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THE SIX-QUARK STATE WIDTHS IN THE NUCLEON-NUCLEON SCATTERING

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The problem of the possible existence of dibaryon states is widely discussed [1]. These states as 6-quark objects are predicted by the quark spectroscopy calculations [2]. Furthermore, one of the sets of phase shifts extracted from the pp-scattering with parallel and antiparallel spins [3] has the resonance behaviour, the fact, which may also be interpreted as an experimental indication of the dibaryon-states existence. However, the investigation of this problem may have some sense if the theory estimates the widths of these states. Remember that the usual quark spectroscopy gives zero widths while experiments apparently deal with real widths, which have large values.

We consider this problem on the base of the coupled channel equations obtained earlier [4], which were used for estimating the 6-quark admixture in the bound state deuteron wave function. Here we solve the equations for the continuum where the resonance behaviour of the pp-amplitude is assumed.

The total wave function of the six-quark system obeys the wave equation $(t_{6q}+V-E)\Psi=0$, where the real potential V generally speaking, is unknown. In the case under investigation this function is usually represented as

$$\Psi = \phi(\vec{R})\chi(\xi) + \sum_{\alpha} C_{\alpha}\Psi_{\alpha}(\vec{R},\xi) . \qquad (1)$$

The first term here describes the nucleon-nucleon channel and includes the relative $\phi(\vec{R})$ and internal motion wave function

$\chi = \chi_1 \chi_2$ of the 3q-clusters which become nucleons at $R \to \infty$ ($(t_{3q} + V^q_{12}(3q) - M)\chi_{1,2} = 0$, M is the mass of a nucleon). The second term in (1) is the true 6q-state existing only inside the bag region $(R, \xi < R_b \approx 1 fm)$, where the quark-quark potential $V^q(6q) = \sum_{i<j}^{6} V_{ij}$ acts $((t_{6q} + V^q(6q) - E_\alpha)\psi_\alpha = 0)$. These 6q-states in (1) have the amplitudes C_α . The function $\phi(\vec{R})$ and the C_α -amplitudes can be derived from the coupled channel equations [4]:

$$(T(\vec{R}) + U(\vec{R}) - \epsilon)\,\phi(\vec{R}) = -\sum_\alpha C_\alpha D_\alpha(\vec{R}) \tag{2a}$$

$$(E - E_\alpha) C_\alpha = \langle B_\alpha(\vec{R}) / \phi(\vec{R}) \rangle \tag{2b}$$

with

$$U(\vec{R}) = \langle \chi_1 \chi_2 / V - V_1^q(3q) - V_2^q(3q) / \chi_1 \chi_2 \rangle \tag{3a}$$

$$D_\alpha(\vec{R}) = \langle \chi_1 \chi_2 / V - V^q(6q) / \psi_\alpha \rangle \tag{3b}$$

$$B_\alpha(\vec{R}) = \langle \psi_\alpha / V - V_1^q(3q) - V_2^q(3q) / \chi_1 \chi_2 \rangle \tag{3c}$$

Here $\epsilon = E - 2M$ and $U(\vec{R})$ are the kinetic and potential energies in the nucleon-nucleon channels, B_α and D_α characterize the space distribution of the transition amplitudes from the 6q-state to the nucleon-nucleon one.

To solve the coupled equations (2) we note that for the energy E near the resonance E_α eq.(2b) demands C_α to be rather large. Thus the right-hand side of eq.(2a) cannot be neglected as has been done in calculating the C_α -admixture for the ground state of a deuteron [4]. Instead, we write the general solution of (2a) as follows

$$\phi^{(+)}(\vec{R}) = \bar{\phi}^{(+)}(\vec{R}) - \sum_\alpha C_\alpha g_\alpha^{(+)}(\vec{R}) \tag{4}$$

with

$$g_\alpha^{(+)}(\vec{R}) = \int d\vec{R}'\, G^{(+)}(\vec{R}\vec{R}')\, D_\alpha(\vec{R}'), \tag{5}$$

where $\bar{\phi}^{(+)}(\vec{R})$ is the solution of the homogeneous eq.(2a) with $C_\alpha = 0$, and $G^{(+)}(\vec{R}\vec{R}')$ is the corresponding Green function. Substituting (4) into (2b) one gets the following set of equations for C_α -coefficients:

$$\sum_{\alpha'}\left[(E-E_{\alpha'})\delta_{\alpha\alpha'}+\langle B_\alpha(\vec{R})|g_{\alpha'}(\vec{R})\rangle\right]C_{\alpha'}=\langle B_\alpha(\vec{R})|\bar{\Phi}^{(+)}(\vec{R})\rangle. \quad (6)$$

For simplicity we consider the one-channel approximation assuming only one 6q-state to exist. Then the solution obtained is the following

$$C_0=\frac{\langle B_0(\vec{R})|\bar{\Phi}^{(+)}(\vec{R})\rangle}{E-E_0+\Delta_0+i\Gamma_0/2} \quad (7)$$

with the width and shift of the E_0 -level

$$\Gamma_0=2\,\mathrm{Im}\langle B_0(\vec{R})|g_0^{(+)}(\vec{R})\rangle\ ;\quad \Delta_0=2\,\mathrm{Re}\langle B_0(\vec{R})|g_0^{(+)}(\vec{R})\rangle \quad (8)$$

For estimating Γ_0 and Δ_0 we restrict ourselves to the case of s-scattering (l=0). First, we need the matrix elements (3). The main difficulty in their calculation is to determine the real interaction potential V. It is natural to suggest that inside the 6q-bag region $(R,\xi<R_6)$ it coincides with the model one

$$V\simeq V^q(6q)=\sum_{i<j}^{6}V_{ij} \qquad (R,r_{ij}<R_6) \quad (9)$$

where one can take the qq-potential in the oscillator form /5/

$$V_{ij}=V_{ij}^{I}+V_{ij}^{II}$$

$$V_{ij}^{I}=-\frac{K}{2}\lambda_i^a\lambda_j^a(\vec{r}_i-\vec{r}_j)^2;\qquad V_{ij}^{II}=\frac{1}{6}\left(\frac{\hbar}{m_q c}\right)^2\vec{\sigma}_i\vec{\sigma}_j\,\nabla^2V_{ij}^{I}. \quad (9a)$$

Here λ^a are the SU(3) colour matrices, the parameters $m_q c^2$ =0.151 GeV and $E_0=\left(\frac{\hbar K}{m_q}\right)^{1/2}$ =0.075 GeV are chosen so as to explain the masses of the p-, Δ -particles and $\langle r^2\rangle_p^{1/2}\simeq 1$fm. However, the model 6q-potential (9) does not include the long range part because of the necessity to confine quarks to the region of an order of the nucleon radius $\langle r^2 \rangle_p^{1/2}$. This means that outside of the 6q-bag region, when the 3q-clusters are separated $(R > R_b > r_{ij})$, we have the following relationship

$$V^q(6q) \approx V_1^q(3q) + V_2^q(3q) \quad (R > R_b). \qquad (10)$$

Inspite of this the real potential V is known to contain the meson tails in the nucleon-nucleon channel and thus can be represented as a sum

$$V \approx U(\vec{R}) + V_1^q(3q) + V_2^q(3q) \qquad \text{at } R > R_b \qquad (11)$$

(For discussion of this problem see ref.[6]). Now one can conclude that an interaction in the nucleon-nucleon channel (3a) may be better considered phenomenologically. We choose $U(\vec{R})$ throughout the region of R in the form of the hard core Reid potential [7]

$$U(R) = V_1\,\theta(R-R_c) + V_2\,\theta(R_c-R)\,v(R) \qquad (12)$$

where V_2 =-0.6 GeV, R_c =0.4 fm and $v(R)$ falls exponentially down to a few MeV at $R = R_c + a$ with $a \approx 0.25$ fm.

The B_α matrix element describes a transition in the internal region $R, r_{ij} < R_b$ because of the presence in (3c) of the 6q-state function ψ_α . Substituting (9), (9a) into (3c) one gets

$$B_\alpha = \langle \psi_\alpha / \sum_{\substack{i=123\\ j=456}} (V_{ij}^I + V_{ij}^{II}) / \chi_1 \chi_2 \rangle \qquad (13)$$

The matrix element of this type was calculated in [4] and for the s^6 -space configuration of ψ_α with the deuteron-like quantum numbers $J=1, (\ell=0), I=0$ one can obtain the following result

$$B_0 = \frac{b_0(R)}{R\sqrt{4\pi}}\,; \qquad b_0 = \tau \cdot \psi_0(R)\,, \qquad (14)$$

$$\psi_0(R) = \sqrt{4\pi}\left(\frac{\Omega_6}{\pi}\right)^{3/4} R\, e^{-\frac{1}{2}\Omega_6 R^2}\,, \qquad (14a)$$

where $\varphi_0(R)$ is the radial s-wave in the bag region, normalized to 1; $\Omega_6 = 24\sqrt{\frac{2}{3}}\, m_q c E_0 = 0.172\ GeV^{-2}$, so that the true 6q-configuration has dimensions of the order $R_6 = \frac{1}{2}\Omega_6^{-1/2} \approx 1 fm$; $\mathcal{E} =$ 0.108 GeV is the calculated colour-spin-isospin part of (13).

As to the D_0 matrix element (3b) we write it with the use of eqs. (10)-(12)

$$\mathcal{D}_0 = \frac{d_0(R)}{R\sqrt{4\pi}}\ ; \qquad d_0 = V_2\, \theta(R_c - R) V(R) \varphi_0(R) \tag{15}$$

Thus, D_α has the meaning of the peripheral transition $(R \approx R_c \div R_c + a)$ from the nucleon-nucleon to the 6q-state. In calculating $\langle B_\alpha | g_\alpha^{(+)} \rangle$ by using eq.(5) we impose the condition $R' > R$ for the Green function taking into account that $D_\alpha(\vec{R})$ and $B_\alpha(\vec{R})$ overlap weakly.

To determine the scattering matrix we substitute (7) into (4) and tend asymptotically $R \to \infty$, where $G^{(+)}(RR') \sim \frac{e^{ikR}}{R}\phi^{(-)*}(R')$. Thus for the s-wave radial part of $\phi(\vec{R})$ one gets

$$\phi_0(R) \approx e^{-ikR} - e^{2i\delta_0} S_R\, e^{ikR} \tag{16}$$

with the resonant part of the S-matrix

$$S_R = \frac{E - E_0 + \Delta_0 - i\Gamma_0/2}{E - E_0 + \Delta_0 + i\Gamma_0/2}\ , \tag{17}$$

where

$$\Gamma_0 = \frac{4\mu}{\hbar k}\int f_0(R) d_0(R)\, dR \int f_0(R) b_0(R)\, dR\ . \tag{18}$$

Here $\mu = M/2$ is the reduced mass and $f_0(R)$ is the regular solution $(f_0(R \to \infty) = \sin(kR + \delta_0))$ of the radial part of the homogeneous equation (2a) with the potential (12). In the most significant region near the core $R_c \div R_c + a$ one can take a simple approximation of an exact numerical solution /8/

$$f_0 = \cos[\bar{k}(R - R_c)]\ ; \qquad \bar{k} = 1.2\ GeV/c \tag{19}$$

Substituting (14), (15), (19) into (18) we obtain for the energy $\epsilon = 0.3$ GeV in the c.m. nucleon-nucleon system (k=0.8 GeV/c):

$$\Gamma_0 \approx 20 MeV . \qquad (20)$$

It is the qualitative result and the influence of relativization should be taken into account. However, it is partly accounted for by the choice of the kinetic and potential term parameters in the nonrelativistic wave equation. The coupling with the other α-channels will lead to an enhancement of Γ towards the discussed value $(\Gamma_{exp} \sim 100 MeV)$ from $\vec{p}\vec{p}$-scattering /3/.

In conclusion we note:

First, the coupled channel method we exploit here may be used successfully for calculating the quark-nuclear wave functions and then for considering the quark-nuclear exotic phenomena (widths and shifts of the multiquark levels in nuclei (if any exist), nuclear form factors, and reactions at high momentum transfers, etc.). Second, the quark spectroscopy calculations based only on the qq-interactions should be revised when an application to nuclei is needed because of the strong coupling of the multiquark and nucleon channels.

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