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G.V.Efimov, M.A.Ivanov, E.A.Nogovitsyn

RADIATIVE DECAYS OF $\eta - \eta'$-MESONS IN QUARK NONLOCAL MODEL

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1. Introduction

A successful description of some strong, weak and electromagnetic decays of the octet of vector and pseudoscalar mesons, of octet and decaplet of baryons within the quark nonlocal model[1] indicates that the model, having only two free parameters, is equally valid in the range of quark confinement.

In the present paper we analyse the radiative decays of $\eta-\eta'$-mesons that have become actual in view of recent experiments[2-6]. The latter include detection and calculation of characteristics of decays: $\pi^0 \to \gamma e^+ e^-$[12], $\eta \to \mu^+\mu^-\gamma$[13], $\eta' \to \mu^+\mu^-\gamma$[14], $\omega \to \pi^0\mu^+\mu^-$[15]. Besides, the angle of $\eta-\eta'$-mixing is still an open question: either $\theta = -11°$ as follows from quadratic mass formulae, or $\theta = -18°$, as follows from the analysis of reaction $\pi^- p \to \eta' n$[7].

The experimental width of decay $\eta \to \pi^0\gamma\gamma$ requires also a careful test since the existing theoretical estimates are much lower[8] than the experimental value[11].

The above listed processes to be described within standard methods require combinations of different approaches and hypothesis (current algebra, vector dominance model (VDM), chiral theories, sum rules, etc.). For instance, chiral theory describes only the widths of decays $\eta \to \gamma\gamma$, $\eta \to \pi^+\pi^-\gamma$, $\eta \to 3\pi$, $\eta \to \pi^0\gamma\gamma$, while the description of the meson electromagnetic form factor in decays $P \to \gamma l^+ l^-$ requires the VDM hypothesis. The quark nonlocal model allows a unique description of all these processes.

In this paper within the above model we calculated:

1) the widths of decays $P\to\gamma\gamma\,(P=\pi^0,\eta,\eta')$, $\eta\to\pi^+\pi^-\gamma$, $\eta\to\pi^0\gamma\gamma$, $\eta'\to V\gamma\,(V=\rho^0,\omega)$,

2) electromagnetic form factors of decays $P\to\gamma\ell^+\ell^-$ $(P=\pi^0,\eta,\eta')$.

The calculations were performed for two values of the $\eta-\eta'$-mixing angle:

$$\theta=-11^\circ,\quad \theta=-18^\circ.$$

The better accord with experiment turns out to be at $\theta=-11^\circ$, the widths of decays $\eta\to\gamma\gamma$, $\eta\to\pi^+\pi^-\gamma$, $\eta'\to\gamma\gamma$, $V\gamma$ being in good agreement with recent experimental data. The $\eta\to\pi^0\gamma\gamma$ -decay width is, as expected, by far lower (~50 times) than the experimental value. Electromagnetic characteristics of the decay $P\to\gamma\ell^+\ell^-$ are in satisfactory agreement with experiment for π^0-meson /2/ and in good agreement/3,4/ for η -meson.

2. Interaction Lagrangian and $\eta-\eta'$-Mixing Angle

In the nonlocal quark model/1/ the $\eta-\eta'$-meson-quark interaction is described by the following Lagrangian

$$\mathcal{L}_I=\frac{ih}{\sqrt{2}}\left[\sqrt{\frac{2}{3}}\,\eta_1(\bar q_a\gamma_5 q_a)+\eta_8(\bar q_a\gamma_5\lambda^8 q_a)\right].$$

Here

$$\eta_1=\eta'\cos\theta-\eta\sin\theta$$
$$\eta_8=\eta'\sin\theta+\eta\cos\theta$$

$$\theta=\begin{cases}-11^\circ\ \text{(mass quadratic formulas)}\\ -18^\circ\ \text{(analysis of reaction } \pi^-p\to\eta' n\ /7/)\end{cases}$$

All parameters of our model have been defined earlier/1/ and equal

$$\xi=1.4,\ L=3.12\ GeV^{-1},\ \lambda=\left(\frac{h}{4\pi}\right)^2=0.13.$$

Results of calculations are expressed in terms of the following functions

$$C_n(\xi)=\frac{2}{n!}\int_0^\infty dt\, t^{2n+1}e^{-t^2}\cos\xi t,$$

$$S_n(\xi)=\frac{2}{n!}\int_0^\infty dt\, t^{2n+1}e^{-t^2}\frac{\sin\xi t}{\xi t}.$$

3. Decays $P\to\gamma\gamma$, $\eta\to\pi^+\pi^-\gamma$, $\eta\to\pi^0\gamma\gamma$.

a) $P\to\gamma\gamma$ $(P=\pi^0,\eta,\eta')$. The diagram of this decay is shown in Fig. 1. The invariant amplitude of this process is written in the standard notation

$$\langle\gamma_1\gamma_2|T|P\rangle=e^2 g_{P\gamma\gamma}\,\epsilon_{\mu\nu\alpha\beta}\,p_1^\mu\epsilon_1^\nu p_2^\alpha\epsilon_2^\beta,$$

$$\Gamma(P\to\gamma\gamma)=\frac{1}{4}\pi\alpha^2 m_P^3\, g^2_{P\gamma\gamma}.$$

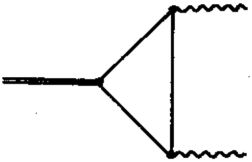


Fig.1

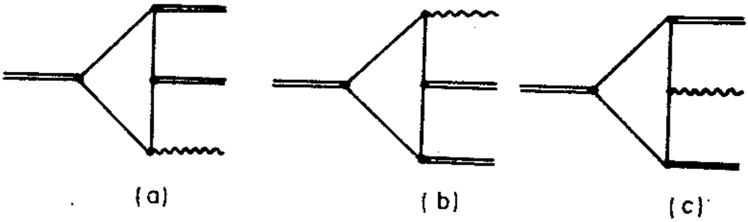
In our model we get

$$g^2_{\pi^0\gamma\gamma}=L^2\frac{\lambda}{2\pi^2};$$

$$g^2_{\eta\gamma\gamma}=L^2\frac{\lambda}{2\pi^2}\cdot\frac{1}{3}\left(\cos\theta-2\sqrt{2}\sin\theta\right)^2;$$

$$g^2_{\eta'\gamma\gamma}=L^2\frac{\lambda}{2\pi^2}\frac{1}{3}\left(2\sqrt{2}\cos\theta+\sin\theta\right)^2.$$

Numerical values are given in Table I. One can observe good agreement with experiment for $\theta \approx -11^\circ$.



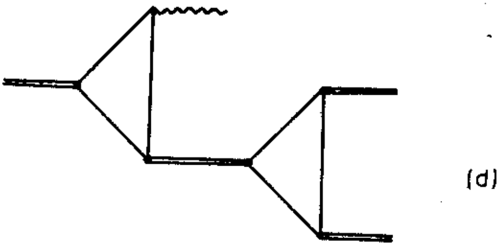


Fig.2

b) $\eta\to\pi^+\pi^-\gamma$. The diagrams for this process are shown in Fig. 2. The invariant amplitude corresponding to diagrams 2a, 2b, 2c is

$$M_1(\eta\to\pi^+\pi^-\gamma)=\epsilon_{\mu\nu\alpha\beta}\,\epsilon^\mu q^\nu P_-^\alpha P_+^\beta\times C.$$

Here

$$C=-e\frac{96\pi}{\sqrt{6}}\lambda^{3/2}L^{3}(\cos\theta-\sqrt{2}\sin\theta)R(\xi),$$

$$R(\xi)=\frac{1}{24}\Big\{\Big(1-\frac{\xi^{2}}{16}\Big)\Big[C_{0}(\sqrt{3}\xi)-C_{0}\Big(\frac{\xi}{\sqrt{3}}\Big)\Big]+\frac{1}{12}\Big[C_{1}(\sqrt{3}\xi)-$$

$$-C_{1}\Big(\frac{\xi}{\sqrt{3}}\Big)\Big]+\frac{27}{16}\xi^{2}\Big[S_{0}(\sqrt{3}\xi)-S_{0}\Big(\frac{\xi}{\sqrt{3}}\Big)\Big]+\frac{\xi^{2}}{4}\Big[S_{1}(\sqrt{3}\xi)-S_{1}\Big(\frac{\xi}{\sqrt{3}}\Big)\Big]\Big\}.$$

$$R(1.4)=-0.12.$$

The amplitude corresponding to diagram 2d has the form

$$M_{2}(\eta\to\pi^{+}\pi^{-}\gamma)=M_{1}(\eta\to\pi^{+}\pi^{-}\gamma)\frac{b(\xi)}{1-(p_{+}+p_{-})^{2}/m_{\rho}^{2}},$$

where

$$b(\xi)=\frac{8\xi^{2}}{9\mu_{\rho}^{2}}\lambda\frac{[1+2S_{1}(\sqrt{2}\xi)-C_{0}(\sqrt{2}\xi)]S_{2}(\xi/\sqrt{3})}{R(\xi)},$$

$$\mu_{\rho}^{2}=\frac{m_{\rho}^{2}L^{2}}{4};\quad b(1.4)=-1.42.$$

The width of this decay is

$$\Gamma(\eta\to\pi^{+}\pi^{-}\gamma)=m_{\eta}\cdot 64\lambda^{3}\alpha(m_{\pi}L)^{6}R^{2}(\xi)(\cos\theta-\sqrt{2}\sin\theta)^{2}\cdot J,$$

$$J=\int_{0}^{3/2}dt\,t^{3}(9-2t)\sqrt{\frac{3-2t}{2(2-t)}}.\Big[1+\frac{2b(\xi)}{1+\frac{1}{2}t}\Big]^{2},$$

where we accept $\frac{4m_{\pi}^{2}}{m_{\eta}^{2}}=\frac{1}{4}$. For θ =-11° we have quite good agreement with experiment (see Table 1).

Table 1

	$\Gamma_{\text{exp.}}$	$\Gamma_{theor.}(\xi=1.4)$ $\theta=-11^\circ$	 $\theta=-18^\circ$
$\eta\to\gamma\gamma$	(323±54) eV[11]	342 eV	492 eV
$\eta\to\pi^+\pi^-\gamma$	(41.6±7.0) eV[11]	41 eV	50 eV
$\eta\to\pi^0\gamma\gamma$	(26±14) eV[11]	0.45 eV	0.55 eV
$\eta'\to\gamma\gamma$	(5.4±2.1) keV[6]	5.3 keV	4.5 keV
$\eta'\to\rho^0\gamma$	(83±52) keV[6]	106 keV	80 keV
$\eta'\to\omega\gamma$	(5.9±3.6) keV[6]	11 keV	8 keV

c) $\eta\to\pi^0\gamma\gamma$. The diagrams of this decay are shown in Fig. 3. The invariant amplitude is

$$M(\eta\to\pi^0\gamma\gamma)=-e^2\lambda L^2\frac{4}{3\sqrt{3}}\left[\cos\theta-\sqrt{2}\sin\theta\right]\cdot$$

$$\cdot\,\epsilon_\mu(k_1)\,\epsilon_\nu(k_2)\left[(k_1k_2)g_{\mu\nu}-k_2^\mu k_1^\nu\right].$$

The width of this process is

$$\Gamma(\eta\to\pi^0\gamma\gamma)=m_\eta\alpha^2\lambda^2(m_\pi L)^4\frac{4}{27}\cdot\frac{I}{\pi}\left(\cos\theta-\sqrt{2}\sin\theta\right)^2,$$

where

$$I=\int\limits_{1}^{M/m_\pi}du\sqrt{u^2-1}\left(u-\frac{M}{m_\pi}\right)^2\simeq 0.391,$$

$$M=\frac{m_\eta^2+m_\pi^2}{2m_\eta}.$$

Fig.3

One can see from Table 1 that the theoretical value is about fifty times smaller than the experimental one though higher than the prediction of chiral theory/8/.

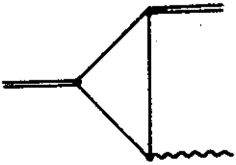


Fig.4

d) $\eta'\to V\gamma$. The diagram of this process is shown in Fig.4. The invariant amplitude is

$$M(\eta'\to V\gamma)=e\,g_{\eta' V\gamma}\,\epsilon_{\mu\nu\alpha\beta}\,\epsilon_\gamma^\mu\,\epsilon_V^\nu\,p_\gamma^\alpha\,p_V^\beta .$$

The width of this decay is

$$\Gamma(\eta'\to V\gamma)=\frac{\alpha}{8}m_{\eta'}^3\left[1-\frac{m_V^2}{m_{\eta'}^2}\right]^3\cdot g_{\eta' V\gamma}^2 .$$

Here

$$g^2_{\eta'\rho^0\gamma}=\lambda^2 L^2\cdot 6\left(\cos\theta+\frac{1}{\sqrt{2}}\sin\theta\right)^2\left[K_{\rho V}(\xi)\right]^2,$$

$$g^2_{\eta'\omega\gamma}=\frac{1}{9}g^2_{\eta'\rho^0\gamma},$$

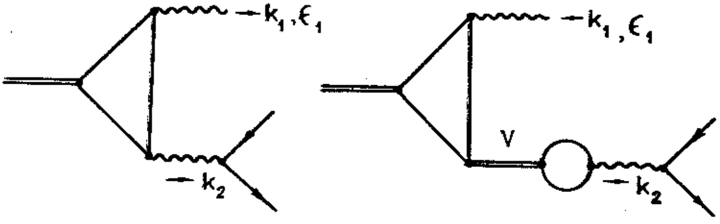
$$K_{\rho V}(\xi)=\xi\left[1+2S_1(\sqrt{2}\xi)-C_0(\sqrt{2}\xi)\right].$$

The obtained results are in good agreement with experiment/6/ (see Table 1).

4. Form Factors of the Decay $P\to\gamma l^+l^-$

The diagrams describing the decay are shown in Fig. 5. The invariant amplitude is

$$M(P\to\gamma l^+l^-)=-e^3\mathcal{F}_P(k_2^2)\epsilon_{\mu\rho\nu\sigma}\epsilon^{\mu}(k_1)k_1^{\rho}k_2^{\sigma}\mathcal{D}^{\nu\alpha}(k_2)j^{\alpha}(k_2).$$



(a) (b)

Fig.5

Here

$$k_1^2=0,\ p^2=m_P^2,\ \mathcal{D}^{\nu\alpha}(k_2)=g^{\nu\alpha}/k_2^2;$$
$$j^{\alpha}(k_2)=\bar{l}(q_1)\gamma^{\alpha}l(q_2)\big|_{q_1+q_2=k_2};\ q_1^2=q_2^2=m_l^2.$$

$$\Phi_P(k_2^2) = g_{P\gamma\gamma}(k_2^2) + k_2^2 \sum_V \frac{g_{PV\gamma}}{f_V} \cdot \frac{1}{m_V^2 - k_2^2} \, .$$

The form factor for small k_2^2 has the following form

$$\Phi_P(k_2^2) = g_{P\gamma\gamma}(0)\left\{1 + \frac{k_2^2}{M_P^2}\right\},$$

where

$$\frac{1}{M_P^2} = \frac{L^2}{4}\,\frac{1}{1 + a(\xi)\mu_P^2}\left\{a(\xi) + F(\xi)\cdot\tau_P\right\},$$

$$\mu_P^2 = \left(\frac{m_P L}{2}\right)^2, \quad a(\xi) = \frac{1}{12}\left(1 + \frac{1}{2}\xi^2\right),$$

$$F(\xi) = \frac{8\lambda}{m_g^2 L^2}\,\xi^2 S_0(\xi)\left[1 + 2S_1(\sqrt{2}\,\xi) - C_0(\sqrt{2}\,\xi)\right],$$

$$\tau_{\pi^0} = 2,$$

$$\tau_\eta = \frac{10}{3}\,\frac{\cos\theta - \sqrt{2}\,\sin\theta}{\cos\theta - 2\sqrt{2}\,\sin\theta},$$

$$\tau_{\eta'} = \frac{10}{3}\,\frac{\sqrt{2}\cos\theta + \sin\theta}{2\sqrt{2}\cos\theta + \sin\theta} \, .$$

It turns out that the contribution of the first diagram to the amplitude M amounts for π^0 - 28%, η - 20%, η' = 40%. The theoretical values for M_P^{-2} are given in Table 2. For π^0-meson our result is about twice as small as the experimental one/2/ and it is of an order of the MVD prediction. For η-meson our result is in good agreement with the recent experiment/3,4/. For η'-meson experimental data are absent.

Table 2.

	M_P^{-2} (GeV^{-2})		
	Experiment	Theory $\theta=-11^\circ$	Theory $\theta=-18^\circ$
$\pi^0 \to \gamma e^+ e^-$	5.5 ± 1.7 [2]	2.3	2.3
$\eta \to \gamma \mu^+ \mu^-$	3 ± 1 [3,4]	2.6	2.5
$\eta' \to \gamma \mu^+ \mu^-$		1.4	1.3

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