объединенный
институт
ядерных
исследований
дубна

E2-80-467

S.Dubnička, D.Krupa, V.A.Meshcheryakov

WHY THE ρ-MESON POLE POSITION COULD NOT BE FOUND FROM THE SPACE-LIKE REGION DATA ON ELECTRIC OR MAGNETIC PROTON FORM FACTORS?

Submitted to "Acta Physica Slovaka"

1980

Дубничка С.,Крупа Д., Мещеряков В.А. E2-80-467

Почему положение ρ-мезонного полюса не может быть определено из данных по электрическому и магнитному формфактору протона в пространственно-подобной области?

С помощью паде-приближения по переменной, выбранной с помощью конформного преобразования, проведен анализ изовекторной части электрического формфактора протона. Обнаружен стабильный полюс на месте короткого разреза на нефизическом листе. Разрез возникает как проекция борновского члена амплитуды πN-рассеяния. Результат анализа показывает, что влияние этого короткого разреза на втором листе римановой поверхности сравнимо со вкладом неупругого канала $K\bar{K}$ и оба они препятствуют определению параметров ρ-мезона из экспериментальных данных по электрическому формфактору протона в пространственно-подобной области.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1980

Dubnička S., Krupa D., Meshcheryakov V.A. E2-80-467

Why the ρ-Meson Pole Position Could Not Be Found from the Space-Like Region Data on Electric or Magnetic Proton Form Factors?

1. INTRODUCTION

The nucleon form factors are believed to be analytic functions in the entire squared four-momentum transfer t-plane except for a cut from t_0 to infinity. There are no poles on the first (physical) sheet and all singularities are restricted only to a sequence of threshold branch points on the positive real axis.

It was 20 years ago when Frazer and Fulco [1] have found, using these analyticity properties, that a resonance of suitable position and width to the $J=1, I=1$ state of the pion-pion system could bring the dispersion-theoretical calculation of the nucleon electromagnetic structure into a qualitative agreement with experiment. At present this resonance, the so-called ρ-meson, is well established and experiments on $e^+e^- \to \pi^+\pi^-$ have demonstrated how much it dominates the pion and hence the nucleon form factors. From the time of Frazer and Fulco a large number of papers have been devoted to the investigation of the electromagnetic structure of nucleon by using the analyticity and a complete review of references can be found in Höhler's paper [2].

As the experimental information on the nucleon form factors in the space-like region $(t<0)$ was improved and as in the near future one can also expect a considerable improvement of the present poor experimental information in the time-like region $(t>0)$, the more realistic models for the nucleon form factors, like in the case of the pion form factor [3], are desirable. In order to construct such a model we have to know, first of all, which of all present singularities have to be taken into account. The only economical way to reveal the most important singularities seems to be to start with the description of the experimental data by means of the Padé-type approximants in the suitably chosen variable which, as we know, are able to reproduce all the analytic structure of the function under consideration [4,5]. So, if the ρ-meson resonance is really dominant in the proton form factors as it follows from Frazer and Fulco analysis [1], then one has to find it by means of the Padé approximations constructed from the coefficients of a polynomial fitting the experimental data as it was done in the case of the pion form factor [5].

It was really claimed by Dumbrajs [6] that the application of Padé approximants to the electric proton form factor gives the stable ρ-meson pole on the second Riemann sheet. This result was surprising with regard to the involved extrapolation through a cut, therefore, Bowcock et al. [7] have decided to apply the above procedure, in order to test it, to a simple model based on a known mathematical function where singularities are easily identifiable. The comparison of the Dumbrajs procedure and the direct fit of simulated data by means of a rational function (we call it Padé-type approximation) clearly demonstrates the deficiencies of the former and the stability of the later. So, the authors of the paper [7] applied the direct fitting procedure to the electric proton form factor data to see what it predicts regarding poles and zeros. However, they found some poles on the first sheet and no ρ-meson poles. By means of this analysis they came to the conclusion that the very precise results obtained for the ρ pole in ref. [1] must be fortuitous. In this papers we present some arguments why the authors in ref. [7] could not find the ρ-meson poles as they expected to find according to the results of Dumbrajs.

2. THE ANALYTIC PROPERTIES OF ELECTRIC AND MAGNETIC PROTON FORM FACTORS

Aside from radiative corrections, electron nucleon elastic scattering, in which the electromagnetic structure of the nucleon is measured, can be described in the one photon exchange approximation. As a consequence the corresponding differential cross section in the laboratory system takes the following form [8]

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_0^2}\,\frac{\cos^2(\frac{\theta}{2})}{\sin^4(\frac{\theta}{2})}\,\frac{1}{1+\frac{2E_0}{m_N}\sin^2(\frac{\theta}{2})}\times \tag{1}$$

$$\times\{F_1^2 - \frac{t}{4m_N^2}[2(F_1+2m_NF_2)^2\,\mathrm{tg}^2(\frac{\theta}{2}) + (2m_NF_2)^2]\},$$

where $F_1(t)$ and $F_2(t)$ are the Dirac and Pauli form factors; α, the fine structure constant; E_0, the incident electron energy; θ, the scattering angle; m_N, the nucleon mass and the approximation $\frac{m_e}{E_0} \approx \frac{m_e}{m_N} \approx 0$ (m_e means the electron mass) was taken into account. $F_1(t)$ and $F_2(t)$ are real valued functions in the space-like region ($t<0$) and different for proton and neutron.

From the theoretical point of view it is convenient both form factors $F_1(t)$, $F_2(t)$ for proton and neutron to decompose into the isovector and isoscalar parts in the following way

$$F_1^p(t) = \frac{1}{2}(F_1^S + F_1^V); \quad F_2^p(t) = \frac{1}{2}(F_2^S + F_2^V);$$
$$F_1^n(t) = \frac{1}{2}(F_1^S - F_1^V); \quad F_2^n(t) = \frac{1}{2}(F_2^S - F_2^V) \tag{2}$$

from which it is straightforward to find

$$F_1^S = F_1^p(t) + F_1^n(t); \quad F_1^V = F_1^p(t) - F_1^n(t);$$
$$F_2^S = F_2^p(t) + F_2^n(t); \quad F_2^V = F_2^p(t) - F_1^n(t). \tag{3}$$

Reason for the decomposition of $F_1(t)$, $F_2(t)$ into the isovector and isoscalar parts is that there is a general belief that the analytic properties of the isovector parts on the physical sheet of t-variable are restricted only to the sequence of threshold branch points at $t = 4m_\pi^2$, $16m_\pi^2$, $4m_K^2$, $4m_N^2$... on the positive real axis and the analytic properties of isoscalar parts to the sequence of threshold branch points at $t = 9m_\pi^2$, $25m_\pi^2$, $4m_K^2$, $4m_N^2$, ..., where m_i ($i=\pi, K, N$) is the mass of the pion, kaon, and nucleon, respectively. Moreover, in the generalized vector meson dominance model the isovector parts are dominated only by pure isovector vector meson like ρ, ρ', ρ'' and the isoscalar parts only by pure isoscalar vector mesons like ω, ϕ, ω', ϕ', ψ,... which in a more realistic form factor model can be taken into account as poles on unphysical sheets of the corresponding Riemann surface.

Now it is not difficult to understand to what extent the analytic structure of Dirac and Pauli form factors is complicated. These complications are transferred also in the electric and magnetic proton form factors $G_E^p(t)$ and $G_M^p(t)$ respectively as they are defined by the relations

$$G_E^p(t) = F_1^p(t) + \frac{t}{4m_p^2}F_2^p(t),$$
$$G_M^p(t) = F_1^p(t) + F_2^p(t) \tag{4}$$

in order to simplify the extraction of the experimental information on the electromagnetic structure of the nucleon from the differential cross section (1).

Dumbrajs[6] and Bowcock et al.[7] by using these complicated analytic properties and not very ingenious conformal mapping (the first sheet is mapped into the interior of the unit circle and the second sheet outside the unit circle) conjectured to determine the ρ-meson pole position on the second Riemann sheet from the data on G^p_E and G^p_M in space-like region. Of course, Bowcock et al.[7] have found some effective poles which in our opinion represent the contributions of important cuts on the unit circle in the conformally mapped plane, the ρ-meson poles placed outside the unit circle and also the contribution of the isoscalar vector mesons like ω, ϕ which are present in both G^p_E and G^p_M form factors. And as to the results of Dumbrajs[6], they seem to be really fortiutous.

In order to bring more light into these problems we have carried out the analysis only with the isovector part of the electric proton form factor that is free of the cut contributions of the isoscalar part and also the contribution of isoscalar vector mesons like ω, ϕ, etc. The results of the analysis are presented in the next section.

3. THE ANALYSIS OF DATA ON THE ISOVECTOR PART OF THE ELECTRIC NUCLEON FORM FACTOR

If we substitute instead of F^p_1 and F^p_2 in (4) the relations (2) and define the isoscalar and isovector parts of the electric nucleon form factor by the relations

$$G^S_E(t) = F^S_1(t) + \frac{t}{4m^2_N} F^S_2(t) ,$$
$$G^V_E(t) = F^V_1(t) + \frac{t}{4m^2_N} F^V_2(t) , \tag{5}$$

one can obtain the decomposition of the electric proton form factor into isoscalar and isovector parts as follows

$$G^p_E(t) = \frac{1}{2}\{G^S_E(t) + G^V_E(t)\} . \tag{6}$$

In a like manner the expression for the electric neutron form factor

$$G^n_E(t) = \frac{1}{2}\{G^S_E(t) - G^V_E(t)\} \tag{7}$$

can be found.

By solving (6) and (7) we get the isovector part of the electric nucleon form factor expressed through the electric proton and electric neutron form factors in the following way

$$G^V_E(t) = G^p_E(t) - G^n_E(t) . \tag{8}$$

By using the data on $G^p_E(t)$ and $G^n_E(t)$ one can obtain the data on $G^V_E(t)$.

For $G^p_E(t)$ we used the same data points[10,11] for $-25.030\ \mathrm{GeV}^2 \leq t \leq -0.0078\ \mathrm{GeV}^2$ as Dumbrajs[1] and Bowcock et al.[7] in order to make a meaningful comparison with their results. However we have the data on $G^n_E(t)$ only for $-1.530\ \mathrm{GeV}^2 \leq t \leq 0.010\ \mathrm{GeV}^2$ at our disposal. Moreover, the data from this interval even do not coincide in t with the data on $G^p_E(t)$. For this reason, for $G^n_E(t)$ we have used the zero width approximation model of Zovko[12] that describes not only the data on $G^n_E(t)$ but also reproduces all the existing data on proton and neutron form factors in space-like region only by four adjustable parameters. The later is a sufficiently justified argument to have a confidence in the expression[12]

$$G^n_E(t) = \{\frac{1}{2} + \frac{\mu^S + 2m^2_M b^S}{4m^2_M} t\}[(1 - \frac{t}{m^2_\omega})(1 - \frac{t}{m^2_\phi})(1 - \frac{t}{m^2_{\omega'}})]^{-1} - \{\frac{1}{2} + \frac{\mu^V + 2m^2_M b^V}{4m^2_M} t\}[(1 - \frac{t}{m^2_\rho})(1 - \frac{t}{m^2_{\rho'}})(1 - \frac{t}{m^2_{\rho''}})]^{-1} \tag{9}$$

with $m^2_\omega = 0.614\ \mathrm{GeV}^2$, $m^2_\phi = 1.039\ \mathrm{GeV}^2$, $m^2_{\omega'} = 1.4\ \mathrm{GeV}^2$, $m^2_\rho = 0.585\ \mathrm{GeV}$, $m^2_{\rho'} = 1.3\ \mathrm{GeV}^2$, $m^2_{\rho''} = 2.1\ \mathrm{GeV}^2$, $\mu^S = -0.12$, $b^S = -0.91\ \mathrm{GeV}^{-2}$, $\mu^V = 0.925$, $b^V = -1.10\ \mathrm{GeV}^{-2}$ and m_M as the neutron mass to be used to give through (8) 68 experimental points on $G^V_E(t)$ in the range of momenta $-25.030\ \mathrm{GeV}^2 \leq t \leq -0.0078\ \mathrm{GeV}^2$ with the errors of $G^V_E(t)$ to be found only from the errors of $G^p_E(t)$.

We emphasize again that by means of the afore-mentioned procedure we have obtained the data on the isovector part of the electric nucleon form factor, the analytic properties of which consist only of the threshold branch points at $t = 4m^2_\pi$, $16m^2_\pi$, $4m^2_K$, $4m^2_N$, etc. According to the vector dominance model it should be dominated first of all by the ρ-meson exchange contributions. So, if one has any chance to find the ρ-meson pole position from the data on the nucleon form factor, then first of all it has to be found from the data on $G^V_E(t)$.

We have carried out two independent analyses of these data by means of the Padé-type approximations for two different variables, on two different computers, and by two different minimization programs.

The first analysis was carried out at Bratislava on the computer SIEMENS by using the CERN minimization program MINUIT [13]. The results of the direct fit of the data on $G_E^V(t)$ by means of the following normalized to 1/2 (then the errors of G_E^V are taken to be a half errors of G_E^p) and respecting the real analyticity Padé-type approximation

$$G_E^V(t) = \frac{1}{2}\,\frac{A_1 + \sum\limits_{n=1}^{M} A_{2n+1}(iq)^n}{1 + \sum\limits_{n=1}^{M} A_{2n}(iq)^n}\;\frac{1 + \sum\limits_{n=1}^{M} A_{2n}(-1)^n}{A_1 + \sum\limits_{n=1}^{M} A_{2n+1}(-1)^n} \qquad (10)$$

on the pion c.m. momentum q-plane are presented in Table 1. The q - plane is obtained from the t-plane by the conformal mapping

$$q = \sqrt{\frac{t - 4m_\pi^2}{4}} \qquad (11)$$

with $m_\pi = 1$.

Table 1

The results of analysis of the data on G_E^V by the Padé-type approximations (10) in q-plane

[N/M]	χ^2	χ^2/ndf	Position of zeros	Position of poles
[2/2]	8I8.607	I2.99	iI4.32I iI2.856	I.640 +i0.624 -I.640 +i0.624
[2/3]	I29.242	2.08	i2I.044 i20.I43	2.I60 +iI.632 -2.I60 +iI.632 -i0.5I3
[3/3]	II2.973	I.85	-i25.9I2 3.678 +iI5.569 -3.678 +iI5.569	2.226 +iI.489 -2.226 +iI.489 -i0.65I
[3/4]	II2.35I	I.87	-i3I.2I8 3.830 +iI5.743 -3.830 +iI5.743	-i757.837 2.236 +iI.497 -2.236 +iI.497 -i0.66I
[4/4]	II2.532	I.9I	+iI97.676 -i 24.6I3 3.9I7 +i I5.848 -3.9I7 +i I5.848	-i289.494 2.230 +iI.504 -2.230 +iI.504 -i0.644
[5/5]	I09.365	I.92	-i 7.977 -i 7.977 -5.4I9 +i I7.79I 5.4I9 +i I7.79I -i 7.977	3.478 -i0.470 -3.478 -i0.470 2.833 +iI.I9I -2.833 +iI.I9I -iI.305

The second analysis was carried out at Dubna on the computer CDC-6500 by using the minimization program FUMILI [14]. In this case a more ingenious conformal mapping was used. The q-plane was first turned at 90° in the anti-clockwise direction and then shifted so that all the data on $G_E^V(t)$ from the range of momenta $-25.0300\ \text{GeV}^2 \leq t \leq -0.0078\ \text{GeV}^2$ be placed symmetrically on the real axis around the origin of this new k-plane. All this could be achieved by the following conformal mapping

$$k = -\sqrt{\frac{4-t}{4}} + k_0 , \qquad (12)$$

where $k_0 \approx 9.5$. The results of the direct fit of the data on $G_E^V(t)$ by the normalized to one and respecting the real analyticity Padé-type approximation

$$G_E^V(t) = \frac{1 + \sum\limits_{n=1}^{M} A_{2n-1}(k_0-1)^n + \sum\limits_{n=1}^{M} A_{2n}\left[k^n - (k_0-1)^n\right]}{1 + \sum\limits_{n=1}^{M} A_{2n-1}\,k^n} \qquad (13)$$

on the k-plane are presented in Table 2.

Table 2

The results of analysis of the data on G_E^V by the Padé-type approximation (13) in k-plane

[N/M]	χ^2	χ^2/ndf	Position of zeros	Position of poles
[I/2]	3932.0	60.49I	-3.2I3	8.7I52 +iI.2459 8.7I52 -iI.2459
[2/2]	735.5	II.492	-3.6422 +i0.92I3 -3.6422 -i0.92I3	8.8908 +iI.6666 8.8908 -iI.6666
[2/3]	I04.8	I.664	-7.03I5 +i4.I079 -7.03I5 -i4.I079	I0.2895 8.0323 +i2.3II5 8.0323 -i2.3II5
[3/2]	225.9	3.586	-7.0690 -I.8304 -i3.5258 -I.8304 +i3.5258	9.0289 +i2.059I 9.0289 -i2.059I
[3/3]	I04.0	I.677	-I2.6486 - 9.2957 -i7.4849 - 9.2957 +i7.4849	I0.4563 8.072I +i2.3888 8.072I -i2.3888
[4/3]	I02.3	I.677	2.8032 +iI0.0472 2.8032 -iI0.0472 - 6.6987 +i2.4955 - 6.6987 -i2.4955	I0.8927 8.0803 +i2.5540 8.0803 -i2.5540

The q-plane and k-plane were prefered to be used in the analysis of the data on $G_E^V(t)$ by the following motivations. First, $G_E^V(t)$ by the mappings (11) and (12) is regularized at the first threshold branch point $t = 4m_\pi^2$ and the contribution of the corresponding two pion out is thus authomatically eliminated. Second, as the $e^+e^- \to 4\pi$ experiments indicate that the cross section remains small below 1 GeV [15], we neglect the four-pion cut and as a consequence, the ρ-meson pole is undoubtedly placed on the second Riemann sheet, that is reached through the two-pion cut from $t = 4m_\pi^2$ to $t = 4m_K^2$. Third, both Riemann sheets are in the q-plane as well as in the k-plane placed equivalently. So, we avoid the difficulty of analytic continuation from the interior of the unit circle through branch cuts to outside the unit circle, what is characteristic for Dumbrajs [6] as well as Bowcock et al. [7] procedures.

As one can see from Tables 1 and 2 immediately, the almost identical results are obtained independent of the variable used and minimization programs. One stable pole in q-plane around the value $q = -i0.65$ that corresponds to $k \approx 10.15$ in k-plane just below the threshold $t = 4m_\pi^2$ in the second Riemann sheet is found. There are other two symmetrically placed stable poles (see Table 1 for $q \approx \pm 2.5 + i1.5$ and Table 2 for $k \approx 8 \pm i2.5$) but no ρ-meson has been found on the second Riemann sheet, which is placed exactly at $q_\rho = \pm 2.59 - i0.30$ and $k_\rho = 9.8 \pm i2.59$ in the q- and k-plane respectively.

The first stable pole at $q \approx -i0.6$ or $k \approx 10$ visibly simulates the contribution of the short branch cut just below the threshold $t = 4m_\pi^2$ in the second Riemann sheet which comes from the partial wave projection of the nucleon Born term in the πN scattering amplitude [2]. The two symmetrically placed poles at $q \approx \pm 2.5 + i1.5$ or $k \approx 8 \pm i2.5$ in our opinion effectively take into account both the ρ-meson contribution as well as $K\bar{K}$ branch cut contribution which in comparison with the ρ-meson seems to be in the isovector part of the electric nucleon form factor considerable.

So, the results of our analysis support the conclusion of Höhler et al. [16] that unlike the pion form factor $G_E^V(t)$ besides the ρ-meson and $K\bar{K}$ branch cut contributions is dominated by the nucleon exchange partial wave projection branch cut in the second Riemann sheet which is responsible, for instance, for the difference between the nucleon and pion charge radii.

4. CONCLUSIONS

By means of a direct least-square fit of the normalized and respecting the real analyticity Padé-tape approximations in a conformally mapped variable to the data on the isovector part of the electric nucleon form factor we have found most important singularities dominating $G_E^V(t)$. Unlike the pion form factor, the isovector part of the electric nucleon form factor is not given mainly by the ρ-exchange contribution but the short branch cut just below the threshold $t = 4m_\pi^2$ on the second Riemann sheet that comes from the partial wave projection of the nucleon Born term in the πN scattering amplitude [2] plays an important role. The later explains the fact why many people [10,17,18] fitting the data on the nucleon form factors in the framework of the extended vector dominance model fixed the ρ-meson mass and did not search it in the fitting procedure.

REFERENCES

1. Frazer W.R., Fulco J.R. Phys.Rev., 1960, 117, p.1609.
2. Höhler G. Lecture given at the Int.Summer Institute on Current Induced Reactions. DESY-Hamburg, September 15-25, 1975.
3. Dubnička S., Dubničkova A.Z., Meshcheryakov V.A. Czech. J.Phys., 1979, B29, p.142.
4. Basdevant J.-L. Fort. d.Phys., 1972, 20, p.283.
5. Dubnička S., Krupa D., Martinovič Ľ. Acta Phys.Slov., 1979, 29, p.201.
6. Dumbrajs O. Rev.Roum.Phys., 1976, 21, p.273.
7. Bowcock J.E., Dacunha N.M.C., Qeen N.M. Rev.Roum.Phys., 1978, 23, p.549.
8. Rosenbluth M.N. Phys.Rev., 1950, 79, p.615.
9. Sachs R.G. Phys.Rev., 1962, 126, p.2256.
10. Blatnik S., Zovko N. Acta Phys.Austriaca, 1974, 39, p.62.
11. Akimov Yu.K. et al. Zh.Eksp.Teor.Fiz., 1972, 62, p.1231; Sov.Phys. JETP, 1972, 35, p.651.
12. Zovko N. Fort. Phys., 1975, 23, p.185.
13. James F., Roos M. Computer Phys.Comm., 1975, 10, No.6, p.343.
14. The Program Library in FORTRAN. JINR, D-520, vol.2 (In Russian), Dubna, 1970.
15. Conversi M. et al. Phys.Lett., 1974, 52B, p.493.
16. Höhler G., Kichlmann H.-D., Schmidt W. Phys.Rev., 1975, D11, p.2667.

17. Mehrotra S., Ross M. University of Helsinki Preprint No.97, Helsinki, 1975.
18. Skachkov N.B., Solovtsov I.L. JINR, E2-9504, Dubna, 1976.

Received by Publishing Department
on August 4 1980.

D1,2-12036	Proceedings of the V International Seminar on High Energy Physics Problems. Dubna, 1978.	15.00
D1,2-12450	Proceedings of the XII International School on High Energy Physics for Young Scientists. Bulgaria, Primorsko, 1978.	18.00
R2-12462	Proceedings of the V International Symposium on Nonlocal Field Theories. Alushta, 1979.	9.50
D-12831	The Proceedings of the International Symposium on Fundamental Problems of Theoretical and Mathematical Physics. Dubna 1979.	8.00
D-12965	The Proceedings of the International School on the Problems of Charged Particle Accelerators for Young Scientists. Minsk 1979.	8.00
D11-80-13	The Proceedings of the International Conference on Systems and Techniques of Analytical Computing and their Applications in Theoretical Physics. Dubna, 1979.	8.00
D4-80-271	The Proceedings of the International Symposium on Few Particle Problems in Nuclear Physics. Dubna, 1979.	8.50
D4-80-385	The Proceedings of the International School on Nuclear Structure. Alushta, 1980.	10.00