all orders.

The first three-loop QCD calculation in the framework of the renormalization group has been performed in [6], where the total cross section of e^+e^--annihilation into hadrons has been computed analytically. This result is confirmed in [7] by a numerical calculation and in [8] also analytically. However, these calculations involve the $\beta(g^2)$ function to order g^6, whereas all other three-loop QCD calculations require the next, $\sim g^8$, contribution to $\beta(g^2)$. The charge renormalization function $\beta(g^2)$ for the non-Abelian gauge theory including fermions is known to g^6 only, i.e., in the two-loop approximation [9]. In the present paper we describe a method which enables one to evaluate $\beta(g^2)$ at the three-loop level. We present the results of these calculations and the full list of needed formulas.

2. RENORMALIZATION GROUP IN THE MINIMAL SUBTRACTION SCHEME

We consider a non-Abelian gauge theory with fermions belonging to the representation R of the gauge group G:

$$\mathfrak{L} = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} - \frac{1}{2\alpha}(\partial_\mu A^a_\mu)^2 - \partial_\mu \bar{\eta}^a \partial_\mu \eta^a +$$

$$+ g f^{abc} \bar{\eta}^a A^b_\mu \partial_\mu \eta^c + i \sum_{m=1}^{f} \bar{\psi}_i^m \hat{\mathfrak{D}} \psi_i^m , \qquad (1)$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu ,$$

$$\mathfrak{D}_\mu \psi_i^m = \partial_\mu \psi_i^m - igR^a_{ij} \psi_j^m A^a_\mu .$$

Here η^a is the ghost field, α is the gauge parameter, and f^{abc} are the totally antisymmetric structure constants of the group G. The indices of the fermion field ψ^m_i specify colour (i) and flavour (m), respectively. The matrices R^a obey the following relations:

$$[R^a, R^b]_- = i f^{abc} R^c , \quad f^{acd} f^{bcd} = C_A \delta^{ab} ,$$

$$R^a R^a = C_F I , \quad tr(R^a R^b) = T \delta^{ab} . \qquad (2)$$

In particular, the values of group invariants C_A, C_F and T in the fundamental (quark) representation of SU(N) are:

$$C_A = N , \quad C_F = \frac{N^2 - 1}{2N} , \quad T = \frac{1}{2} . \qquad (3)$$

The underlying gauge symmetry of the Lagrangian (1) gives rise to the well-known Slavnov-Taylor identities /10/ extensively used throughout the paper. In particular, a transversality of the radiative corrections to the gluon propagator allows one to compute such a correction in the scalar form, i.e., with its Lorentz indices contracted.

We now turn to a brief discussion of the renormalization procedure. In this paper we adopt the renormalization prescription by 't Hooft /11/, the so-called "minimal subtraction scheme", which by definition subtracts only pole parts in ϵ from a given diagram. The renormalization constants Z_Γ relating the dimensionally regularized 1PI Green function with the renormalized one,

$$\Gamma_R(\frac{Q^2}{\mu^2}, \alpha, g^2) = \lim_{\epsilon \to 0} Z_\Gamma(\frac{1}{\epsilon}, \alpha, g^2) \Gamma(Q^2, \alpha_B, g_B^2, \epsilon) , \qquad (4)$$

look in this scheme like

$$Z_\Gamma(\frac{1}{\epsilon}, \alpha, g^2) = 1 + \sum_{n=1}^{\infty} c_\Gamma^{(n)}(\alpha, g^2) \epsilon^{-n} , \qquad (5)$$

with $\epsilon = \frac{4-d}{2}$, d being the space-time dimension. In (4) μ is the renormalization parameter with the dimension of mass. The bare charge g^2_B is to be constructed from appropriate Z's. The most convenient choice is as follows:

$$g_B^2 = \mu^{2\epsilon} g^2 \tilde{Z}_1^2 Z_3^{-1} \tilde{Z}_3^{-2} . \qquad (6)$$

Here $\tilde{Z}_1$ is the renormalization constant of the ghost-ghost-gluon vertex, Z_3 and $\tilde{Z}_3$ being those of inverted gluon and ghost propagators, respectively. Note also α_B in (4) to be given by $\alpha_B = \alpha Z_3$. The Green function $\Gamma_R(\frac{Q^2}{\mu^2}, \alpha, g^2)$ satisfies the renormalization group equation

$$[Q^2 \frac{\partial}{\partial Q^2} - \beta(g^2) \frac{\partial}{\partial g^2} - \gamma_3(\alpha, g^2) \alpha \frac{\partial}{\partial \alpha} - \gamma_\Gamma(\alpha, g^2)] \Gamma_R(\frac{Q^2}{\mu^2}, \alpha, g^2) = 0 \qquad (7)$$

and the normalization condition $\Gamma_R(\frac{Q^2}{\mu^2}, \alpha, 0) = 1$. The anomalous dimensions γ_Γ are given by the relation

$$\gamma_\Gamma(\alpha, g^2) = g^2 \frac{\partial}{\partial g^2} c_\Gamma^{(1)}(\alpha, g^2) . \qquad (8)$$

Similarly, from

$$g_B^2 = \mu^{2\epsilon} [g^2 + \sum_{n=1}^{\infty} a^{(n)}(g^2) \epsilon^{-n}] \qquad (9)$$

one obtains the charge renormalization function β,

$$\beta(g^2) = (g^2 \frac{\partial}{\partial g^2} - 1) a^{(1)}(g^2) = g^2 [2\tilde{\gamma}_1(\alpha, g^2) - \gamma_3(\alpha, g^2) - 2\tilde{\gamma}_3(\alpha, g^2)] , \qquad (10)$$

which is known to be gauge independent /12/. Thus, the computation of $\gamma_\Gamma(\alpha, g^2)$ and $\beta(g^2)$ requires the functions $c_\Gamma^{(1)}(\alpha, g^2)$ for the renormalization constants in the right-hand side of (6).

The residues of higher-order poles in the expansions (5) and (9) are related with $c^{(1)}$ and $a^{(1)}$ by the equalities

$$[\beta(g^2) \frac{\partial}{\partial g^2} + \gamma_3(\alpha, g^2) \alpha \frac{\partial}{\partial \alpha} + \gamma_\Gamma(\alpha, g^2)] c_\Gamma^{(n)}(\alpha, g^2) = g^2 \frac{\partial}{\partial g^2} c_\Gamma^{(n+1)}(\alpha, g^2) , \qquad (11)$$

$$\beta(g^2)\frac{\partial}{\partial g^2}a^{(n)}(g^2) = (g^2\frac{\partial}{\partial g^2} - 1)\,a^{(n+1)}(g^2). \qquad (12)$$

We choose to work in the Feynman gauge $a=1$ throughout this paper. For checking the higher residues by means of (11), one may use the results of the corresponding two-loop calculations [13] performed in a general gauge.

According to the minimal subtraction prescription[11], the renormalization constants are uniquely determined by requiring that all the divergences in ϵ disappear from the product $Z_\Gamma(\frac{1}{\epsilon}, a, g^2)\,\Gamma(Q^2, a_B, g_B^2, \epsilon)$, so that the limit $\epsilon \to 0$ in (4) does exist. However, we find a somewhat different (but equivalent) definition[14] to be more convenient:

$$Z_\Gamma = 1 - \mathcal{K}R'\Gamma. \qquad (13)$$

An operator $\mathcal{K}$ picks out all the pole terms in ϵ,

$$\mathcal{K}\sum_n b_n\epsilon^n = \sum_{n<0} b_n\epsilon^n . \qquad (14)$$

R' is the BPHZ minimal subtraction procedure (R -operation) with its final subtraction missing: $R=(1-\mathcal{K})R'$. In other words, the R'-operation subtracts all the divergences of internal subgraphs but does not subtract an overall divergence of a diagram. To construct R' explicitly one can employ the following recursion relation [15]:

$$R'G = G + \Sigma(-\mathcal{K}R'G_1)\cdot\ldots\cdot(-\mathcal{K}R'G_m)\cdot G/(G_1+\ldots+G_m), \qquad (15)$$

where the sum is over all sets of disjoint 1PI divergent subgraphs of the diagram G, and $G/(G_1+\ldots+G_m)$ is the diagram obtained from G by contracting $G_1,\ldots,G_m$ to points (as an example see Fig.1).

The $\mathcal{K}R'G$ is the negative of a contribution from G to an appropriate renormalization constant. The computation of $\mathcal{K}R'G$ is simplified drastically owing to the following fact[16].

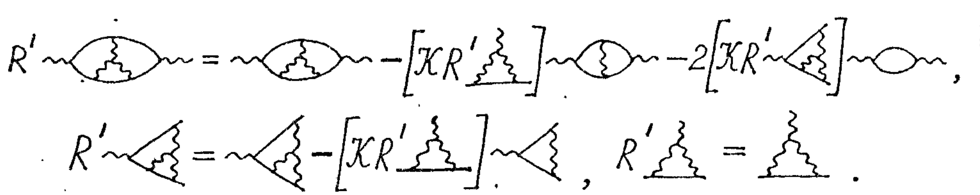


Fig.1

Let a diagram G be infrared finite in a range of external momenta k_i and internal masses m_j. Then in this range $\mathcal{K}R'G$ is a polynomial in k_i and m_j. Therefore, it either is independent of k_i and m_j (for the logarithmically divergent diagram G) or loses such a dependence after differentiating once or twice with respect to k_i.

3. A METHOD FOR COMPUTING THREE-LOOP INTEGRALS

This feature of $\mathcal{K}R'G$ provides the basis for a simple and efficient computational technique developed in [15], which enables one to evaluate analytically all three-loop contributions to the renormalization group functions γ and β in any renormalizable theory. It is shown in [15] that one may calculate $\mathcal{K}R'G$ (properly differentiated, if necessary) with all its external momenta equal to zero and with an auxiliary mass $m \neq 0$ introduced into one of its internal lines (which is sufficient to remove all infrared divergences). The momentum integration corresponding to this line is chosen to be the last one. It looks like

$$\int \frac{dp}{(p^2)^a(p^2+m^2)} \qquad (16)$$

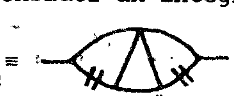
and is readily done using Eq. (A8) in Appendix. We thus show the last momentum integration to be trivial. Therefore, the problem of three-loop calculations reduces to computing the two-loop massless integrals depending on a single momentum p^2,

$$\int \frac{dt\,dq}{t^{2a}q^{2\beta}(p-t)^{2\gamma}(p-q)^{2\sigma}(t-q)^{2\rho}} \qquad (17)$$

with a,β,γ,σ and ρ being integers. If one of the denominators is missing (e.g., $\rho=0,-1,-2,\ldots$) the integral (17) can be evaluated by sequential use of Eq.(A9). Otherwise one needs the non-trivial two-loop integration formulas deduced in [17] through the x-space Gegenbauer polynomial technique. In Appendix we give a list of the relevant integrals of the type (17).

As an illustrative example we consider an integral

$$J = \int \frac{dp\,dq\,dt\,(qt)^2}{p^2q^2t^2(p-q)^2(p-t)^2(k-q)^2(k-t)^2} \equiv \qquad (18)$$



Due to quadratic divergence, it should be differentiated twice with respect to k. Using the relation

$$\frac{\partial^2}{\partial k_\mu \partial k_\mu}\left[\frac{1}{(k-q)^2(k-t)^2}\right] = \frac{8(k-q)(k-t)+4\epsilon[(k-q)^2+(k-t)^2]}{(k-q)^4(k-t)^4} \qquad (19)$$

we obtain $\mathcal{K}\partial^2 R'J$ as displayed in Fig.2 in self-evident notation. Since $\mathcal{K}R'J = k^2 A(\frac{1}{\epsilon})$, we finally get

$$A(\frac{1}{\epsilon}) = \mathcal{K}\frac{1}{8-4\epsilon}\mathcal{K}\partial^2 R'J = -(i\pi^2)^3\left(\frac{1}{12\epsilon^2}+\frac{25}{32\epsilon}\right). \qquad (20)$$

The last two diagrams in Fig.2 diverge logarithmically so that one can compute them with $k = 0$ provided that a non-zero mass

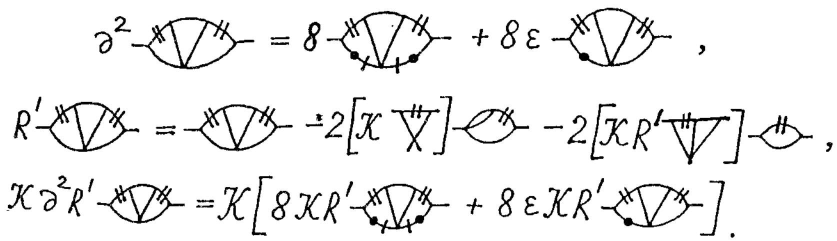


Fig.2

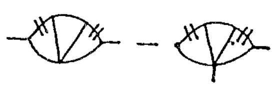


Fig.3

is introduced into one of the differentiated lines, i.e., into that with a blob. The problem of evaluating $\mathcal{K}R'G$ at the three-loop level thus reduces to the integrations (16) and (17). The described procedure has been employed in a considerable part of the calculations presented in this paper.

One can also determine the pole part of (18), $\mathcal{K}J$, by means of a somewhat different method, which involves transferring an external momentum to the other vertex in order to simplify the denominator. Consider the difference (Fig.3)

$$\int\frac{dp\,dq\,dt\,(qt)^2}{p^2q^2t^2(p-q)^2(p-t)^2(k-q)^2(k-t)^2}\left[1-\frac{(k-q)^2}{q^2}\right] = \qquad (21)$$

$$= \int\frac{dp\,dq\,dt\,(qt)^2\,(2k_\mu q_\mu - k^2)}{p^2t^2q^4(p-q)^2(p-t)^2(k-q)^2(k-t)^2} \equiv 2k_\mu J_\mu - k^2 J_1 .$$

Let us further subtract from J_μ the other integral having a more simple structure of the denominator:

$$J_\mu - \int\frac{dp\,dq\,dt\,q_\mu(qt)^2}{p^2t^4q^4(p-q)^2(p-t)^2(k-q)^2} =$$

$$= \int\frac{dp\,dq\,dt\,q_\mu(qt)^2[2k_\nu t_\nu - k^2]}{p^2q^4t^4(p-q)^2(p-t)^2(k-q)^2(k-t)^2} . \qquad (22)$$

There is only one (logarithmically) divergent integral in the right-hand side of (22), namely

$$\int\frac{dp\,dq\,dt\,2q_\mu t_\nu(qt)^2}{p^2q^4t^4(p-q)^2(p-t)^2(k-q)^2(k-t)^2} . \qquad (23)$$

Due to the absence of divergent subgraphs, its pole part does not depend on k and coincides with

$$\mathcal{K}\int\frac{dp\,dq\,dt\,2q_\mu t_\nu(qt)^2}{p^2q^4t^6(p-q)^2(p-t)^2(k-q)^2} . \qquad (24)$$

As to the integral J_1, it diverges logarithmically and contains divergent subgraphs. We note the difference

$$J_1 - \int\frac{dp\,dq\,dt\,(qt)^2}{p^2q^4t^4(p-q)^2(p-t)^2(k-q)^2} \qquad (25)$$

to be convergent, and combining the last five relations finally obtain

$$\mathcal{K}J = \mathcal{K}\int\frac{dp\,dq\,dt\,(qt)^2[4(kt)(kq)+2q^2t^2-t^2(k-q)^2]}{p^2q^4t^6(k-q)^2(p-q)^2(p-t)^2} . \qquad (26)$$

This integral is easy to evaluate with the use of formulas listed in Appendix and gives the same answer as in (20).

The essence of the procedure presented above is as follows. One subtracts from the initial integral J an infrared finite integral J′ with a more simple denominator reducing thus the degree of divergence. Such a subtraction is to be repeated until the difference becomes convergent.

4. CALCULATION OF SPECIFIC DIAGRAMS

It is now seen that the three-loop momentum integrals contributing to Z's are always calculable. However, one must introduce an auxiliary mass into the diagram (which as a rule represents a sum of distinct integrals similar to (18)) and into all its counterterms in a consistent fashion. For the most complicated diagrams of the gluon propagator this task appears to be unmanageable. Therefore, we deal with the diagrams of the topological type, depicted in Fig.4, as follows. We reduce the numerator of the integrand to the scalar form and then decompose it into a sum of invariants like $k^2(q-t)^4$, $p^2q^2(p-t)^2, \ldots$. Cancelling numerator against denominator and taking symmetry into account results in at most 66 distinct three-loop massless integrals. Their pole parts are to be found either by the direct use of (A9-A14) or by differentiating, introducing a mass, and then converting KR' into K through the compensating subtraction. The latter pole parts are given in Appendix.

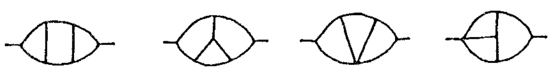


Fig.4

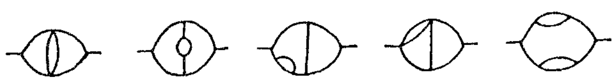


Fig.5

The propagator diagrams of more simple ("nested") topology (Fig.5) can be computed straightforwardly using (A9-A14). The remaining topological type is represented by a single diagram (all others equal zero owing to the antisymmetry of the group structure constants) which can be easily calculated by means of differentiation:

$g_{\mu\nu}$  =

$$= \frac{g^6 T(C_F - C_A)(C_F - \frac{C_A}{2})}{(4\pi)^6 (k^2)^{3\varepsilon - 1}} \left(\frac{16}{3\epsilon^2} + \frac{20}{\epsilon} - \frac{32}{\epsilon}\zeta(3) + O(1) \right). \tag{27}$$

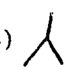
All the diagrams of the ghost-ghost-gluon vertex diverge logarithmically. We evaluate them setting all external momenta to be zero and introducing an auxiliary mass into one of the internal lines. For each particular diagram this "potentially infrared" line is easy to identify.

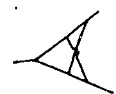
Thus, all the diagrams of a certain Green function are calculated in the same fashion: with an auxiliary mass for the vertices and without it for the propagators. It enables one to perform the subtractions either following 't Hooft [11] or determining $KR'G$ for each individual diagram. In order to check the intermediate results, we choose the latter way.

The problem of evaluating the group weights appears to be of no substantial difficulty. Mostly it reduces to making contractions in the products of several structure constants f^{abc}. The following graphical representation is here of great use [18].

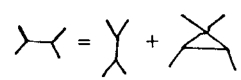
 $= f^{abc}$ —— $= \delta^{ab}$

 $= (-C_A)$ —— $\Longrightarrow f^{cad} f^{dbc} = -C_A \delta^{ab}$

 $= (-\frac{C_A}{2})$  $\Longrightarrow f^{dae} f^{ebg} f^{gcd} = -\frac{C_A}{2} f^{abc}$ (28)

 $= 0 \Longrightarrow f^{gai} f^{ijd} f^{jbh} f^{heg} f^{dce} = 0$.

The last two relations are derived from the Jacobi identity

 $\Longrightarrow f^{abc} f^{ade} + f^{abe} f^{acd} + f^{abd} f^{aec} = 0.$ (29)

The only products of structure constants which cannot be contracted by the sequential use of (28) are the following (Fig.6).

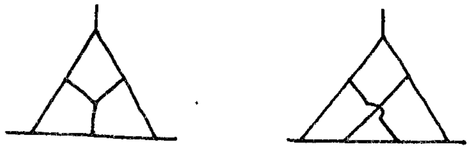
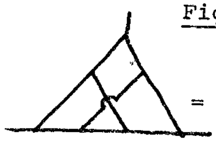
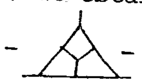
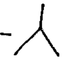


Fig.6

 $= \frac{3N}{2}$  (31)

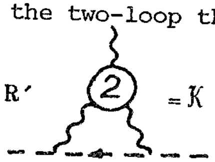
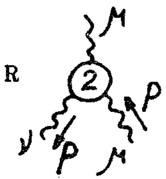
From (29) we obtain

 $= -$  $- \frac{C_A^3}{8}$  (30)

However, one fails to express the graphs of Fig.6 separately in terms of C_A. In a specific case of the SU(N) group, we have found

Fortunately, the relation (30) is quite sufficient for the three-loop calculations of the renormalization group functions. Only the sum of the diagrams of Fig.6 contributes to the final answer. This fact is easy to explain. The non-trivial products (Fig.6) might contribute to the vertex anomalous dimension, $\tilde{\gamma}_1(a, g^2)$, only. But it is known to vanish in the Landau gauge: $\tilde{\gamma}_1(0, g^2)=0$. Hence these products do not contribute to the gauge independent function $\beta(g^2)$ and consequently, to $\tilde{\gamma}_1(a, g^2)$ in arbitrary gauge as well.

Concluding this section we wish to discuss one more example where the Slavnov-Taylor identities [10] have been fruitfully used. To facilitate the computation of the vertex diagram with the two-loop three-gluon vertex insertion

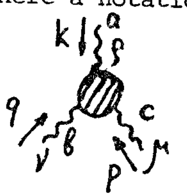
KR'  $= K \frac{1}{4-2\epsilon} \int \frac{dp\ p_\nu}{(2\pi)^4 p^4 (p^2+m^2)} [\, R$  $]$ (32)

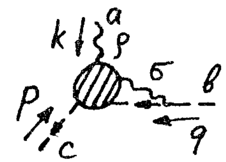
we employ an identity

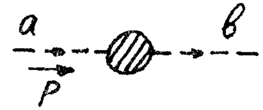
$$p_\mu \Gamma^{abc}_{\rho\nu\mu}(k,q,p) =$$

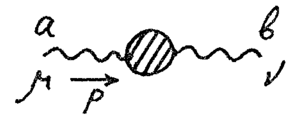
$$= G(p^2)\,[\,M^{abc}_{\sigma\rho}(k,q,p)\,\mathfrak{D}^{-1}(q^2)(q^2 g_{\sigma\nu} - q_\sigma q_\nu) + \begin{pmatrix} b \leftrightarrow a \\ \rho \leftrightarrow \nu \\ q \leftrightarrow k \end{pmatrix}\,], \quad (33)$$

where a notation is as follows:

 $= \Gamma^{abc}_{\rho\nu\mu}(k,q,p)$,

 $= q_\sigma M^{abc}_{\sigma\rho}(k,q,p)$,

 $= -i\frac{\delta^{ab}}{p^2} G(p^2)$,

 $= -i\frac{\delta^{ab}}{p^2}[(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2})\,\mathfrak{D}(p^2) + a\frac{p_\mu p_\nu}{p^2}]$. (34)

In our case k=0 so that (33) transforms into

$$p_\mu \Gamma^{abc}_{\rho\nu\mu}(0,-p,p) = G(p^2)\,\mathfrak{D}^{-1}(p^2)(p^2 g_{\nu\sigma} - p_\nu p_\sigma) M^{abc}_{\sigma\rho}(0,-p,p)\,. \quad (35)$$

Identity (35) allows us to calculate $M^{abc}_{\sigma\rho}$ rather than fairly complicated three-gluon vertex $\Gamma^{abc}_{\rho\nu\mu}$.

5. THREE-LOOP RESULTS FOR QCD

A total number of topologically distinct three-loop diagrams contributing to $\beta(g^2)$ amounts to 440 (without counting opposite directions of the ghost and fermion lines). For performing the Lorentz and Dirac algebra, reducing the integrands, decomposing the scalar products, evaluating and summing standard integrals, the computer program SCHOONSCHIP [19] has been substantially used. The total execution time is rather difficult to estimate. Here we only indicate that the diagrams of Fig.7 require 110 and 90 seconds, respectively, at the CDC-6500 computer.

Fig.7

Our final results obtained in collaboration with A.Yu. Zharkov are as follows (f is the number of flavours, $h = \frac{g^2}{(4\pi)^2}$):

$$\tilde{\gamma}_1(1,h) = -\frac{C_A}{2}h - \frac{3}{4}C_A^2 h^2 + h^3\left(-\frac{125}{32}C_A^3 + \frac{15}{8}C_A^2 Tf\right), \tag{36}$$

$$\gamma_3(1,h) = h\left(\frac{5}{3}C_A - \frac{4}{3}Tf\right) + h^2\left(\frac{23}{4}C_A^2 - 5C_A Tf - 4C_F Tf\right) +$$
$$+ h^3\Big[\left(\frac{4051}{144} - \frac{3}{2}\zeta(3)\right)C_A^3 + \left(-\frac{875}{18} + 18\zeta(3)\right)C_A^2 Tf - \tag{37}$$
$$-\left(\frac{5}{18} + 24\zeta(3)\right)C_A C_F Tf + 2C_F^2 Tf + \frac{76}{9}C_A T^2 f^2 + \frac{44}{9}C_F T^2 f^2\Big],$$

$$\tilde{\gamma}_3(1,h) = \frac{C_A}{2}h + h^2\left(\frac{49}{24}C_A^2 - \frac{5}{6}C_A Tf\right) + h^3\Big[\left(\frac{229}{27} + \frac{3}{4}\zeta(3)\right)C_A^3 -$$
$$-\left(\frac{5}{216} + 9\zeta(3)\right)C_A^2 Tf + \left(-\frac{45}{4} + 12\zeta(3)\right)C_A C_F Tf - \frac{35}{27}C_A T^2 f^2\Big], \tag{38}$$

$$\beta(h) = h^2\left(-\frac{11}{3}C_A + \frac{4}{3}Tf\right) + h^3\left(-\frac{34}{3}C_A^2 + \frac{20}{3}C_A Tf + 4C_F Tf\right) +$$
$$+ h^4\left(-\frac{2857}{54}C_A^3 + \frac{1415}{27}C_A^2 Tf - \frac{158}{27}C_A T^2 f^2 +\right.$$
$$\left. + \frac{205}{9}C_A C_F Tf - \frac{44}{9}C_F T^2 f^2 - 2C_F^2 Tf\right). \tag{39}$$

The cancellation of the transcendental $\zeta(3)$ in the expression for $\beta(h)$ is in complete analogy with QED treated in the minimal subtraction scheme, where /20/

$$\beta_{QED}(\alpha) = \frac{4}{3}\frac{\alpha^2}{4\pi} + 4\frac{\alpha^3}{(4\pi)^2} - \frac{62}{9}\frac{\alpha^4}{(4\pi)^3}. \tag{40}$$

In a particular case of QCD, when fermions transform according to the fundamental representation of SU(3), $\beta(h)$ reads

$$\beta_{QCD}(h) = h^2\left(-11 + \frac{2}{3}f\right) + h^3\left(-102 + \frac{38}{3}f\right) +$$
$$+ h^4\left(-\frac{2857}{2} + \frac{5033}{18}f - \frac{325}{54}f^2\right). \tag{41}$$

Now we are in a position to find an effective charge $\bar{h}\left(\frac{Q^2}{\mu^2}, h\right)$ from

$$\ln\frac{Q^2}{\mu^2} = \int_h^{\bar{h}} \frac{dx}{\beta(x)} = \psi(\bar{h}) - \psi(h), \tag{42}$$

where $\psi(h)$ represents an indefinite integral $\int^h \frac{dx}{\beta(x)}$. Let us express $\bar{h}$ in terms of the renormalization group invariant quantity $\ln\frac{Q^2}{\mu^2} + \psi(h) \equiv \ln\frac{Q^2}{\Lambda^2} \equiv L$, where Λ is the momentum scale. Assuming

$$\beta(x) = -\beta_0 x^2 - \beta_1 x^3 - \beta_2 x^4 + O(x^5) \tag{43}$$

we arrive at

$$\psi(h) = \frac{1}{\beta_0 h} + \frac{\beta_1}{\beta_0^2}\ln h + \delta + \frac{\beta_2\beta_0 - \beta_1^2}{\beta_0^3}h + O(h^2) \tag{44}$$

and obtain from (42)

$$\bar{h}(L) = \frac{1}{\beta_0 L} - \frac{\beta_1}{\beta_0^3}\frac{\ln L}{L^2} + \frac{\delta\beta_0^2 - \beta_1\ln\beta_0}{\beta_0^3 L^2} + \frac{\beta_1^2\ln^2 L}{\beta_0^5 L^3} -$$
$$-\frac{\ln L}{L^3}\left[\frac{\beta_1^2}{\beta_0^5} + \frac{2\beta_1}{\beta_0^5}(\delta\beta_0^2 - \beta_1\ln\beta_0)\right] + \tag{45}$$
$$+\frac{1}{L^3\beta_0^5}\left[\beta_2\beta_0 - \beta_1^2 + \beta_1(\delta\beta_0^2 - \beta_1\ln\beta_0) + (\delta\beta_0^2 - \beta_1\ln\beta_0)^2\right] + O\left(\frac{\ln^3 L}{L^4}\right)$$

with δ being an arbitrary constant. Fixing the momentum scale Λ by choosing, as usual, $\delta = \frac{\beta_1\ln\beta_0}{\beta_0^2}$, we finally get

$$\bar{h}(L) = \frac{1}{\beta_0 L} - \frac{\beta_1}{\beta_0^3}\frac{\ln L}{L^2} + \frac{\beta_1^2(\ln^2 L - \ln L)}{\beta_0^5 L^3} + \frac{\beta_2\beta_0 - \beta_1^2}{\beta_0^5 L^3} + O\left(\frac{\ln^3 L}{L^4}\right). \tag{46}$$

Using (41), (43) and (46) one readily finds the QCD effective charge in the three-loop approximation.

6. VANISHING OF $\beta(g^2)$ IN A SUPERSYMMETRIC GAUGE MODEL

Some time ago a very interesting SU(4)-supersymmetric non-Abelian gauge model has been derived [3,4] which exhibits the vanishing charge renormalization effects, since its charge renormalization function $\beta(g^2)$ proves to be zero through the two-loop order [5]. The Lagrangian is [4]:

$$\mathfrak{L} = \mathfrak{L}_{YM} + \frac{i}{2}\bar{\lambda}^a_m \hat{\mathfrak{D}}\lambda^a_m + \frac{1}{2}(\mathfrak{D}_\mu \phi^a_r)^2 + \frac{1}{2}(\mathfrak{D}_\mu \chi^a_r)^2 -$$

$$- \frac{g}{2} f^{abc}\bar{\lambda}^a_m [\alpha^r_{mn}\phi^b_r + \gamma_5 \beta^r_{mn}\chi^b_r]\lambda^c_n - \qquad (47)$$

$$- \frac{g^2}{4}[(f^{abc}\phi^b_r\phi^c_t)^2 + (f^{abc}\chi^b_r\chi^c_t)^2 + 2(f^{abc}\phi^b_r\chi^c_t)^2],$$

with $a,b,c = 1,\dots,N^2-1$; $m,n = 1,\dots,4$; $r,t = 1,2,3$. Here $\mathfrak{L}_{YM}$ is the pure Yang-Mills Lagrangian with SU(N) gauge symmetry. The matter fields (Majorana spinors λ^a_m, scalars ϕ^a_r and pseudoscalars χ^a_r) transform according to the adjoint (regular) representation of SU(N). Hence

$$\mathfrak{D}_\mu \lambda^a_m = \partial_\mu \lambda^a_m + g f^{abc} A^b_\mu \lambda^c_m$$

with similar expressions for $\mathfrak{D}_\mu \phi^a_r$ and $\mathfrak{D}_\mu \chi^a_r$. The six real antisymmetric 4x4 matrices α^r, β^r obey the relations

$$[\alpha^r, \alpha^t]_+ = [\beta^r, \beta^t]_+ = -2\delta^{rt}, \quad [\alpha^r, \beta^t]_- = 0. \qquad (48)$$

The other properties of these matrices and their explicit form are given in Appendix.

To determine the contributions to the renormalization group functions of the model (47) from the diagrams without scalar and pseudoscalar particles, one may use the results (36-39) with

$$C_A = C_F = N, \quad Tf = 2N. \qquad (49)$$

This leads to

$$\beta(h)_{\text{without scalars}} = -Nh^2 + 10N^2h^3 + \frac{101}{2}N^3h^4. \qquad (50)$$

Now an appropriate scalar contribution must be added to (50). In the two-loop approximation it has been done in [5] with the intriguing result $\beta(h) = 0$.

The method of our three-loop calculations is described above. Here we shall only consider the issue of applicability of the standard dimensional regularization to supersymmetric theories. This subject has been discussed by various authors [21]. Proceeding in the spirit of ref. [21] we write down the following rules of the "supersymmetric dimensional regularization" which is to maintain both gauge invariance and global supersymmetry: The relations defining the Dirac matrices look as in four dimensions (see Appendix) while the numbers of scalar and pseudoscalar fields equal $3+\epsilon$ rather than 3. This modification of the regularization maintains equal (and integral) total numbers of Bose and Fermi degrees of freedom even in $4-2\epsilon$ dimensions: 8 components of four Majorana spinors correspond to $(2-2\epsilon)$ massless vectors $+(3+\epsilon)$ scalars $+(3+\epsilon)$ pseudoscalars $= 8$ bosons. It is this matching of the Fermi and Bose field components that is crucial for preserving supersymmetry [21].

For lack of a rigorous proof, we have verified the invariance of the supersymmetric dimensional regularization by direct calculation of $\beta(h)$ at the two-loop level in two different ways:

$$\beta(h) = h[2\tilde{\gamma}_1(h) - \gamma_3(h) - 2\tilde{\gamma}_3(h)] \qquad (51)$$

and

$$\beta(h) = h[2\gamma_4(h) - \gamma_\phi(h) - 2\gamma_\lambda(h)]. \qquad (52)$$

Here $\tilde{\gamma}_1$ and γ_4 are the anomalous dimensions of the ghost-ghost-gluon and fermion-fermion-scalar vertices, and γ_3, $\tilde{\gamma}_3$, γ_ϕ and γ_λ are those of gluon, ghost, scalar and fermion propagators, respectively. In the standard (with $\delta^{rr} = 3$) dimensional regularization, these anomalous dimensions are (in the Feynman gauge):

$$\tilde{\gamma}_1 = -\frac{Nh}{2} - \frac{3}{4}N^2h^2, \qquad \gamma_4 = -5Nh + 5N^2h^2,$$

$$\gamma_3 = -2Nh + \frac{N^2h^2}{2}, \qquad \gamma_\phi = -2Nh, \tag{53}$$

$$\tilde{\gamma}_3 = \frac{Nh}{2} - N^2h^2, \qquad \gamma_\lambda = -4Nh + 6N^2h^2.$$

With the use of supersymmetric dimensional regularization (with $\delta^{rr} = 3 + \epsilon$), we obtain

$$\tilde{\gamma}_1 = -\frac{Nh}{2} - \frac{3}{4}N^2h^2, \qquad \gamma_4 = -5Nh + \frac{11}{2}N^2h^2,$$

$$\gamma_3 = -2Nh + N^2h^2, \qquad \gamma_\phi = -2Nh - N^2h^2, \tag{54}$$

$$\tilde{\gamma}_3 = \frac{Nh}{2} - \frac{5}{4}N^2h^2, \qquad \gamma_\lambda = -4Nh + 6N^2h^2.$$

Using (51) gives $\beta(h) = 0$ for both regularizations while relation (52) leads to $\beta(h) = -2N^2h^3$ for the standard regularization and to $\beta(h) = 0$ for the supersymmetric one. This discrepancy shows the former regularization to be noninvariant under supersymmetry transformations.

For our three-loop calculations we employ formula (51). Below we write down the scalar contributions to anomalous dimensions through the three-loop order calculated in the supersymmetric dimensional regularization scheme (in collaboration with L.V.Avdeev):

$$\gamma_3^{scal} = -Nh + \frac{53}{4}N^2h^2 + \left(\frac{69}{8} - \frac{9}{4}\zeta(3)\right)N^3h^3,$$

$$\tilde{\gamma}_3^{scal} = -\frac{13}{8}N^2h^2 + \left(\frac{771}{32} + \frac{9}{8}\zeta(3)\right)N^3h^3, \tag{55}$$

$$\tilde{\gamma}_1^{scal} = \frac{101}{32}N^3h^3.$$

From (55) and (51) we obtain

$$\beta^{scal}(h) = Nh^2 - 10N^2h^3 - \frac{101}{2}N^3h^4 \tag{56}$$

and using (50), arrive at the final result

$$\beta(h)_{\text{three loops}} = 0. \tag{57}$$

It is worth mentioning that the use of the standard dimensional regularization yields

$$\gamma_3^{scal} = -Nh + \frac{51}{4}N^2h^2 + \left(\frac{193}{48} - \frac{9}{4}\zeta(3)\right)N^3h^3,$$

$$\tilde{\gamma}_3^{scal} = -\frac{11}{8}N^2h^2 + \left(\frac{527}{24} + \frac{9}{8}\zeta(3)\right)N^3h^3, \quad \tilde{\gamma}_1^{scal} = \frac{87}{32}N^3h^3, \tag{58}$$

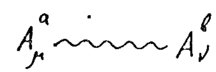
$$\beta(h)_{\text{three loops}} = 8N^3h^4.$$

The result (57) implies the absence of the charge renormalization effects in the model (47) to the three-loop order. It confirms a conjecture that $\beta(h)$ in this model vanishes to all orders. If it were the case, the model (47) would be unique in the four dimensional quantum field theory. The vanishing $\beta(h)$ might imply, for instance, that this model would be free of supercurrent anomalies /22/. In any case, a rigorous argument proving this conjecture on symmetry ground is now a great urgency.

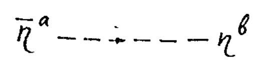
We would like to thank L.V.Avdeev, G.A.Chochia and A.Yu.Zharkov for the help in some calculations.

APPENDIX

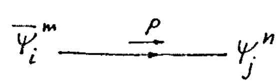
1. Feynman rules for the model (1)



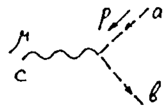
$$-\frac{i}{p^2}\delta^{ab}\left(g_{\mu\nu} + (\alpha - 1)\frac{p_\mu p_\nu}{p^2}\right),$$



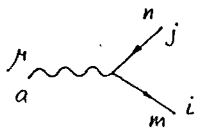
$$-\frac{i}{p^2}\delta^{ab},$$



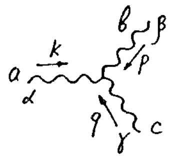
$$\frac{i\hat{p}}{p^2}\delta^{mn}\delta_{ij},$$



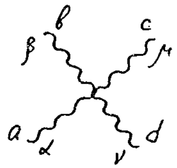
$g p_\mu f^{abc}$,



$i g \gamma_\mu \delta^{mn} R^a_{ij}$,



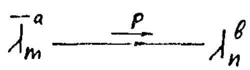
$g f^{abc}[(p-q)_\alpha g_{\beta\gamma} + (q-k)_\beta g_{\alpha\gamma} + (k-p)_\gamma g_{\alpha\beta}]$,



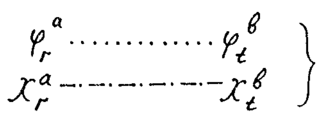
$-ig^2[f^{abe} f^{cde}(2g_{\alpha\mu} g_{\beta\nu} - g_{\alpha\nu} g_{\beta\mu} - g_{\alpha\beta} g_{\mu\nu}) +$

$+ f^{ace} f^{bde} (2g_{\alpha\beta} g_{\mu\nu} - g_{\alpha\nu} g_{\beta\mu} - g_{\alpha\mu} g_{\beta\nu})]$,

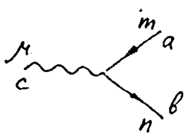
2. Additional Feynman rules for the model (47)



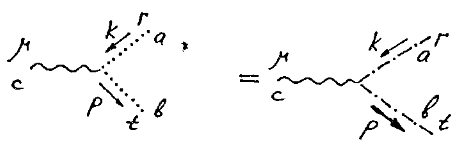
$\frac{i\hat{p}}{p^2} \delta^{ab} \delta_{mn}$,



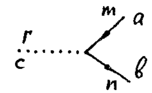
$\frac{i}{p^2} \delta^{ab} \delta_{rt}$,



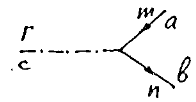
$-g\gamma_\mu f^{abc} \delta_{mn}$,



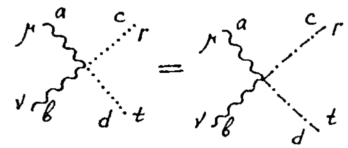
$-g(k+p)_\mu f^{abc} \delta_{rt}$



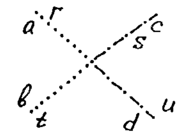
$-igf^{abc} \alpha^r_{nm}$



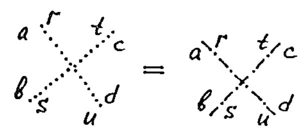
$-igf^{abc} \gamma_5 \beta^r_{nm}$



$ig^2 g_{\mu\nu} \delta_{rt} (f^{ace} f^{bde} + f^{ade} f^{bce})$



$-ig^2 \delta_{rt} \delta_{su} (f^{ace} f^{bde} + f^{ade} f^{bce})$



$-ig^2[f^{abe} f^{cde} (2\delta_{rt}\delta_{su} - \delta_{rs}\delta_{tu} - \delta_{ru}\delta_{ts}) +$

$+ f^{ace} f^{bde} (2\delta_{rs}\delta_{tu} - \delta_{rt}\delta_{su} - \delta_{ru}\delta_{ts})]$

In addition to this:
a) each closed loop brings a factor $(2\pi)^{-4}$,
b) each fermion or ghost loop gives an extra minus sign,
c) arrows on the Majorana spinor lines should be ignored in calculating the symmetry factors.

3. Dirac matrices in $4-2\epsilon$ dimensions

We use the metric $g_{\mu\nu} = (1, -1, -1, \ldots)$, $g_{\mu\mu} = 4-2\epsilon$.

$$[\gamma_\mu, \gamma_\nu]_+ = 2g_{\mu\nu}, \quad \gamma_\mu\gamma_\mu = 4-2\epsilon, \quad \gamma_\mu\gamma_\nu\gamma_\mu = (2\epsilon-2)\gamma_\nu,$$

$$\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\mu = 4g_{\nu\rho} - 2\epsilon\gamma_\nu\gamma_\rho, \quad \gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_\mu = 2\epsilon\gamma_\nu\gamma_\rho\gamma_\sigma - 2\gamma_\sigma\gamma_\rho\gamma_\nu, \tag{A.1}$$

$$[\gamma_5, \gamma_\mu]_+ = 0, \ \gamma_5^2 = -1, \quad \mathrm{tr}\,\gamma_5 = 0, \quad \mathrm{tr}\,I = 4, \quad \mathrm{tr}(\gamma_\mu\gamma_\nu) = 4g_{\mu\nu},$$

$$\mathrm{tr}(\gamma_\mu\gamma_\nu\gamma_\alpha\gamma_\beta) = 4(g_{\mu\nu}g_{\alpha\beta} - g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha}), \quad \mathrm{tr}(\gamma_{\mu_1} \ldots \gamma_{\mu_{2N+1}}) = 0.$$

4. α-and β-matrices of the model (47)

These real antisymmetric 4x4 matrices have an explicit representation in terms of the Pauli matrices:

$$\alpha^1 = \begin{pmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{pmatrix}, \quad \alpha^2 = \begin{pmatrix} 0 & -\sigma_3 \\ \sigma_3 & 0 \end{pmatrix}, \quad \alpha^3 = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & i\sigma_2 \end{pmatrix},$$

$$\beta^1 = \begin{pmatrix} 0 & i\sigma_2 \\ i\sigma_2 & 0 \end{pmatrix}, \quad \beta^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \beta^3 = \begin{pmatrix} -i\sigma_2 & 0 \\ 0 & i\sigma_2 \end{pmatrix}, \tag{A.2}$$

Their relevant properties are

$$[\alpha^r, \alpha^t]_+ = [\beta^r, \beta^t]_+ = -2\delta^{rt}, \quad [\alpha^r, \beta^t]_- = 0,$$

$$\mathrm{tr}\,\alpha^r = \mathrm{tr}\,\beta^r = \mathrm{tr}(\alpha^r\beta^t) = 0, \qquad \mathrm{tr}(\alpha^r\alpha^t) = \mathrm{tr}(\beta^r\beta^t) = -4\delta^{rt}. \tag{A.3}$$

The supersymmetric regularization used in section 6 implies $\delta^{rr} = 3+\epsilon$ giving rise to the following relations:

$$\alpha^r\alpha^r = \beta^r\beta^r = -3-\epsilon, \quad \alpha^r\alpha^t\alpha^r = (1+\epsilon)\alpha^t, \; \beta^r\beta^t\beta^r = (1+\epsilon)\beta^t, \tag{A.4}$$

whereas the standard dimensional regularization prescribes

$$\delta^{rr} = 3, \quad \alpha^r\alpha^r = \beta^r\beta^r = -3, \quad \alpha^r\alpha^t\alpha^r = \alpha^t, \; \beta^r\beta^t\beta^r = \beta^t. \tag{A.5}$$

5. Properties of the Euler Γ-function

$$\Gamma(z+1) = z\Gamma(z), \quad \Gamma(1) = \Gamma(2) = 1, \quad \Gamma(N+1) = N!,$$

$$\Gamma(1+x) = \exp[-\gamma x + \sum_{n=2}^{\infty} (-)^n \frac{\zeta(n)}{n} x^n], \tag{A.6}$$

where γ is the Euler constant and ζ the Riemann function. We note that γ and $\zeta(2)$ do not occur in $\bar{K}R'G$, and consequently in the renormalization group functions.

6. One-loop integration formulas

We choose a volume of the unit sphere in $4-2\epsilon$ dimensions to be $\frac{2\pi^2}{1-\epsilon}$.

$$\int dp\,(p^2)^\lambda = 0 \qquad \text{for any } \lambda, \tag{A.7}$$

$$\int \frac{dp}{p^{2\alpha}(p^2+m^2)^\beta} = \frac{i\pi^2\,\Gamma(\alpha+\beta-2+\epsilon)\Gamma(2-\alpha-\epsilon)}{(m^2)^{\alpha+\beta-2+\epsilon}(1-\epsilon)\Gamma(\beta)}, \tag{A.8}$$

$$\int \frac{dq}{q^{2\alpha}(p-q)^{2\beta}} = \frac{i\pi^2\Gamma(1-\epsilon)\Gamma(\alpha+\beta-2+\epsilon)\Gamma(2-\alpha-\epsilon)\Gamma(2-\beta-\epsilon)}{(p^2)^{\alpha+\beta-2+\epsilon}\,\Gamma(\alpha)\Gamma(\beta)\Gamma(4-\alpha-\beta-2\epsilon)}, \tag{A.9}$$

$$\int \frac{dq\; q_\mu}{q^{2\alpha}(p-q)^{2\beta}} = \frac{i\pi^2 p_\mu\Gamma(1-\epsilon)\Gamma(\alpha+\beta-2+\epsilon)\Gamma(3-\alpha-\epsilon)\Gamma(2-\beta-\epsilon)}{(p^2)^{\alpha+\beta-2+\epsilon}\Gamma(\alpha)\Gamma(\beta)\Gamma(5-\alpha-\beta-2\epsilon)}, \tag{A.10}$$

$$\int \frac{dq\; q_\mu q_\nu}{q^{2\alpha}(p-q)^{2\beta}} = \frac{i\pi^2\Gamma(1-\epsilon)\Gamma(\alpha+\beta-3+\epsilon)\Gamma(3-\alpha-\epsilon)\Gamma(2-\beta-\epsilon)}{(p^2)^{\alpha+\beta-2+\epsilon}\,\Gamma(\alpha)\Gamma(\beta)\Gamma(6-\alpha-\beta-2\epsilon)} \times \tag{A.11}$$

$$\times[(\alpha+\beta-3+\epsilon)(3-\alpha-\epsilon)p_\mu p_\nu + \frac{1}{2}(2-\beta-\epsilon)g_{\mu\nu}p^2],$$

$$\int \frac{dq\; q_\mu q_\nu q_\lambda}{q^{2\alpha}(p-q)^{2\beta}} = \frac{i\pi^2\Gamma(1-\epsilon)\Gamma(\alpha+\beta-3+\epsilon)\Gamma(4-\alpha-\epsilon)\Gamma(2-\beta-\epsilon)}{(p^2)^{\alpha+\beta-2+\epsilon}\;\Gamma(\alpha)\Gamma(\beta)\Gamma(7-\alpha-\beta-2\epsilon)} \times \tag{A.12}$$

$$\times[(\alpha+\beta-3+\epsilon)(4-\alpha-\epsilon)p_\mu p_\nu p_\lambda + \frac{1}{2}(2-\beta-\epsilon)p^2(p_\mu g_{\nu\lambda} + p_\nu g_{\mu\lambda} + p_\lambda g_{\mu\nu})].$$

7. Two-loop integration formulas [17]

$$\frac{(p^2)^{\alpha+\beta+\gamma+\sigma+\rho-4+2\epsilon}}{(i\pi^2)^2}\int \frac{dq\; dt}{t^{2\alpha}q^{2\beta}(p-t)^{2\gamma}(p-q)^{2\sigma}(t-q)^{2\rho}} \equiv V(\alpha,\beta,\gamma,\sigma,\rho).$$

$$V(\alpha,1,\gamma,1,1) = \frac{\Gamma^3(1-\epsilon)\Gamma(-1+2\epsilon)\Gamma(1-\alpha-\epsilon)\Gamma(1-\gamma-\epsilon)\Gamma(\alpha+\gamma-2+2\epsilon)}{\Gamma(\alpha)\Gamma(\gamma)\Gamma(3-\alpha-\gamma-3\epsilon)} \times$$

$$\times[\frac{\Gamma(3-\alpha-\gamma-3\epsilon)}{\Gamma(2-\alpha-\gamma-\epsilon)} - \frac{\Gamma(\alpha+\gamma-1+\epsilon)}{\Gamma(\alpha+\gamma-2+3\epsilon)} + \frac{\Gamma(\alpha)}{\Gamma(\alpha-1+2\epsilon)} + \tag{A.13}$$

$$+\frac{\Gamma(\gamma)}{\Gamma(\gamma-1+2\epsilon)} - \frac{\Gamma(2-\alpha-2\epsilon)}{\Gamma(1-\alpha)} - \frac{\Gamma(2-\gamma-2\epsilon)}{\Gamma(1-\gamma)}],$$

$$V(\alpha,\beta,1,1,\rho)=\frac{\Gamma^3(1-\epsilon)\Gamma(2-\alpha-\epsilon)\Gamma(2-\beta-\epsilon)\Gamma(2-\rho-\epsilon)}{\Gamma(2-2\epsilon)\Gamma(\alpha)\Gamma(\beta)\Gamma(\rho)}\times$$

$$\times\sum_{m,n=0}^{\infty}\frac{(-)^m\Gamma(n+2-2\epsilon)\Gamma(m+n+\alpha+\beta+\rho-2+2\epsilon)}{m!\,n!\,(n+1-\epsilon)\Gamma(4-m-\alpha-\beta-\rho-3\epsilon)\Gamma(m+n+2-\epsilon)}\times$$

$$\times\Big[\frac{1}{(n+\rho)(m+n+\alpha+\rho-1+\epsilon)}+\frac{1}{(n+\rho)(m+n+\beta+\rho-1+\epsilon)}+\frac{1}{(m+n+\alpha)(m+n+\alpha+\rho-1+\epsilon)}+$$

$$+\frac{1}{(m+n+\beta)(m+n+\beta+\rho-1+\epsilon)}+\frac{1}{(m+n+\alpha)(n+2-\rho-2\epsilon)}+\frac{1}{(m+n+\beta)(n+2-\rho-2\epsilon)}\Big]. \qquad \text{(A.14)}$$

8. Individual two-loop integrals

Here we write down the relevant integrals $V(\alpha,\beta,\gamma,\sigma,\rho)$ with all the arguments being positive integers, retaining the $\frac{1}{\epsilon^2},\frac{1}{\epsilon}$ and $O(1)$ terms.

$$V(1,1,1,1,1)=6\zeta(3),$$

$$V(2,1,1,1,1)=\frac{1}{2\epsilon^2}-\frac{1}{2\epsilon}+\frac{1}{2},$$

$$V(1,1,1,1,2)=\frac{1}{\epsilon^2}+\frac{1}{\epsilon}-3,$$

$$V(2,2,1,1,1)=\frac{1}{\epsilon}-\frac{5}{2},$$

$$V(2,1,2,1,1)=\frac{1}{\epsilon^2}-\frac{1}{\epsilon}-1,$$

$$V(2,1,1,2,1)=\frac{2}{\epsilon^2}+\frac{3}{\epsilon}-1,$$

$$V(3,1,1,1,1)=\frac{1}{4\epsilon^2}+\frac{5}{8\epsilon}+\frac{11}{16}.$$

9. Pole parts of the essentially three-loop integrals of the form

$$\frac{(k^2)^{3\epsilon-1}}{(i\pi^2)^3}\int\frac{dp\,dq\,dt\,Y(p,q,t,k)}{p^2q^2t^2(k-p)^2(k-q)^2(k-t)^2(p-q)^2(p-t)^2(q-t)^2}.$$

$Y=(p-t)^8 \Longrightarrow$	$-\frac{2}{3\epsilon^3}-\frac{61}{18\epsilon^2}-\frac{877}{108\epsilon}+\frac{4}{\epsilon}\zeta(3)$,
$(p-t)^6k^2$	$\frac{1}{\epsilon^3}+\frac{41}{6\epsilon^2}+\frac{31}{\epsilon}-\frac{6}{\epsilon}\zeta(3)$,
$(p-t)^4k^4$	$\frac{12}{\epsilon}\zeta(3)$,
$(k-q)^8$	$\frac{2}{3\epsilon^2}+\frac{49}{6\epsilon}+\frac{4}{\epsilon}\zeta(3)$,
$(k-q)^6k^2$	$\frac{1}{3\epsilon^2}+\frac{4}{\epsilon}+\frac{4}{\epsilon}\zeta(3)$,
$(k-q)^4k^4$	$\frac{4}{\epsilon}\zeta(3)$,
$(k-q)^2k^6$	$-\frac{2}{\epsilon}\zeta(3)$,
$(k-q)^4(p-t)^4$	$\frac{1}{2\epsilon^2}+\frac{17}{3\epsilon}$,
$(k-q)^6(p-t)^2$	$\frac{5}{12\epsilon^3}+\frac{73}{24\epsilon^2}+\frac{661}{48\epsilon}$,
$(k-q)^2(p-t)^6$	$-\frac{1}{4\epsilon^3}-\frac{65}{24\epsilon^2}-\frac{865}{48\epsilon}$,
$(k-q)^4(p-t)^2k^2$	$\frac{1}{3\epsilon^3}+\frac{7}{3\epsilon^2}+\frac{31}{3\epsilon}$,
$(k-q)^2(p-t)^4k^2$	$\frac{1}{3\epsilon^3}+\frac{3}{\epsilon^2}+\frac{53}{3\epsilon}$,
$(k-q)^4p^4$	$\frac{1}{6\epsilon^3}+\frac{17}{12\epsilon^2}+\frac{199}{24\epsilon}$,
$(k-q)^6p^2$	$\frac{1}{8\epsilon^3}+\frac{49}{48\epsilon^2}+\frac{531}{96\epsilon}$,
$(k-q)^4p^2k^2$	$\frac{1}{6\epsilon^3}+\frac{3}{2\epsilon^2}+\frac{55}{6\epsilon}$.

REFERENCES

1. Buras A.J. Rev.Mod.Phys., 1980, 52, p.199.
2. Le Guillou J.C., Zinn-Justin J. Phys.Rev.Lett., 1977, 39, p.95; Vladimirov A.A., Kazakov D.I., Tarasov O.V. ZhETF, 1979, 77, p.1035.
3. Brink L., Schwarz J.H., Scherk J. Nucl.Phys., 1977, B121, p.77.
4. Gliozzi F., Scherk J., Olive D. Nucl.Phys., 1977, B122, p.253.
5. Jones D.R.T. Phys.Lett., 1977, 72B, p.199; Poggio E.C., Pendleton H.N. Phys.Lett., 1977, 72B, p.200.
6. Chetyrkin K.G., Kataev A.L., Tkachov F.V. Phys.Lett., 1979, 85B, p.277.
7. Dine M., Sapirstein J. Phys.Rev.Lett., 1979, 43, p.668.
8. Celmaster W., Gonsalves R.J. Phys.Rev.Lett., 1980, 44, p.560.
9. Caswell W.E. Phys.Rev.Lett., 1974, 33, p.244; Jones D.R.T. Nucl.Phys., 1974, B75, p.531.
10. Slavnov A.A. TMF, 1972, 10, p.153; Taylor J.C. Nucl.Phys., 1971, B33, p.436.
11. 't Hooft G. Nucl.Phys., 1973, B61, p.455; Collins J.C., Macfarlane A.J. Phys.Rev., 1974, D10, p.1201.
12. Caswell W.E., Wilczek F. Phys.Lett., 1974, 49B, p.291; Kallosh R.E., Tyutin I.V. Yad.Fiz., 1974, 20, p.1247.
13. Egorian Ed.Sh., Tarasov O.V. TMF, 1979, 41, p.26.
14. Vladimirov A.A. TMF, 1978, 36, p.271.
15. Vladimirov A.A. TMF, 1980, 43, p.210.
16. Speer E.R. J.Math.Phys., 1974, 15, p.1; Collins J.C. Nucl.Phys., 1974, B80, p.341; Breitenlohner P., Maison D. Comm.Math.Phys., 1977, 52, p.55.
17. Chetyrkin K.G., Tkachov F.V. Preprint INR П-0118, Moscow, 1979.
18. Cvitanovic P. Phys.Rev., 1976, D14, p.1536.
19. Strubbe H. Comp.Phys.Comm., 1974, 8, p.1.
20. Vladimirov A.A., Shirkov D.V. Usp.Fiz.Nauk., 1979, 129, p.407.
21. Curtright T., Ghandour G. Ann.Phys., 1977, 106, p.209; Townsend P.K., Nieuwenhuizen van P. Phys.Rev., 1979, D20, p.1832; Sezgin E. Nucl.Phys., 1980, B162, p.1; Siegel W. Phys.Lett., 1979, 84B, p.193.
22. Abbott L., Grisaru M., Schnitzer H. Phys.Rev., 1977, D16, p.2995; Phys.Lett., 1977, 71B, p.161; Curtright T. Phys.Lett., 1977, 71B, p.185.

Received by Publishing Department
on July 9 1980.