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S.V.Zenkin

ON DERIVATION OF THE PARITY-VIOLATING INTERNUCLEON POTENTIAL

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Introduction

At the present time there is no complete clarity in the description of the parity-violating N-N interactions[1,2]. Usually at the low energy these interactions are described by P-odd OBEP, which are derived by standard methods[1] on the basis of the diagram of Fig.1. The problem consists in the calculation of the effective parity-violating NNB vertices.

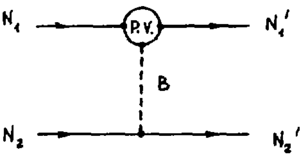

Fig. 1

In the framework of current algebras we consider the P-odd amplitude $N \to N' B_\mu$, where B_μ stands for vector mesons ρ, ω, φ. According to modern ideas[3] diagrams of Fig. 2 may contribute into this vertex (wavy lines are W, Z).

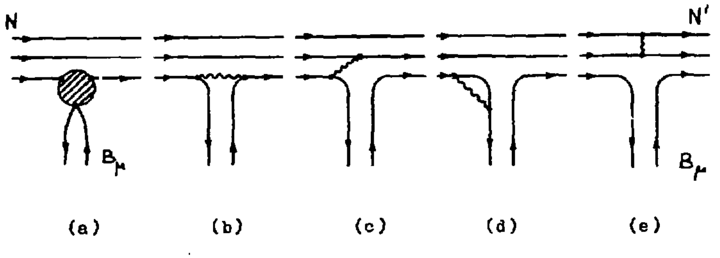

Fig. 2

The framework of current algebras (more exactly that of the current field identity) restricts the consideration to diagram (a), where the shaded circle means the complete set of permissible states. Usually the contribution of (a) is estimated within factorization approximation[4] (see also[2] and[5]), i.e., only hadron vacuum states are taken into consideration, Fig.3. Then one can

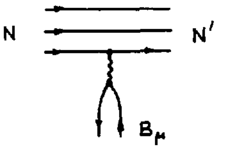

Fig. 3

regard the gluon corrections as subsequent approximations[6]. Such a situation is attractive from the standpoint of QCD. Therefore it is important to verify the factorization approximation in the framework of other approach.

As a rule calculations based on current algebras contain uncertainties caused by the Schwinger terms (S.T.) and by the seagulls. So, Olesen and Rao[7] suggesting the cancellation of S.T. by seagulls came to a negligible contribution of the diagram of Fig.2(a). At the same time Fischbach, Tadic and Trabert[8] assumed that seagull terms do not change but only covariantize the amplitude, calculated without their consideration, and they obtained the result coinciding with that of the factorization approximation.

This uncertainty may be considered in terms of more simple objects than amplitudes. Covariant T^*-products of two currents (correlation functions) or Ward identities may be used as such objects. Then in case of c-number S.T. the general solution can be found to the equation for seagull terms. Considering the covariant amplitude in terms of correlation functions one can show that for small momentum transfer this amplitude is completely determined by the time components of seagulls and the results of refs.[7,8] correspond to different particular solutions of this equation.

We show that the use of the current field identity within such an approach should be completed by certain selfconsistency condition. This condition allows us to remove arbitrariness in the choice of the solution of the equation for seagulls, moreover the final result coincides with that of the factorization approximation.

The paper is organized as follows. In Sec.2 the general solution of the equation for seagulls is found, covariant Ward identities are quoted and the amplitude is calculated within the framework of the adopted current algebra. In Sec.3 the self-consistency condition is formulated and the effective P-odd $NN'B_\mu$ vertex is calculated.

2. Ward Identities and Covariant Amplitude

According to the standard reduction technique the parity-violating amplitude $N \to N'B_\mu$ can be represented as [8]

$$\langle N' B^a_\mu | \mathcal{L}^{P.V.}(0) | N \rangle = \varepsilon^{*\mu} M^a_\mu \quad ,$$

$$M^a_\mu = -\frac{i}{\sqrt{(2\pi)^3 2k^0}} \int d^4x\, e^{ikx} (\Box + m^2) \langle N' | T\left(B^a_\mu(x), \mathcal{L}^{P.V.}(0)\right) | N \rangle . \tag{1}$$

Here ε^μ is the polarization vector and $B^a_\mu(x)$ is the interpolating field of a vector meson (a=1,2,3 correspond to ρ isotriplet, a=0,8 correspond to linear combinations of ω and φ), $\mathcal{L}^{P.V.}(0)$ is the parity-violating part of the weak interaction Lagrangian with $\Delta S = 0$ [2]

$$\mathcal{L}^{P.V.}(0) = \frac{G}{2\sqrt{2}}\, u_{ab} \left\{ V^a_\mu(0), A^{b\mu}(0) \right\} .$$

Within the framework of current algebras it is convenient to consider the longitudinal part of amplitude M^a_μ , i.e., the product $k^\mu M^a_\mu$. Using the current field identity of the form

$$B^a_\mu(x) = \frac{f}{m^2} V^a_\mu(x)$$

and conservation of the vector currents $\partial_\mu V^{a\mu}(x) = 0$ we find

$$k^\mu M^a_\mu = C(k)\, u_{bc} \int d^4x\, e^{ikx} \langle N' | \left\{ \partial^\mu T\left(V^a_\mu(x), V^b_\lambda(0)\right), A^{c\lambda}(0) \right\} + \tag{2}$$
$$+ \left\{ \partial^\mu T\left(V^a_\mu(x), A^{c\lambda}(0)\right), V^b_\lambda(0) \right\} | N \rangle ,$$

where

$$C(k) = -\frac{Gf}{2\sqrt{2}\, m^2} \frac{k^2 - m^2}{\sqrt{(2\pi)^3 2k^0}} \quad .$$

Thus the longitudinal part of the amplitude is expressed in terms of the divergences of T-products of two currents and can be represented according to Ward identities. We find the general form of these identities.

Let the currents from (2) belong to the following algebra.

(A). All S.T. are c-numbers. For instance the algebra of fields, free quark model and also quark models with vector gluons satisfy this assumption /9/.

(B). Equal time commutators (E.T.C.) $[V_0^a, V_0^b]$ do not contain S.T.

$$[V_0^a(x), V_0^b(0)]_{x^0=0} = i\,\delta^3(x)\, f^{abc}\, V_{c0}(0) \quad ,$$

f^{abc} are SU(3) structure constants. These E.T.C. are model-independent and are a consequence of the canonical formalism /9,10/.

(C). Mixed E.T.C. $[V_0^a, A^b_\mu]$ do not contain S.T.

$$[V^a_0(x), A^b_\mu(0)]_{x^0=0} = i\,\delta^3(x)\, f^{abc} A_{c\mu}(0) \; .$$

This assumption seems to be natural too by virtue of (A). We assume nothing about other E.T.C.

From (B) it follows that E.T.C. $[V^a_0, V^b_\ell]$ contain not more than one S.T. /9,10/

$$[V^a_0(x), V^b_\ell(0)]_{x^0=0} = i\,\delta^3(x)\, f^{abc}\, V_{c\ell}(0) + S^{ab}_{\ell m}\, \partial^m \delta^3(x) \quad (3)$$

with

$$S^{ab}_{\ell m} = S^{ba}_{m\ell} \; .$$

Let us consider the T-product of two currents from such an algebra.

$$T^{ab}_{\mu\nu}(x) = \theta(x^0)\,[V^a_\mu(x), V^b_\nu(0)] + V^b_\nu(0)\, V^a_\mu(x) \; . \qquad (4)$$

It is known that S.T. break the Lorentz covariance (L.C.) of this product/9,10/.

$$[M_{lo}, T^{ab}_{\mu\nu}(x)] = X_{lo} T^{ab}_{\mu\nu}(x) + G^{\lambda}_{\mu,lo} T^{ab}_{\lambda\nu}(x) +$$

$$+ G^{\lambda}_{\nu,lo} T^{ab}_{\mu\lambda}(x) - i x_l \delta(x^0) [V^a_\mu(x), V^b_\nu(0)] .$$

Here M_{lo} is a generator of the Lorentz rotation $X_{lo} = i(x_l\partial_o - x_o\partial_l)$, $G^{\lambda}_{\mu,lo} = i(g_{\mu l}\delta^{\lambda}_{o} - g_{\mu o}\delta^{\lambda}_{l})$. The term breaking the covariance (proportional to x_l) is nonzero by virtue of (3).

Following ref./10/, for constructing the covariant T^*-product we redefine (4) for the coinciding arguments

$$T^{*ab}_{\mu\nu}(x) = T^{ab}_{\mu\nu}(x) + \delta^4(x)\, \Upsilon^{ab}_{\mu\nu} . \qquad (5)$$

Requiring $T^{*ab}_{\mu\nu}(x)$ to be the tensor of second rank, one obtains the equation for the seagulls $\Upsilon^{ab}_{\mu\nu}$ /10/

$$\delta^4(x)[M_{lo}, \Upsilon^{ab}_{\mu\nu}] = i x_l\, \delta(x^0) [V^a_\mu(x), V^b_\nu(0)] +$$

$$+ \delta^4(x)\left(G^{\lambda}_{\mu,lo} \Upsilon^{ab}_{\lambda\nu} + G^{\lambda}_{\nu,lo} \Upsilon^{ab}_{\mu\lambda}\right). \qquad (6)$$

By the assumption (A) $\Upsilon^{ab}_{\mu\nu}$ may be regarded as a c-number. Then the general solution of (6) has the form

$$\left.\begin{aligned} &\Upsilon^{ab}_{om} = \Upsilon^{ab}_{mo} = 0 \\ &\Upsilon^{ab}_{mn} - g_{mn}\Upsilon^{ab}_{oo} = -S^{ab}_{nm} \end{aligned}\right\} . \qquad (7)$$

Furthermore, if $S^{ab}_{lm} = 0$ we will always choose $\Upsilon^{ab}_{\mu\nu} = 0$. We mention that for $\Upsilon^{ab}_{oo} = 0$ (7) is a solution of (6) also in the case when S.T. are q-number/10/.

Using (7), we find the general form of covariant Ward identities

$$\partial^\mu T^{*ab}_{\mu\nu}(x) = i\delta^4(x) f^{abc} V_{c\nu}(0) + \partial_\nu \delta^4(x)\, \Upsilon^{ab}_{oo} ,$$

$$\partial^\mu T^*(V^a_\mu(x), A^b_\nu(0)) = i\,\delta^4(x) f^{abc} A_{c\nu}(0) . \qquad (8)$$

We return now to the calculation of the amplitude. Since one of the principal requirements for the amplitude is its L.C., we ought to realize T-products in (2) as correlation functions defined by (5) and (7). Then inserting (8) into (2) we obtain

$$K^\mu M^a_\mu = C(\kappa) \langle N' | \left[Q^a, \mathcal{L}^{P.V.}(0) \right] | N \rangle - \qquad (9)$$
$$- 2i C(\kappa) \kappa_\lambda \Upsilon^{ab}_{00} u_{bc} \langle N' | A^{c\lambda}(0) | N \rangle .$$

Here $Q^a = \int d^3x \, V^a_0(x)$ and the first term in the right-hand side of (9) vanishes [7,8] (owing to the strangeness conservation for a=0,8 and owing to the even CP-parity of $\mathcal{L}^{P.V.}(0)$ for a=1,2,3).

Thus the longitudinal part of the amplitude is completely determined by the time components Υ^{ab}_{00} of seagull terms. From solution (7) it is clear that the requirement of L.C. is not sufficient for complete determining of these components. Choosing the solution $\Upsilon^{ab}_{00} = 0$ we obtain the result of Olesen and Rao [7], choosing $^{00}\Upsilon^{ab}_{\ell m} = 0$ within the framework of algebra of fields $\left(S^{ab}_{\ell m} = i \delta^{ab} m^2 / f^2 g_{\ell m} \right)$ we obtain the result of Fischbach, Tadic and Trabert [8]. The choice of the relevant value of Υ^{ab}_{00} requires additional assumptions. So, for example, regarding the Ward identities (8) as the universal relation we may hope to measure Υ^{ab}_{00} in strong or electromagnetic processes described by the relevant currents. However, in the considered scheme of calculation of potentials the equal status of solutions (7) is broken for the intrinsic reasons. Let us consider this problem.

3. Effective Vertex and Selfconsistency Condition

Usually the parity-violating N-N potential is derived from the effective Lagrangian determining the diagram of Fig.1 [2].

$$\mathcal{L}_{eff}(x) = \upsilon_{ab} \, B^a_\mu(x) \, \bar{N}(x) \gamma^\mu \gamma_5 \frac{\tau^b}{2} N(x) +$$
$$+ f_a \, B^a_\mu(x) \, \bar{N}(x) \gamma^\mu \frac{\tau_a}{2} N(x) + \cdots \qquad (10)$$

Here τ^a are Pauli matrices $\left(\tau^0 = \tau^8 = \mathbb{1} \right)$, the dots stand for nonminimal interactions. The main problem here is to find the matrix υ . This matrix can be found in the following way. Inserting (10) into (1) and using the standard (Wick) definition of the T-product of vector fields /11/

$$T\left(B^a_\mu(x), B^b_\nu(0)\right) = \theta(x^0)\left[B^a_\mu(x), B^b_\nu(0)\right] +$$

$$+ B^b_\nu(0) B^a_\mu(x) + \frac{i}{m^2}\delta^{ab}\delta_{\mu 0}\delta_{\nu 0}\delta^4(x) \qquad (11)$$

we obtain

$$M^a_\mu = \frac{1}{\sqrt{(2\pi)^3 2k^0}}\,\upsilon^a_c\left(g_{\mu\lambda} - \frac{1}{m^2}k_\mu k_\nu\right)\bar{N}'\gamma^\lambda\gamma_5\frac{\tau^c}{2}N\,. \qquad (12)$$

At small momentum transfers $(k^2 \to 0)$ one can find the matrix R /2/

$$\langle N'| A^c_\lambda(0)|N\rangle = R^c_{c'}\,\bar{N}'\gamma_\lambda\gamma_5\frac{\tau^{c'}}{2}N\quad . \qquad (13)$$

Comparing (12) with (9) and taking into consideration (13), we find

$$\upsilon^a_c = -i\,\frac{Gf}{\sqrt{2}}\,\Upsilon^{ab}_{00}\,u_{bc'}R^{c'}_c \quad . \qquad (14)$$

Let us notice now that while deriving the relation (14), the fields $B^a_\mu(x)$ have been regarded,on the one hand, as quantized vector fields within the Lagrangian formalism, on the other hand, as vector currents belonging to an algebra. Since the quantized vector fields belong to certain algebra (with $S^{ab}_{\ell m} = \frac{i}{m^2}\delta^{ab}g_{\ell m}$) too, selfconsistency of the whole method of calculation requires that at least two conditions be fulfilled:
(i) vector currents must belong to the algebra with $S^{ab}_{\ell m} = i\, m^2/f^2\,\delta^{ab}g_{\ell m}$ (algebra of fields);
(ii) according to (11) the solution

$$\Upsilon^{ab}_{00} = i\,\frac{m^2}{f^2}\,\delta^{ab} \qquad (15)$$

must be chosen.

We mention that the choice $\Gamma^{ab}_{\ell m} = 0$ follows from (i), (ii), and (7) thus confirming the assumptions of works [1,8]. Inserting (15) into (14) we obtain finally:

$$v^a{}_c = \frac{G m^2}{\sqrt{2} f} u^a{}_{c'} R^{c'}{}_c ,$$

that in our notation coincides with the result of the factorization approximation.

In conclusion we mention that the considered method of calculation of the effective vertex is based on the two fundamental hypotheses: current field identity and conservation of vector currents and therefore may be regarded as a substantiation of the factorization approximation.

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