объединенный
институт
ядерных
исследований
дубна

E2-80-352

G.A.Kozlov, S.P.Kuleshov, R.P.Zaikov

ON LINEAR QUASIPOTENTIAL IN THE FRAMEWORK OF QUANTUM FIELD THEORY

Submitted to "Письма в ЖЭТФ"

1980

1. Consider the Logunov-Tavkhelidze quasipotential equation[1] for the two-particle wave function in two-dimensional space-time

$$(E^2-\vec{p}^{\,2}-m^2)\,\phi(E;\vec{p}) = \int \frac{dk}{\sqrt{\vec{k}^2+m^2}}\, V(E;\vec{p},\vec{k})\,\phi(E;\vec{q}), \tag{1}$$

where E is the total energy of two-particle equal mass system, $\vec{p}$ and $\vec{q}$ are space components of relative momentum in the c.m.s., $V(E;\vec{p},\vec{k})$ is the quasipotential. This potential can be determined in the framework of Quantum Field Theory (QFT) by summing the ladder diagrams[1]. Consider a theory with the following interaction Lagrangian

$$\mathcal{L}_{int}(x) = g:\phi^2(x)\chi(x):, \tag{2}$$

where $\phi(x)$ is the scalar "quark" field, $\chi(x)$ is the scalar "gluon" field, and g is the coupling constant with dimension of the squared mass. For the "gluon" free field the massless Klein-Gordon equation is

$$\Box\chi(x) = 0. \tag{3}$$

Consequently, the propagator of this field is given by

$$D_\chi(p) = \frac{1}{p^2 - i\epsilon}. \tag{4}$$

Then the first term corresponding to one-particle exchange of the decomposition of the quasipotential V is given by

$$V_2(E;\vec{p},\vec{k}) = -\frac{g^2}{(\vec{p}-\vec{k})^2 + i\epsilon}, \tag{5}$$

where $\vec{p}-\vec{k}$ is the space component of the relative momentum. No radiative corrections are made for the propagator in (5). In the configuration space from eq.(5) we have

$$V_2(r) = -g^2 \int dk \frac{\exp(i\vec{k}\vec{x})}{\vec{k}^2 + i\epsilon} = \pi g^2 r, \tag{6}$$

where $r = |\vec{x}|$. Consequently, in the second order in g, we have the quasipotential that increases with r, i.e., with the relative displacement between particles.

As is well known, the potential of that linearity is a sufficient condition for existence of confinement. It can be checked, that the spectrum of energy, obtained from the solutions of eq.(1), coincides with that of energy, found in ref.[2] for the s-wave in 4-dimensional space-time.

2. In the four-dimensional space-time, eq.(1) has the form[1]

$$(E^2 - \vec{p}^2 - m^2)\phi(E;\vec{p},\vec{k}) = \int \frac{d^3k}{\sqrt{\vec{k}^2 + m^2}} V(E;\vec{p},\vec{k})\phi(E,\vec{k}). \tag{1'}$$

The potential $V(E;\vec{p},\vec{k})$ can be determined in the same manner as in the two-dimensional case, i.e., as the ladder decomposition, where the interaction Lagrangian is given by (2).

If the "gluon" field $\chi(x)$ satisfies the second-order equation in the four-dimensional space-time, the potential will decrease as $1/r$ at large r. Such a modification is possible for the theory that the quasipotential has form (6) if we assume that the free field $\chi(x)$ satisfies the equation

$$\Box^c \chi(x) = 0, \tag{7}$$

where $c = 1,2,3,\ldots$. Then the corresponding propagator for the field χ is given by

$$D_\chi(p) = \frac{1}{(p^2 + i\epsilon)^c}. \tag{8}$$

Substituting (8) into the first term of the decomposition for we have

$$V_2(\vec{p}) = -\frac{g^2}{(\vec{p}^2 + i\epsilon)^c}. \tag{9}$$

In the configuration space we have

$$V_2(r) = -\int d^3k \frac{\exp(i\vec{k}\vec{x})}{(\vec{k}^2 + i\epsilon)^c} = -\frac{g^2\pi^{3/2}}{(c-1)!}\Gamma\left(\frac{3}{2} - c\right)\left(\frac{r}{2}\right)^{2c-3}, \tag{10}$$

where $r = \sqrt{x_1^2 + x_2^2 + x_3^2}$. Formula (10) implies the distribution function for $c \neq 3/2, 5/2, \ldots$ Consequently, in the cases $c \geq 2$ the quasipotential (10) increases at large r as a linear function ($c = 2$) or more fastly ($c > 2$). Consequently, the four-order equation (7) for the "gluon" field is sufficient for confining the "quarks". It can be pointed out that in the case of linear potential as has been shown in[2], the quasipotential approach gives the correct mass spectrum for particles. On the other hand, the problem of confinement was considered in the framework of QFT based on the fourth-order equation in paper[3].

3. The propagators (4) and (8) have infrared divergences. Following paper[3] we use the infrared regularized propagator

$$D_{\mathfrak{D}/2}(p) = \lim_{\lambda^2 \to 0} \left[\frac{(-1)^{\mathfrak{D}/2 - 1}}{(p^2 - \lambda^2 - i\epsilon)^{\mathfrak{D}/2}} + i\pi^{\mathfrak{D}/2} \ln \frac{\lambda^2}{\mu^2} \delta_{\mathfrak{D}}(p)\right], \tag{11}$$

where $\mathfrak{D}$ is the dimensionality of space-time, λ and μ are parameters with the dimension of mass. In the configuration space the second term of (11) gives the following additional terms for the quasipotentials (6) and (10)

$$g^2 r \ln \mu r$$

and, consequently,

$$V_2(r) = g^2 r(\pi^{\mathfrak{D}/2} + \ln \mu r). \tag{12}$$

Using the iterative procedure given in ref.[1] with the quasipotential (12) the two-particle wave function can be derived. The parameters of the theory m, g, and μ can be found from the mass spectrum and decay probabilities of particles.

It can be checked that the radiative corrections to the propagator (8) do not break the character of the potential (12) at large distances.

Acknowledgements are due to Profs. I.S.Zlatev and V.A.Meshcherjakov for stimulating interest, and to Prof. V.A.Matveev, to Dr. N.B.Skachkov, and S.I.Zlatev for valuable discussions and remarks.

REFERENCES

1. Logunov A.A., Tavkhelidze A.N. Nuovo Cim., 1963, 29, p.380; Faustov R.N. Physics of Elementary Particles and Atomic Nuclei, Atomizdat, M., 1972, v.3, p.238; Rizov V.A., Todorov I.T. Physics of Elementary Particles and Atomic Nuclei, Atomizdat, M., 1975, v.6, p.669.
2. Goloskokov S.V., Kuleshov S.P., Sidorov A.V. Pis'ma v JETF, 1980, 31, p.154.
3. D'Emilio E., Mintchev M. Phys.Lett., 1980, 89B, p.207.

Received by Publishing Department
on May 15 1980.