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MAJORANA AND DIRAC MASSES, NEUTRINO OSCILLATIONS AND THE NUMBER OF CHARGED LEPTONS

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1. The question as to whether the neutrino masses are identically equal to zero and the related question about lepton mixing are extremely important. Recently a direct experiment to measure the $\bar{\nu}_e$ -mass from ^3H -decay [1] and a number of search experiments [2-6] for neutrino oscillations [7,8] increased the interest of the question as they seem to give some indications for nonzero neutrino masses. It is worth to notice that the recent developments of grand unified models nicely accomodate finite neutrino masses [9].

There are several schemes of lepton mixing and neutrino oscillations, a Table of which was given for example in ref. [10] for the case when the initial neutrinos are ν_e . In this note we wish to discuss one possibility which does not appear in the Table. Namely the following one: In the lepton sector Nature has just chosen among various possibilities that which makes the description in terms of Majorana masses on the same footing as the description in terms of Dirac ones. This possibility seems to us attractive for the following reasons:

i) The fact that classical massless Dirac longitudinal neutrino is indistinguishable [11] from the Majorana one is rather suggestive.

ii) In a recent paper [12] it was shown that in the most general case it is impossible to observe a different behaviour of neutrino oscillations for the case of Majorana and Dirac masses.

Before we expose the new scheme let us briefly summarize all the lepton mixing schemes, having in mind that the charged lepton current has the form

$$j_\alpha = 2(\bar{\nu}'_L \gamma_\alpha \ell_L), \tag{1}$$

where

$$\nu' = \begin{pmatrix} \nu'_e \\ \nu'_\mu \\ \nu'_\tau \\ \vdots \end{pmatrix}, \qquad \ell = \begin{pmatrix} e \\ \mu \\ \tau \\ \vdots \end{pmatrix} \tag{2}$$

and ν'_ℓ are the eigenvalues of the weak interaction. The mixing is determined by the structure of the mass term. Until now only neutrino mass terms of the following type have been considered.

1) Dirac mass term [13]

$$\mathfrak{L} = -\bar{\nu}'_R M \nu'_L + \text{h.c.}, \tag{3}$$

where M is a complex $N \times N$ matrix. After diagonalization of the matrix M we get

$$\mathfrak{L} = -\sum_{i=1}^{N} m_i \bar{\nu}_i \nu_i \quad , \tag{4}$$

where ν_i is Dirac field of mass m_i. We have

$$\nu'_{\ell L} = \sum_{i=1}^{N} U_{\ell i} \nu_{iL} \tag{5}$$

$(\ell = e, \mu, \tau, ..)$; here U is the $N \times N$ mixing matrix. Notice that in this scheme the right components of neutrino fields have no function in the usual weak current (1), although they are present in the mass term.

2) Majorana mass term [14]

$$\mathfrak{L} = -\overline{(\nu'_L)}^c M \nu'_L + \text{h.c.} = \nu'_L C^{-1} M \nu'_L + \text{h.c.} \tag{6}$$

($(\nu'_L)^c = C\bar{\nu}'_L = \nu'^c_R$; C is the charge conjugation matrix, $C\gamma_a^T C^{-1} = -\gamma_a$, $C^T = -C$).After the diagonalization of the matrix M we get

$$\mathfrak{L} = -\sum_{i=1}^{N} m_i \bar{\chi}_i \chi_i \ , \tag{7}$$

where $\chi_i = C\bar{\chi}_i$ is the Majorana field of mass m_i. Clearly

$$\nu'_{\ell L} = \sum_{i=1}^{N} U_{\ell i} \chi_i \quad .. \tag{8}$$

Notice that in this scheme the neutrino field components which are present in the mass term are present as well in the ordinary weak current.

In both cases considered above with Dirac and Majorana mass terms the number of neutrinos $\nu_e, \nu_\mu, ...$* are equal to the number of neutrinos $\nu_1, \nu_2, ...$ with definite masses. This means that in both schemes only oscillations of the type $\nu_\ell \rightleftarrows \nu_{\ell'}$, $\bar{\nu}_\ell \rightleftarrows \bar{\nu}_{\ell'}$, $(\ell' \neq \ell)$ take place. Moreover it can be shown [12] that it is impossible to observe a different behaviour* of oscillations for the case of Dirac and Majorana masses even in the most general case of PC-violation [15]. This once again suggests that a natural possibility would be the coexistence of Dirac and Majorana mass terms. A possible scheme of such a type was considered in ref. [16]:

3)

$$\mathfrak{L} = -\overline{(\nu''_L)}^c M \nu''_L + \text{h.c.}, \tag{9}$$

where

$$\nu'' = \begin{pmatrix} \nu' \\ \nu'^c \end{pmatrix}$$

is a 2N -column, and M is a symmetric $2N \times 2N$ complex matrix. Clearly in (9) Dirac as well as Majorana mass terms are present. After diagonalization we have

$$\mathfrak{L} = -\sum_{i=1}^{2N} m_i \bar{\chi}_i \chi_i \tag{10}$$

and

$$\nu_{\ell L} = \sum_{i=1}^{2N} U_{\ell i} \chi_{iL} \quad , \tag{11}$$

where χ_i $(i = 1, ...2N)$ are Majorana fields of masses m_i and the index ℓ may have N values $e, \mu, \tau, ...$. It should be noticed that in such a scheme the number of Majorana neutrinos with definite masses is twice as large as the number of phenomenological neutrinos. As a result, in addition to the usual oscillations $\nu_{\ell L} \rightleftarrows \nu_{\ell' L}$ $(\bar{\nu}_{\ell R} \rightleftarrows \bar{\nu}_{\ell R})$ there appear rather "exotic" oscillations such as $\nu_{\ell L} \rightleftarrows \bar{\nu}_{\ell' L}$ $(\bar{\nu}_{\ell R} \rightleftarrows \nu_{\ell' R})$.

2. The scheme discussed above in 3) is not "economical" in the sense that in the mass term there appear right-handed components of neutrino fields. Here we construct the most economical scheme with Dirac and Majorana mass terms in such a way that only "physical" components of neutrino fields are present in the mass terms. An embryo of our scheme is well known: It is the old lepton scheme suggested for massless neutrinos by Zeldovich and Konopinsky and Mahmoud in ref. [17].According to such

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* Notice that here and later we conserve the usual notation $\nu_e, \nu_\mu, ...$ for the phenomenological particles undergoing the weak interaction.

* Clearly such processes as neutrinoless double β-decay are possible in principle in scheme 2) but not in scheme 1).

a scheme conceived at a time when only electrons and muons were known, there is a four-component neutrino with only one additive lepton number, which takes opposite values for e^- and μ^- (and of course ν_e and ν_μ). In this scheme the charged current is

$$j_\alpha = 2(\bar{\nu}'_L \gamma_\alpha \ell_L), \qquad (12)$$

where

$$\nu' = \begin{pmatrix} \nu \\ \nu^c \end{pmatrix}, \quad \ell = \begin{pmatrix} e \\ \mu \end{pmatrix}. \qquad (13)$$

Incidentally notice that in order to construct a $SU(2)\times U(1)$ Glashow-Weinberg-Salam theory with the current (12) we must assume that

$$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} \nu^c_L \\ \mu_L \end{pmatrix}$$

transform as doublets.* Let us consider now the possibility that the lepton charge (only one for the case considered of the two charged lepton e and μ) is violated by neutrino mass terms (let us say in the Higgs sector). The most general neutrino mass term acquires the form

$$\mathfrak{L} = \overline{(\nu'_L)^c} M \nu'_L + \text{h.c.}, \qquad (14)$$

where M is a 2x2 complex matrix. The usual diagonalization yields

$$\mathfrak{L} = -\sum_{i=1,2} m_i \chi_i \chi_i, \qquad (15)$$

$$\nu'_L = U \chi_L, \qquad (16)$$

where χ_i is the Majorana field of mass m_i.

In the scheme considered here the four phenomenological neutrinos ν_e, ν_μ, $\bar{\nu}_e$, $\bar{\nu}_\mu$ correspond to the four states of two Majorana neutrino with different masses, the only "physical" components of neutrino fields being present in the current and in the mass term. In this sense the above scheme is identical for the case of two charged leptons to the scheme first proposed in ref.[14] (see 2 above). An essential difference between these two schemes arises when the number of charged lepton is larger than 2. The scheme of the type [14] can be constructed for any number of charged leptons whereas the scheme that we are proposing requires an even number of charged leptons.

* See for example, I.G.Jafarov. Yad.Fiz.,1979, 30, p.215.

The expressions (14)-(16) without difficulties can be generalized to the case of 2N charged leptons:

$$j_\alpha = 2(\bar{\nu}'_L \gamma_\alpha \ell_L), \qquad (17)$$

where

$$\nu' = \begin{pmatrix} \nu_1 \\ \nu_1^c \\ \nu_2 \\ \nu_2^c \\ \vdots \end{pmatrix}, \quad \ell = \begin{pmatrix} e \\ \mu \\ \tau \\ \xi \\ \vdots \end{pmatrix}. \qquad (18)$$

The neutrino mass term has the form

$$\mathfrak{L} = -\overline{(\nu'_L)^c} M \nu'_L + \text{h.c.} = -\sum_{i=1}^{2N} m_i \bar{\chi}_i \chi_i \qquad (19)$$

and

$$\nu'_L = U \chi_L, \qquad (20)$$

where χ_i is the Majorana field of mass m_i and U is a $2N\times 2N$ mixing matrix.

Thus the scheme which we are proposing requires the existence of a new charged lepton in addition to the well known τ lepton and the existence of a new neutrino type in addition to ν_τ. Obviously, in the scheme considered there are possible only oscillations among phenomenological neutrinos of the type $\nu_\ell \rightleftarrows \nu_{\ell'}, \bar{\nu}_\ell \rightleftarrows \bar{\nu}_{\ell'}$. However, oscillations of the type $\nu_e \rightleftarrows \nu_\mu$ involve a change of 2 in the lepton number. Thus, it is reasonable to expect that such oscillations are suppressed with respect to oscillations of type $\nu_e \rightleftarrows \nu_\tau$ and/or $\nu_e \rightleftarrows \nu_\xi^*$.

3. Our proposal, in which the usual conception of flavour is taken up by two distinct charged leptons leads to the following:

1) It requires at least one more charged lepton and one more neutrino in addition to the well known ones e, μ, τ, ν_e, ν_μ, ν_τ. This is not excluded by cosmological evidence.

2) It gives arguments in favour of some hierarchy of oscillation types, the suppressed ones ($\nu_e \rightleftarrows \nu_\mu$?) and favoured ($\nu_e \rightleftarrows \nu_\tau$?). This does not contradict the few data on oscillations which are available [3,4].

* Similarly the decay $\tau \to e\gamma$ might have a probability quite different from $\tau \to \mu\gamma$.

3) It is the most economical in as much as all the neutrino field components fulfill their function.

4) In the Lagrangian the Dirac and Majorana mass terms are present on equal footing, which makes in our scheme the distinction between Majorana and Dirac description rather meaningless.

5) Clearly, the scheme proposed requires the existence of at leat one pair of quarks in addition to the well known u, d , s , c , b(t).

In conclusion, this scheme seems to us quite attractive if it turns out that in Nature there are no right-handed currents.

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REFERENCES

1. Lubimov V.A. et al. Preprint ITEP, No.62, Moscow, 1980.
2. Reines F., Sobel H.W., Pasierb E. Preprint Univ. of Calif., Irvine, 1980.
3. Barger V. et al. Preprints COO-881-135 (1980), COO-881-146 (1980).
4. Wachsmuth H. CERN-EP/79-115 (1979). De Rujula A. et al. Preprint TH 2788-CERN, 1979.
5. Davis R., Jr., Evans J.C., Cleveland B.T. Proc.Conf. Neutrino-78, Purdue Univ. Press, 1978. Bilenky S.M., Pontecorvo B. Comments on Nuclear and Particle Physics, 1977, 7, p.149.
6. Crouch M.F. et al. Phys.Rev., 1978, D18, p.2239; Volkova L.V., Zatsepin G.T. Sov.J.Nucl.Phys., 1972, 14, p.117.
7. Pontecorvo B. JETP (Sov.Fiz.), 1967, 53, p.1717.
8. Bilenky S.M., Pontecorvo B. Phys.Rep., 1978, C41, p.226.
9. See, for example: Barbieri H. et al. Preprint TH 2776-CERN, 1979; Barbieri H., Ellis J., Gaillard M.K. Preprint TH-2787-CERN, 1980.
10. Bilenky S.M., Pontecorvo B. Phys.Rep., 1978, C41, p.255.
11. Case K.M. Phys.Rev., 1957, 107, p.307; Ryan C., Okubo S. Nuovo Cimento Suppl., 1964, 2, p.234.
12. Bilenky S.M., Hošek J., Petcov S.T. JINR, E2-80-373, Dubna, 1980.
13. Bilenky S.M., Pontecorvo B. Phys.Lett., 1976, 61B, p.248; Fritzsch H., Minkowsky P. Phys.Lett., 1976, 62B, p.72.
14. Gribov V., Pontecorvo B. Phys.Lett., 1969, 28B, p.493; Pontecorvo B. JETP Lett. (Pis'ma JETP), 1971, 13, p.281.
15. Cabibbo N. Phys.Lett., 1978, 72B, p.333.

16. Bilenky S.M., Pontecorvo B. Lett. Nuovo Cim., 1976, 17, p.569; see also ref. 10.
17. Zeldovich Ya.B. DAN SSSR, 1952, 86, p.505; Konopinsky E.J., Mahmoud H. Phys.Rev., 1953, 92, p.1045.

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