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THE DATA ON THE ELECTROMAGNETIC PION FORM FACTOR AND P-WAVE ISOVECTOR $\pi\pi$ PHASE SHIFT ARE DEFINITELY CONSISTENT

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О согласованности данных по пионному формфактору и по изовекторной фазе P-волнового $\pi\pi$-рассеяния

Данные по абсолютному значению формфактора пиона /свободные от вклада ω-мезона/ объединены с данными по P-волновому изовекторному $\pi\pi$-фазовому сдвигу. Полученные в результате действительная и мнимая части пионного формфактора описываются посредством паде-аппроксимации. Результат проведенного анализа троякий: 1/ установлено, что все данные, а именно, экспериментальные точки по пионному формфактору из интервала импульсов $-0{,}8432$ ГэВ$^2 \leq t \leq 1$ ГэВ^2, пионный зарядовый радиус и P-волновой изовекторный $\pi\pi$-фазовый сдвиг в упругой области /с учетом общепринятого значения длины рассеяния a_1^1/, взаимно совместны; 2/ анализ этих данных с помощью паде-аппроксимации показал, что вышеупомянутую совместность можно достигнуть только тогда, когда учитывается левый разрез пионного формфактора на втором римановом листе; 3/ почти во всех рассмотренных паде-аппроксимациях обнаружен один устойчивый полюс пионного формфактора в пространственно-подобной области, что могло бы служить признаком существования дифракционного минимума в дифференциальном сечении упругого $e^-\pi$-рассеяния, обусловленного составной структурой пиона, подобно тому, как это имеет место в случае упругого рассеяния электронов ядрами.

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The Data on the Electromagnetic Pion Form Factor and P-Wave Isovector $\pi\pi$ Phase Shift are Definitely Consistent

1. INTRODUCTION

The pion form factor model proposed in the papers [1,2] reflects all the fundamental properties and describes all the data both in spacelike and in timelike regions simultaneously. However, the pion form factor phase $\delta_\pi(t)$ (t being four momentum transfer squared) as well as the value of the $\pi\pi$-scattering length a_1^1 do not agree (see ref. [2]) with the data on P-wave isovector $\pi\pi$ phase shift $\delta_1^1(t)$ in the elastic region and the generally accepted value of a_1^1, respectively. This could be explained naturally by inconsistency of the pion form factor data and the data on the P-wave isovector $\pi\pi$ phase shift demonstrated recently by Hammer et al. [3]. However, this question was investigated carefully in paper [4], and the following results were obtained. If in the analysis the pion form factor data from the range of momenta -0.29 GeV$^2 \leq t \leq 0.490$ GeV^2 are used only, the description of which can be achieved by a polynomial parametrization respecting all the basis principles, no discrepancy is found.

So, in this paper by using the Pade-type approximation for the description of unified data on the pion form factor and the P-wave isovector $\pi\pi$ phase shift we carry out a more detailed analysis, in which we confirm the conclusion of paper [4] unambiguously and find under what assumptions the experimental information on the pion form factor, the pion charge radius, and the P-wave isovector $\pi\pi$ phase shift with the correct scattering length is consistent in the framework of a pion form factor model.

2. UNIFIED DATA ON PION FORM FACTOR AND P-WAVE ISOVECTOR $\pi\pi$ PHASE SHIFT

It is well known that the three pion vector isoscalar resonance, ω-meson, decays through isospin nonconserving electromagnetic interactions into two pions. So, in the process $e^+e^- \to \pi^+\pi^-$ together with the pion form factor the contributions of $\omega \to \pi^+\pi^-$ are measured which causes the so-called ρ-ω interference effect.

As we would like to unite the experimental information on the pion form factor and P-wave isovector $\pi\pi$ phase shift, first of all we have to eliminate from the data on $|f(t)|$ obtained in $e^+e^- \to \pi^+\pi^-$ the contributions of ω-meson. We realize it by using the well established phenomenological formula [5] for description of $\rho-\omega$ interference effect, which consists of the sum of the pure pion form factor $F_\pi(t)$ and the ω-meson Breit-Wigner form multiplied by the amplitude R and the phase factor $e^{i\phi}$ in the following way

$$|f(t)| = |F_\pi(t) + R e^{i\phi} \frac{m_\omega^2}{m_\omega^2 - t + i m_\omega \Gamma_\omega}|, \qquad (1)$$

where R is related to the branching ratio $B(\omega\to 2\pi) = \frac{\Gamma(\omega\to 2\pi)}{\Gamma(\omega\to tot)}$ by the relation [5]

$$R = \frac{6\Gamma_\omega}{\alpha m_\omega \beta^{3/2}} \{B(\omega\to e^+e^-) B(\omega\to 2\pi)\}^{1/2} \qquad (2)$$

with

$$\beta = [1 - \frac{4m_\pi^2}{m_\omega^2}]^{1/2} \quad \text{and} \quad \alpha \approx \frac{1}{137}.$$

Taking into account the identity $\delta_\pi(t) = \delta_1^1(t)$ following from the pion form factor elastic unitarity condition and substituting the expression

$$F_\pi(t) = |F_\pi(t)| e^{i\delta_1^1(t)} \qquad (3)$$

into (1), one gets the quadratic equation

$$|F_\pi(t)|^2 + |F_\pi(t)| \frac{2Rm_\omega^2}{(m_\omega^2-t)^2 + m_\omega^2\Gamma_\omega^2}\{(m_\omega^2 - t)\cos(\phi-\delta_1^1(t)) +$$
$$+ m_\omega\Gamma_\omega \sin(\phi-\delta_1^1(t))\} + \{\frac{R^2 m_\omega^4}{(m_\omega^2-t)^2 + m_\omega^2\Gamma_\omega^2} - |f(t)|^1\} = 0 \qquad (4)$$

the solution of which gives the pion form factor absolute value expressed through experimentally measured $|f(t)|$ in the following way

$$|F_\pi(t)| = -Z(t) + \{Z^2(t) + |f(t)|^2 - \frac{R^2 m_\omega^4}{(m_\omega^2-t)^2 + m_\omega^2\Gamma_\omega^2}\}^{1/2}, \qquad (5)$$

where

$$Z(t) = \frac{R m_\omega^2}{(m_\omega^2-t)^2 + m_\omega^2\Gamma_\omega^2}\{(m_\omega^2-t)\cos(\phi-\delta_1^1(t)) + m_\omega\Gamma_\omega \sin(\phi-\delta_1^1(t))\}.$$

Provided that we know R, ϕ, $\delta_1^1(t)$ and m_ω, Γ_ω are taken from Review of Particle Properties [6], by using (5) and (3) one can calculate pure $\mathrm{Re}\,F_\pi(t)$ and $\mathrm{Im}\,F_\pi(t)$.

The values of R and ϕ were determined by three different experimental groups [7,8,9] in the process $\gamma A \to \pi^+\pi^- A$, where A means hydrogen, corbon, aluminium, and lead. The results of these experiments are summarized in Table 1. In the calculation of (5) we have used statistically averaged values

$$R = 0.01157 + 0.00041,$$
$$\phi = 90.86^\circ + 2.56^\circ, \qquad (6)$$

obtained by means of the relations

$$\xi = (\Delta\xi)^2 \sum_i \frac{\xi_i}{(\Delta\xi_i)^2}, \qquad (\Delta\xi)^2 = 1/\sum_i \frac{1}{(\Delta\xi_i)^2}. \qquad (7)$$

We note that the value of R in (6) is in perfect agreement with the value following from (2) by using the world averages for m_ω, Γ_ω as well as for $\Gamma(\omega\to tot)$, $\Gamma(\omega\to e^+e^-)$ and $\Gamma(\omega\to 2\pi)$.

For the $\pi\pi$ phase shift $\delta_1^1(t)$ we used the parametrization

$$\delta_1^1(t) = \frac{1}{2i}\ln\frac{(1 + A_2q^2 + A_4q^4) + i(A_3q^3 + A_5q^5)}{(1 + A_2q^2 + A_4q^4) - i(A_3q^3 + A_5q^5)} \qquad (8)$$

with the values of the coefficients

$$A_2 = 0.15142 + 0.01444$$
$$A_3 = 0.04146 + 0.00119$$
$$A_4 = -0.04507 + 0.00216 \qquad (9)$$
$$A_5 = -0.00025 + 0.00013$$

Table 1

The values of the $\rho - \omega$ interference parameters R and ϕ obtained by three different experimental groups from the process $\gamma A \to \pi^+ \pi^- A$, where A means hydrogen, carbon, aluminium and lead.

Experimental group	hydrogen	carbon	aluminium	lead
H.Alvenshben et al.[7]	$R = 0.131 \pm .0013$ $\phi = 85^\circ \pm 16^\circ$	$R = .0105 \pm .0007$ $\phi = 92^\circ \pm 10^\circ$		$R = .0120 \pm .0013$ $\phi = 96^\circ \pm 14^\circ$
H.-J.Behrend et al.[8]		$R = .0124 \pm .0012$ $\phi = 92.4^\circ \pm 5^\circ$	$R = .0138 \pm .0018$ $\phi = 80^\circ \pm 5.6^\circ$	$R = .0151 \pm .0020$ $\phi = 80.2^\circ \pm 6.6^\circ$
P.J.Biggs et al.[9]		$R = .0097 \pm .0008$ $\phi = 104.0^\circ \pm 5.1^\circ$		

which is the result of an optimal fit of three different experimental sets [12,13,14] on this phase in the elastic region simultaneously and the requirement to get through the pion form factor phase representation a Pade-type approximation of the pion form factor in the pion centre of mass momentum q -plane. This plane is obtained from the t -plane by means of the conformal mapping

$$q = \sqrt{\frac{t-4}{4}}, \tag{10}$$

where we have put the pion mass equal to one.

By using (8), (7), (5) and (3) the $\mathrm{Re}\,F_\pi(t)$ and $\mathrm{Im}\,F_\pi(t)$ are obtained in the whole elastic region which is specified by the circle of the radius from the elastic threshold to the first inelastic threshold (four pions branch point) in q -plane. As the P -wave isovector inelasticity $\eta_1^1(t)$ in the $\pi\pi$ phase shift analysis [3] starts to be different from one only for $t > 1\ \mathrm{GeV}^2$, the four pions cut contribution can be neglected and the elastic region is enlarged. As a consequence, we obtain $\mathrm{Re}\,F_\pi(t)$ and $\mathrm{Im}\,F_\pi(t)$ at 98 different values of t from the interval $-0.8432\ \mathrm{GeV}^2 < t < 1\ \mathrm{GeV}^2$ which represent the unified data on the pion form factor absolute value and the P -wave isovector $\pi\pi$ phase shift.

3. ANALYSIS OF UNIFIED DATA BY MEANS OF PADE-TYPE APPROXIMATIONS

It is well known that a function of a complex variable in an analytic domain can be represented by the Taylor series expansion from the coefficients of which different Pade approximants can be constructed [15,16]. Though exact mathematical proof does not exist [15], by the experience we know these Pade approximants converge also outside of the convergence circle of the original Taylor series and the best convergency for the type [M-1/M], [M/M], [M+1/M] approximants is observed. However, the improved convergency in comparison with the Taylor series expansion is not only the priority of the Pade approximations. They are able to reproduce essential singularities of the analytic function under consideration. Poles and zeros are reproduced by poles and zeros of Pade approximants and the branch singularities are taken into account by an alternate sequence of poles and zeros on the place of a corresponding cut [16,17].

In the case that we have instead of a function a set of experimental points with errors then the direct fit of these data by a rational function /LIN/ (in this case we call it a Pade-type approximation) gives better results in comparison with the fit of the same data by means of Taylor series expansion and the subsequent construction on Pade approximants /18/.

In this paper the situation is still more complicated. In order to reach the essential singularities of $F_\pi(t)$ not only on the physical sheet (we have in mind threshold branch points at $t = 4m_\pi^2, 16m_\pi^2, 4m_K^2, 4m_N^2, \dots$, where m_π, m_K and m_N is the mass of the pion, kaon, and nucleon, respectively) but also the singularities (two conjugate ρ -meson poles, eventually the left-hand cut) on the second Riemann sheet, we analyze the unified data on the pion form factor absolute value and the P-wave isovector $\pi\pi$ phase shift in q-plane. Here the conformal mapping (10) regularizes $F_\pi(t)$ at the elastic threshold $q = 0$ and one can decompose $F_\pi(t)$ into the Taylor series around this point. However, its convergence is restricted strictly speaking to the circle of the radius from $q = 0$ to the branch point $q = -i$ which corresponds to the left-hand cut on the second Riemann sheet. As a consequence, not all the data from the range of momenta -0.8432 $-0.8432\ \mathrm{GeV}^2 < t < 1\ \mathrm{GeV}^2$ can be used to determine the coefficient of the corresponding Taylor series expansion and the results for all the data obtained by the Pade approximants are expected to be even worse than in the previous case.

So, for the analysis of all the unified data from Section 2 a direct fit by means of the Pade-type approximation

$$F_\pi(t) = \frac{A_1(1+iA_2q) + \sum_{n=2}^{L} A_{2n-1}(iq)^n}{1 + \sum_{n=1}^{N} A_{2n}(iq)^n} \cdot \frac{1 + \sum_{n=1}^{N} A_{2n}(-1)^n}{A_1(1-A_2) + \sum_{n=2}^{L} A_{2n-1}(-1)^n} \qquad (11)$$

with real coefficients A_i and respecting fundamental properties /2/ of the pion form factor like analyticity, reality condition, normalization and threshold behaviour is favored unambiguously. In this way all the data are used to adjust the free parameters in (11) and an optimal description of the unified experimental information can be achieved.

The results of the analysis by means of the $[M-1/M]$, $[M/M]$ and $[M+1/M]$ Pade-type approximations (11) are summarized in Table 2. Here, in columns from the left to the right we give the Pade-type approximation, chi-squared obtained in the simultaneous least-squared fit of the 98 values of $\mathrm{Re}\,F_\pi(t)$

Table 2

The results of the analysis of the unified data by means of the Pade-type approximations

[L/N]	χ^2	χ^2/ndf	$\langle r_\pi^2 \rangle^{1/2}$	a_1^1	zeros	poles
[2/2]	240.514	2.559	0.649	0.006	$-i5.174$ $+i8.933$	$2.515 - i0.260$ $-2.515 - i0.260$
[3/2]	165.010	1.774	0.645	0.015	$6.105 - i5.782$ $-6.105 + i5.782$ $i3.989$	$2.522 - i0.281$ $-2.522 - i0.281$
[2/3]	124.487	1.339	0.605	0.016	$-i18.435$ $-i3.276$	$2.538 - i0.294$ $-2.538 - i0.294$ $-i3.722$
[3/3]	121.509	1.321	0.662	0.037	$-i13.379$ $+i12.270$ $-i1.828$	$2.543 - i0.293$ $-2.543 - i0.293$ $-i2.218$
[4/3]	119.564	1.314	0.663	0.036	$10.174 - i5.292$ $-10.174 - i5.292$ $+i9.136$ $-i1.995$	$2.544 - i0.294$ $-2.544 - i0.294$ $-i2.614$
[3/4]	121.183	1.332	0.677	0.039	$-i15.886$ $i9.638$ $-i1.752$	$+i27.063$ $-i2.092$ $2.542 - i0.291$ $-2.542 - i0.291$
[4/4]	118.958	1.321	0.663	0.034	$9.096 - i4.005$ $-9.096 - i4.005$ $+i8.892$ $-i2.140$	$-i95.070$ $2.543 - i0.294$ $-2.543 - i0.294$ $-i2.979$
[4/5]	118.557	1.332	0.664	0.036	$-i1.771$ $-i1.772$ $+i9.769$ $-i1.771$	$-i49.885$ $+i63.756$ $2.545 - i0.292$ $-2.545 - i0.292$ $-i2.157$

and $\text{Im}\,F_\pi(t)$ with errors from Section 2, chi-squared over the number of degrees of freedom (ndf), the pion charge radius defined by the relation

$$\langle r_\pi^2\rangle = 6\,\frac{dF_\pi(t)}{dt}, \tag{12}$$

the P-wave isovector $\pi\pi$ scattering length determined by

$$a_1^1 = \frac{\dfrac{\partial^3 \text{Im}\,F_\pi(t)}{\partial q}}{6\,|F_\pi(t)|}\Bigg|_{q=0}, \tag{13}$$

the position of zeros found as the solutions of the numerator and finally the position of poles given by the roots of the denominator of (11).

The P-wave isovector $\pi\pi$ phase shift in the elastic region is determined by means of the relation

$$\delta_1^1(t) = \text{arctg}\,\frac{\text{Im}\,F_\pi(t)}{\text{Re}\,F_\pi(t)}. \tag{14}$$

As all the phases from [3/3] - to [4/5] - Pyde-type approximations exhibit to deviate from each other maximally at a half degree, we present only the phase of the [4/3] Pade-type approximation in the figure, which gives the best description of all the data from Section 2.

By comparing χ^2/ndf, $\langle r_\pi^2\rangle^{1/2}$, a_1^1 in Table 2 one can see immediately that by means of [3/3] and higher order of Pade-type approximations a good description of the unified data is achieved and the corresponding values $\langle r_\pi^2\rangle^{1/2}$ and a_1^1 are in a perfect agreement with the world averaged [19] value of the pion charge radius and a number of evaluations [20-24] of the P-wave isovector $\pi\pi$ scattering length $0.027 < m_\pi^3 a_1^1 < 0.045$, respectively. These results together with the figure clearly demonstrate that in contradiction with the conclusion of Hammer et al.[3] the pion form factor data, the pion charge radius, and the P-wave isovector $\pi\pi$ phase shift with generally accepted value of the scattering lengh are definitely consistent.

By a more careful study of the position of zeros and poles in the last two columns in Table 2 we find that besides two ρ-meson poles (to be compared with the Particle data group

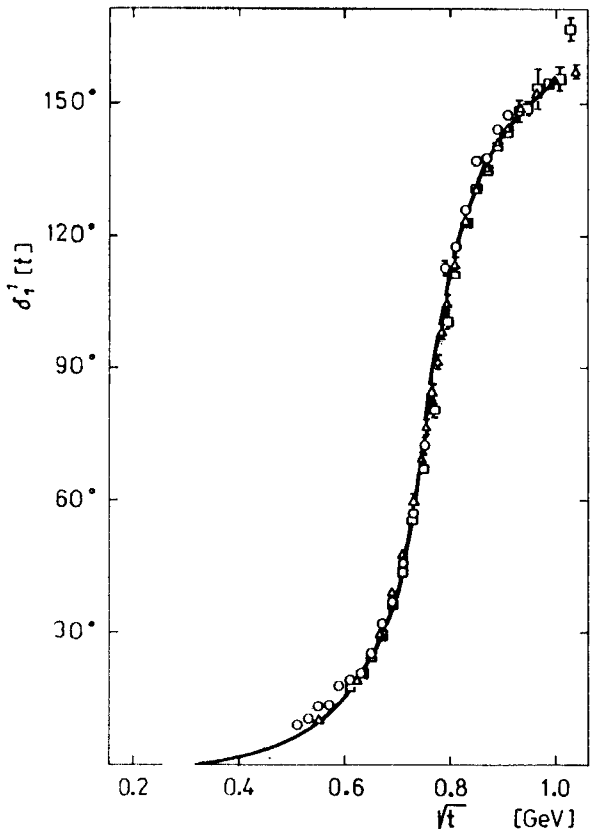

Figure. The P-wave isovector $\pi\pi$ phase shift predicted by our [4/3] Pade-type approximation and compared with the data. O - the data of [12], Δ - the data of [14] and □ - the data of [15].

positions of ρ-meson $q_\rho = +2.59 - i\,0.30$) in the case of the consistency of all mentioned data one stable zero and one stable pole on the negative imaginary axis of q-plane, into which the pion form factor left-hand cut is mapped, are present in corresponding Pade-type approximation. What is more interesting the position of these zeros and poles is almost identical with the positions obtained in paper [11] in a different way.

These results clearly exhibit that the consistency among the pion form factor data, the pion charge radius, and the P-wave isovector $\pi\pi$ phase shift with the generally accepted value of the scattering length in the frame work of a pion form factor model can be achieved if only the pion form factor left-hand cut from the second Riemann sheet is taken into account.

There is another zero on the positive imaginary axis of the plane seen in Table 2, which moves in the interval $-11.725\ GeV^2 \leq t \leq -6.120\ GeV^2$ of the four momentum transfer squared in the spacelike region. This zero might indicate the existence of a diffraction minimum in the differential cross section for elastic $e^-\pi$ scattering as a consequence of a constituent structure of the pion, like in the case of the electron elastic scattering on nuclei. However, to do a more stringent conclusion about this pion form factor zero, more dence and more reliable data in spacelike region are necessary.

4. CONCLUSION

The consistency of the experimental data which involve the pion form factor absolute values from the range of momenta $-0.8432\ GeV^2 < t < 1\ GeV^2$, the pion charge radius and the P-wave isovector $\pi\pi$ phase shift in the elastic region (including also the generally accepted value of the scattering length a_1^1) was investigated in detail.

As there are data obtained in $e^+e^- \to \pi^+\pi^-$ only on a mixture of the pion form factor and of the electromagnetic decay of ω-meson into two pions, first of all by using the well established phenomenological formula for the description of the $\rho-\omega$ interference effect we eliminated the omega meson contributions. Then, combining the absolute values of the pure pion form factor with the P-wave isovector $\pi\pi$ phase shift we obtained the unified data represented by $\mathrm{Re}\,F_\pi(t)$ and $\mathrm{Im}\,F_\pi(t)$ which subsequently by means of various Pade-type approximations, respecting all the fundamental properties of the pion form factor, were fitted.

On the base of this very detailed analysis we came to the following conclusions:

a) The pion form factor data, the pion charge radius, and the P-wave isovector $\pi\pi$ phase shift as well as the corresponding scattering length are in contradiction with the conclusion of /3/ definitely consistent.

b) The consistency of the afore-mentioned sets of experimental data in the framework of a pion form factor model can be achieved if only the contribution of the pion form factor

left-hand cut from the second Riemann sheet is taken into account.

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