объединенный
институт
ядерных
исследований
дубна

E2-80-373

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ON OSCILLATIONS OF NEUTRINOS WITH DIRAC AND MAJORANA MASSES

Submitted to "Physics Letters B"

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1980

1. The possibility of oscillations in neutrino beams was considered first by Pontecorvo[1]. The first phenomenological theory of neutrino oscillations was constructed by Gribov and Pontecorvo[2]. Quite recently, the interest in neutrino oscillations remarkably increased due to the progress in grand unified theories. In these theories the leptonic charge is not conserved and neutrinos do have finite masses in most of the schemes[3].

The neutrino oscillations become possible provided: (i) The neutrino masses are non-zero and different from each other; (ii) The neutrino fields enter into the charged current in the mixed form. It should be mentioned that the presently available experimental data are entirely compatible with the assumption that neutrinos have either Dirac or Majorana masses[4,5].

Oscillations of two types of neutrons with Majorana masses were elaborated in ref[2]. The work is based on the two-component neutrino theory. The scheme is maximally economical: to four particles ν_e, $\bar{\nu}_e$, ν_μ, and $\bar{\nu}_\mu$ there correspond four spin states of two Majorana neutrons. Oscillations of neutrons with Dirac masses were considered in refs.[6,7]. These papers are tightly connected with the electroweak gauge theories and are based on an analogy between leptons and quarks. Finally, in grand unified theories, depending on the scheme considered and on the choice of admissible Higgs representations, Dirac and Majorana neutrino mass terms can emerge.

If it turns out that the neutrino masses are really different from zero and that the lepton mixing takes place*, the question will naturally arise about the type of neutrino masses. Obviously, in the case of Majorana masses and of mixing, there is no lepton charge. Consequently, the processes like neutrinoless double β-decay, $K^+ \to \pi^- + \mu^+ + e^+$ decay, etc., which are forbidden in theories with Dirac neutrinos, become possible. However, the existing limits on the neutrino masses impose stringent constraints on the probabilities of these processes, which turn out to be[9] by many orders of magnitude

*Possible indications of neutrino oscillations were obtained recently in the beam dump experiments in CERN[5] and in the experiments with the reactor antineutrinos[8].

smaller than the existing experimental upper bounds (for double β-decay, e.g., by 4-5 orders below the upper found in [10]).

The oscillations of neutrinos are a subtle interference effect. Are the experiments on neutrino oscillations able to clarify the type of neutrino masses? In this note we shall consider neutrino oscillations from this point of view. No model assumptions will be made on values of neutrino masses, leptonic mixing angles and CP-violating phases.

2. In SU(2) x U(1) theory of Glashow, Weinberg and Salam the leptonic charged current is given by

$$j_\alpha = 2 \sum_{\ell=e,\mu,\tau,\dots} (\bar{\nu}_{\ell L} \gamma_\alpha \ell_L). \tag{1}$$

We will assume that the neutrino masses are different from zero. For neutrinos with Dirac masses

$$\nu_{\ell L} = \sum_{i=1}^{n} U_{\ell i} \nu_{iL}, \tag{2}$$

where U is the unitary mixing matrix, ν_i is the field operator of the four-component neutrino with the mass m_i, the mass term being

$$\mathcal{L}^{\nu}_M = -\sum_i \bar{\nu}_i \nu_i m_i . \tag{3}$$

Note that the expressions (2) and (3) emerge naturally in gauge theories with the Higgs mechanism as a result of diagonalization (with the help of bi-unitary transformation) of the Lagrangian

$$\mathcal{L}_M = -\sum_{\ell',\ell} \bar{\nu}_{\ell' L} M_{\ell'\ell} \nu_{\ell R} - \sum_{\ell'\ell} \bar{\ell}'_L N_{\ell'\ell} \ell_R + \text{h.c.} \tag{4}$$

Here M and N are arbitrary complex matrices. For neutrinos with Majorana masses

$$\nu_{\ell L} = \sum_i U_{\ell i} \chi_{iL}, \tag{5}$$

where $\chi_i = C\bar{\chi}_i$ is the field operator of Majorana neutrino with the mass m_i (C is the charge conjugation matrix; $C\gamma_\alpha^T C^{-1} = -\gamma_\alpha$, $C^T = -C$) and U is the unitary mixing matrix. The corresponding mass term

$$\mathcal{L}^{\nu}_M = -\sum_i \bar{\chi}_i \chi_i m_i . \tag{6}$$

Eqs.(5) and (6) can be obtained by a proper diagonalization of the Lagrangian*

$$\mathcal{L}_M = \sum_{\ell'\ell} \nu_{\ell' L} C^{-1} M_{\ell'\ell} \nu_{\ell L} - \sum_{\ell',\ell} \bar{\ell}'_L N_{\ell'\ell} \ell_R + \text{h.c.} \tag{7}$$

Notice that the mass terms of the type (7) emerge naturally in grand unified theories [8].

3. Let us proceed now to the discussion of the oscillations in neutrino beams. For the state vector of the neutrino and antineutrino** it follows from eqs.(2) and (5) respectively

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*Obviously, the matrix M has to be symmetric due to Pauli principle. In such a case we have [11]

$$M = V^T m V,$$

where m is a real diagonal matrix ($m_i > 0$, $m_i \neq m_k$), V is an unitary matrix. Thus

$$\sum_{\ell',\ell,i} \nu_{\ell' L} C^{-1} V^T_{\ell' i} m_i V_{i\ell} \nu_{\ell i} + \text{h.c.} = \sum \nu_{iL} C^{-1} \nu_{iL} m_i + \text{h.c.}$$

$$= -\sum \overline{(\nu_{iL})^c} \nu_{iL} m_i + \text{h.c.} = -\sum_i \bar{\chi}_i \chi_i m_i ,$$

where $\nu_{iL} = \sum_\ell V_{i\ell} \nu_{\ell L}$ and $\chi_i = \nu_{iL} + (\nu_{iL})^c = \nu_{iL} + \nu^c_{iR}$ is the field operator of Majorana neutrino.

** We consider simultaneously the oscillations with Dirac and with Majorana masses. In both cases we will call neutrino (antineutrino) that particle which is emitted in the usual weak interaction together with $\ell^+(\ell^-)$.

$$|\nu_\ell > = \sum_i U^*_{\ell i} \; |i,L>,$$

$$|\bar\nu_\ell > = \sum_i U_{\ell i} \, |i,R> . \qquad (8)$$

Here $|\nu_\ell>$ is the state vector of the neutrino with momentum $\vec p$ and negative helicity, $|\bar\nu_\ell>$ is the state vector of antineutrino with momentum $\vec p$ and positive helicity, $|i,L>$ and $|i,R>$ are the corresponding state vectors of neutrino with the mass m_i ($|\vec p| \gg m_i$).

If at the moment $t = 0$ the neutrino beam was described by the vector $|\nu_\ell>$ ($|\bar\nu_\ell>$) then for the state vector of the beam at the time t we get

$$|\nu_\ell>_t = \sum_{i,\ell'} X^i_{\ell';\ell} \, e^{-iE_i t} \, |\nu_{\ell'}>, \qquad (9)$$

$$|\bar\nu_\ell>_t = \sum_{i,\ell'} X^{i*}_{\ell';\ell} \, e^{-iE_i t} \, |\bar\nu_{\ell'}>, \qquad (10)$$

respectively. Here

$$X^i_{\ell';\ell} = U_{\ell' i} \, U^*_{\ell i} \qquad (11)$$

with the property

$$X^i_{\ell';\ell} = X^{i*}_{\ell;\ell'} . \qquad (12)$$

The corresponding probabilities are given by

$$P_{\nu_{\ell'};\nu_\ell}(t) = \sum_{i,k} X^i_{\ell';\ell} \, X^{k*}_{\ell';\ell} \, e^{-i(E_i - E_k)t},$$

$$P_{\bar\nu_{\ell'};\bar\nu_\ell}(t) = \sum_{i,k} X^{i*}_{\ell';\ell} \, X^k_{\ell';\ell} \, e^{-i(E_i - E_k)t}. \qquad (13)$$

With the help of eq.(12) we get the relation

$$P_{\nu_{\ell'};\nu_\ell}(t) = P_{\nu_\ell;\nu_{\ell'}}(t). \qquad (14)$$

Clearly it is the consequence of the CPT-invariance. Provided the leptonic weak interaction is CP- and therefore T-invariant*, the following relation holds true:

$$X^{i*}_{\ell';\ell} = X^i_{\ell';\ell} = X^i_{\ell;\ell'} . \qquad (15)$$

In such a case from the eqs.(13) it follows

$$P_{\nu_{\ell'};\nu_\ell}(t) = P_{\bar\nu_{\ell'};\bar\nu_\ell}(t) \qquad (16)$$

and

$$P_{\nu_{\ell'};\nu_\ell}(t) = P_{\nu_\ell;\nu_{\ell'}}(t), \qquad P_{\bar\nu_{\ell'};\bar\nu_\ell}(t) = P_{\bar\nu_\ell;\bar\nu_{\ell'}}(t). \qquad (17)$$

The relation (16) was obtained in ref.[7]. The check of the validity of the relation (16) for $\ell' \neq \ell$ is a test of CP-invariance of leptonic weak interaction (for $\ell' = \ell$ the relation (16) is satisfied due to CPT).

Let us rewrite eqs.(13) in the form

$$P_{\nu_{\ell'};\nu_\ell}(r) = \sum_i |X^i_{\ell';\ell}|^2 + 2\sum_{i<k} |X^i_{\ell';\ell}| \, |X^k_{\ell';\ell}| \cos\left(2\pi \frac{r}{L_{ik}} \xi_{ik} - \eta^{\ell';\ell}_{ik}\right),$$

$$P_{\bar\nu_{\ell'};\bar\nu_\ell}(r) = \sum_i |X^i_{\ell';\ell}|^2 + 2\sum_{i<k} |X^i_{\ell';\ell}| \, |X^k_{\ell';\ell}| \cos\left(2\pi \frac{r}{L_{ik}} \xi_{ik} + \eta^{\ell';\ell}_{ik}\right). \qquad (18)$$

Here r is the distance between the source and the detector of the neutrinos, $\xi_{ik} = (m_i^2 - m_k^2)/|m_i^2 - m_k^2|$,

$$L_{ik} = 4\pi \frac{p}{|m_i^2 - m_k^2|} \qquad (19)$$

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*The question of CP-violation in neutrino oscillations was discussed first by Cabibbo [7].

is the oscillation length,

$$\eta_{ik}^{\ell';\ell} = \phi_i^{\ell';\ell} - \phi_k^{\ell';\ell} , \qquad (20)$$

where $\phi_i^{\ell';\ell} = \arg X^i_{\ell';\ell}$.

If as a result of averaging (over the neutrino spectrum, over the neutrino production region, etc.) the r - and p-dependent terms in eq.(18) vanish, the averaged probabilities become

$$\langle P_{\nu_{\ell'};\nu_\ell} \rangle = \sum_i |X^i_{\ell';\ell}|^2 ,$$

$$\langle P_{\bar\nu_{\ell'};\bar\nu_\ell} \rangle = \sum_i |X^i_{\ell';\ell}|^2 . \qquad (21)$$

Therefore the averaged probabilities satisfy the conditions (16) and (17) even if the CP-invariance is violated. Thus checking of CP invariance of leptinic weak interactions requires performance of extremely difficult experiments, capable to yield the cosine dependence in oscillation probabilities.

4. Now we turn to the comparison of oscillations of neutrinos with Dirac versus with Majorana masses. Clearly the unitary mixing matrices U differ in these two cases only in number of CP-violating phases. Namely, for N neutrino species, the number of these is (N-1)(N-2)/2 in the former case while N(N-1)/2 in the latter one (the Majorana fields are unable to absorb phases). However, this effect cannot be observed in any experiment searching for CP-violation in neutrino oscillations. Indeed, instead of the mixing matrix U let us introduce the matrix

$$U' = S^*(\alpha) U S(\beta), \qquad (22)$$

where $S_{\ell'\ell}(\alpha) = \delta_{\ell'\ell} e^{i\alpha_\ell}$, $S_{ik}(\beta) = \delta_{ik} e^{i\beta_k}$, α_ℓ and β_k are arbitrary real parameters. It is easy to see that the oscillation probabilities (13) are invariant with respect to the transformation (22). This invariance implies that the number of CP-violating phases in the case of oscillations of neutrinos with Dirac or Majorana masses is always the same and it is equal to (N-1)(N-2)/2.

5. Assume now that both Dirac and Majorana mass terms together are present in the Lagrangian *. In the most general case of N neutrino types they take the form

$$\mathcal{L}^\nu_M = -(\bar\nu^c_R M_1 \nu_L + \bar\nu_R M_2 \nu^c_L + \bar\nu_R M_3 \nu_L + \bar\nu^c_R M_3^T \nu^c_L + h.c.), \qquad (23)$$

where

$$\nu = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \vdots \end{pmatrix} , \quad \nu^c = C\bar\nu = \begin{pmatrix} \nu_e^c \\ \nu_\mu^c \\ \nu_\tau^c \\ \vdots \end{pmatrix}$$

and M_i (i=1,2,3) are N x N complex matrices, M_1 and M_2 being symmetrical. The expression (23) can be written as

$$\mathcal{L}^\nu_M = -\bar n^c_R M n_R + h.c. \qquad (24)$$

where $n = \begin{pmatrix} \nu \\ \nu^c \end{pmatrix}$ and

$$M = \begin{pmatrix} M_1 & M_3^T \\ M_3 & M_2 \end{pmatrix}$$

is 2N x 2N symmetric matrix. The analogous treatment as in the case of Majorana masses alone leads to

$$\mathcal{L}^\nu_M = -\bar\chi m \chi , \qquad (25)$$

where

$$\chi = V n_L + (V n_L)^c . \qquad (26)$$

m is real diagonal 2N x 2N matrix $(m_i > 0, m_i \neq m_k)$. V is the unitary matrix. Thus as a result of diagonalization of the Lagrangian (23) we came to the mass term of 2N Majorana neutrinos which differ from each other only in masses. Including the effect of diagonalization of the leptonic mass matrix we get

* Here we generalize the paper /12/ for the case of possible CP-violation.

$$\nu_{\ell L} = \sum_{i=1}^{2N} U_{\ell i} \chi_{iL} , \tag{27}$$

$$\nu^{c}_{\ell L} = \sum_{i=1}^{2N} U_{\bar{\ell} i} \chi_{iL} . \tag{28}$$

Here U is a 2N x 2N unitary mixing matrix. The N-component column $\nu_{\ell L}$ is understood to enter into the usual charge leptonic current (1).

Let us consider briefly the oscillations of neutrinos in the scheme with 2N Majorana neutrinos. If at the moment $t = 0$ the neutrino beam has described by the vector $|\nu_{\ell L}>$, then at the moment t we have

$$|\nu_{\ell L}>_t = \sum_{\ell'} \sum_{i=1}^{2N} U^*_{\ell i} e^{-iE_i t} U_{\ell' i} |\nu_{\ell' L}> + \sum_{\ell'} \sum_{i=1}^{2N} U^*_{\ell i} e^{-iE_i t} U_{\bar{\ell}' i} |\bar{\nu}_{\ell' L}> \tag{29}$$

($|\bar{\nu}_{\ell L}>$ is the state vector of the left-handed antineutrino). The present case differs from those considered above in several aspects. (i) Together with $\nu_{\ell L} \to \nu_{\ell' L}$ ($\bar{\nu}_{\ell R} \to \bar{\nu}_{\ell' R}$) the oscillations $\nu_{\ell L} \to \bar{\nu}_{\ell' L}$ ($\bar{\nu}_{\ell R} \to \nu_{\ell' R}$) become possible. In principle these oscillations can be observed provided the right-handed currents are present in the interaction Lagrangian. The available data, however, testify, that the interaction with right-handed currents, even if it exists, is severely suppressed [13,14]. (ii) Oscillation probabilites $P_{\nu_{\ell' L};\nu_{\ell L}}(r)$ and $P_{\bar{\nu}_{\ell' R};\bar{\nu}_{\ell R}}(r)$ depend upon 2N(2N-1) parameters (N(2N-1) mixing angles, (N-1)(2N-1) phases and 2N-1 differences of neutrino masses squared). Detailed experiments on neutrino oscillations, in particular the Fourier analysis of the dependence of oscillation probabilities on neutrino energy and/or on the distance r could in principle allow one to get information about the number of massive neutrinos. (iii) It follows from the unitarity of the matrix U that

$$\sum_{\ell'} < P_{\nu_{\ell' L};\nu_{\ell L}} > + \sum_{\ell'} < P_{\bar{\nu}_{\ell' L};\nu_{\ell L}} > = 1. \tag{30}$$

The measurement of probabilities $<P_{\nu_{\ell' L};\nu_{\ell L}}>$ for $\ell' = e, \mu, \tau, \ldots$ and ℓ held fixed might give the possibility of obtaining information about the presence (or absence) of the second term in eq. (30).

In conclusion we express our deep gratitude to B.M.Pontecorvo for useful discussions of the questions considered here.

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Received by Publishing Department
on May 27 1980.