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исследований
дубна

E2-80-255

R.P.Zaikov

ON A CLASS OF RELATIVISTIC EQUATIONS FOR FIELDS WITH ARBITRARY SPIN

Submitted to ЯФ

1980

1. INTRODUCTION

When dealing with relativistic wave equations for fields with spin greater than one [1,2] inconsistencies and difficulties arise connected with problems for including external fields [3], quantization [4], positivity and existence of states with nonphysical masses [5] (a comprehensive list of references is given in ref. [6]). In the last years, self-consistent equations have been proposed for fields with spins $S = 2$ [7], $5/2$ [8] and 3 [9]. So, if fields with spin greater than two are considered, it is necessary to include auxiliary fields. However, the theory based on these equations is nonrenormalizable. Another way to avoid the above-mentioned difficulties was proposed in paper [10]. There, the main requirement for the writing of first order relativistic equations was the correctness of the Cauchy problem, i.e., nonsingularity of the coefficient of the time derivative. In the last case, it is necessary to consider reducible representations of SL(2,C) group and the mass operator is a degenerate matrix. In such a way, it is possible to construct a great number of relativistic first order equations. However, the method for explicit writing of these equations given in [10] is connected with great technical difficulties.

In the present paper another method of constructing self-consistent relativistic equations for the fields with arbitrary spin is given. These equations can be transformed, without any difficulties, in the form of equations proposed in [10]. Our method is based on the following assumptions:

(i). There exists a local positive-definite relativistic invariant action, from which equations can be derived.

(ii). In the limiting case of zero mass and dimensional coupling constants this action must be a conformal invariant.

(iii). The coefficients of the highest-order time derivatives, must be invertible matrices.

(iv). The spectrum of energy, in the free case, is positive-definite and the masses are physically admissible.

Assumption (i) is natural. Assumption (ii) quarantees the conformal invariance of the massless theory with dimensionless coupling constants. This condition is very restrictive. The condition (iii) quarantees the correctness of the Cauchy problem [10] because, if (iii) is satisfied, the equations can be solved for the highest-order time derivative. The free field equations with nondefinite sign of energy and nonphysical values of mass of particles are excluded by assumption (iv).

For the representations of conformal group SU(2,2) under consideration, (we restrict ourselves only to such irreducible representations of SU(2,2), which are irreducible with respect to SL(2,C) subgroup, too), the so found equations are of order higher than two. These equations can be reduced to a set of equations of first or second order. The mass term is introduced in such a way that the states with nonphysical values of mass do not arise. As a consequence of the sufficiently high order of equations of motion for the initial fields the theory is ultraviolet renormalizable. However, there appeared an indefinite metric. A realization of the reducible representations of SL(2,C), very convenient for writing the equations for arbitrary spin in a compact form, is the one, in terms of homogeneous polynomials of two component complex spinor $z=(z_1,z_2)$ /11/.

In the second section the free field equations for the fields with arbitrary spin are constructed. As an example the equations for fields with spin $S = 0, 1/2, 1, 3/2$ and 2 are considered.

In the third section the propagator for the fields with arbitrary spin is given in terms of spin projection operators.

The problems of quantization and switching on of interactions will be considered separately.

2. FREE EQUATIONS FOR FIELDS WITH AN ARBITRARY SPIN

Consider first the kinetic part of action S_k. The mass term will be included later. From assumption (ii), it follows that S_k must be a conformal invariant. It is known /12/, that the general form of conformal-invariant bilinear form is given by

$$S_k = \frac{1}{(2j_1)!(2j_2)!}\int d^4x\, d^4y\, \psi_{\chi_1}(x;\frac{\partial}{\partial z}) F_{\tilde{\chi}_1\tilde{\chi}_2}(x-y; z, \frac{\partial}{\partial w})\psi_{\chi_2}(y;w), \quad (2.1)$$

where $\chi = \{d, j_1, j_2\}$ label the irreducible representations (IR) of SU(2,2) (d is the scale dimension, $j_1, j_2 = 0, 1/2, 1, \ldots$ label the finite dimensional IR of SL(2,C) subgroup), $\tilde{\chi} = \{4-d, j_1, j_2\}$ is the dual representation of χ /12/, z is a complex two component spinor /11/. The fields $\psi(x;z)$ transforming according to finite-dimensional IR of SL(2,C) are homegeneous polynomials in $z(\bar{z})$ of degree $2j_1(2j_2)$. The scalar product in the spinor space is given by

$$(f,g) = \frac{1}{(2j_1)!(2j_2)!} f(\frac{\partial}{\partial z}, \frac{\partial}{\partial \bar{z}}) g(z,\bar{z}) . \tag{2.2}$$

In (2.1) $F(x;z,w)$ is a conformal invariant two-point function (intertwining operator) satisfying the normalization condition

$$\frac{1}{(2j_1)!(2j_2)!} \int d^4x_2 F_{\chi_1,\chi_2}(x_1 - x_2; z_1, \frac{\partial}{\partial z_2}) F_{\tilde{\chi}_2,\tilde{\chi}_1}(x_2 - x_3; z_2, z_3) = \tag{2.3}$$

$$= \delta(x_1 - x_3) I(z_1; z_3) ,$$

where $I(z,w)$ is the identity operator in the space of homogeneous polynomials. The general form of $F(x;z,w)$ for the arbitrary IR of $SU(2,2)$, which is irreducible with respect to the $SL(2,C)$ subgroup also, is found in [12-14]. Its Fourier kernel, in the case when $j_1 \geq j_2$, is given by

$$F(p;z,w) = \frac{(2j_2)!(d_0-2)_{2j_2}}{(2j_2+c+1)!} (p^2)^{d_0-2} (z\underset{\sim}{p}\bar{w})^{2(j_1-j_2)} \times$$

$$\times (z\underset{\sim}{p}\bar{z}w\underset{\sim}{p}\bar{w})^{2j_2} P_{2j_2}^{(d_0-2,\, j_1-j_2)}\left(1 - 2\frac{p^2 z\epsilon w \bar{z}\epsilon\bar{w}}{z\underset{\sim}{p}\bar{z}\, w\underset{\sim}{p}\bar{w}}\right), \tag{2.4}$$

where the normalization factor is determined by (2.3), $P_n^{(\alpha,\beta)}(x)$ are Jacobi polynomials, $d_0 = d - j_1 - j_2$, $(a)_k = a(a+1)\ldots(a+k-1)$. The two-point function (2.4) is nonvanishing only if the following conditions $d_1 = d_2 = d$, $j_1^1 = j_2^2 = j_1$ and $j_2^1 = j_1^2 = j_2$ are satisfied [15]. In the case, when $j_2 > j_1$, the corresponding two-point function can be obtained from (2.4) with the substitutions $j_1 \to j_2$ and $z \to w$.

In the case when $d_0 = 2$ only the maximal spin $s_{max} = j_1 + j_2$ is presented in (2.4). In all other cases ($d_0 \neq 2$), all spin states $s = |j_1 - j_2|, \ldots, j_1 + j_2$ are presented. In the last one, the decomposition of the intertwining operator (2.4) over spin projection operators [12-14] is given in the Appendix.

From the locality requirement of action (2.1) (assumption (i)), we receive that $d_0 - 2 = c$ is an integer nonnegative number. Consider first, the case when $c = 0$, i.e., $d_0 = 2$, which is the case of canonical dimensions of fields $\psi(x;z)$. In this case, as was mentioned above, (2.4) is a projection operator on the subspace with spin $s = j_1 + j_2$, i.e., the maximal spin contained in the representation $\{j_1, j_2\}$. If $j_1(j_2) \neq 0$, then $F(p;z,w)$ has no an inverse operator, and consequently, the assumption (iii) is not satisfied. In the last case, the mentioned difficulties can be avoided introducing auxiliary fields in lower spin-tensor ranks and using the root method proposed in[7]. In the case when $j_1(j_2) = 0$ we deal with the representations considered in[15], which are nonconsistent because the corresponding equations admit nonphysical masses.

In the present paper we shall choose another way. If $c \geq 1$, then all spin states $s = |j_1 - j_2|; |j_1 - j_2| + 1, \ldots, j_1 + j_2$ are presented in $F(p;z,w)$ and consequently $F(p;z,w)$ is invertible. From the invertibility of the coefficients of the highest-order time derivatives there follows, consequently, the correctness of the Cauchy problem, too. Because of the fact that all s-in states are presented, there are no constraints. Moreover, when $d_0 \geq 2$ (2.4) is positive-definite[12], from which follow the positivity of energy and the right signs of the charges[16].

Varying the action (2.1) we derive the following equations for free massless fields:

$$(iz\partial\nabla_{\bar{w}})^{2(j_1-j_2)}\,[(z\partial\bar{z})(\nabla_w\partial\nabla_{\bar{w}})]^{2j_2}\times$$
$$\times P^{(c,j_1-j_2)}_{2j_2}\Big(1-2\frac{(z\epsilon\nabla_w)(\bar{z}\epsilon\nabla_{\bar{w}})}{(z\partial\bar{z})(\nabla_w\partial\nabla_{\bar{w}})}\Box\Big)\Box^c\,\psi(x;w) = 0\,, \tag{2.5}$$

where $\nabla_w = \frac{\partial}{\partial w}$. These equations, with respect to $\frac{\partial}{\partial x}$, are $2(j_1+j_2+c)$ order differential equations. As was pointed out, in the case when $c = 0$, there are constraints excluding all lower spin states.

For any c equation (2.5) can be transformed to a set of equations of first or second order using the identity

$$x^n P^{(\alpha,\beta)}_n\Big(1-\frac{y}{x}\Big) = \prod_{k=1}^{n}[(1-a_k)x-y] = \prod_{k=1}^{n} L_k\,, \tag{2.6}$$

where $a_k\,(k=1,\ldots,n)$ are the zeros of the Jacobi polynomials. Then introducing the fields

$$\psi_k = L_k \psi_{k-1}, \quad \psi_0 = \psi \quad (k=1,\dots,n) \tag{2.7}$$

the equation (2.5) can be written in the following equvalent form

$$(\tilde{L} - \tilde{\kappa})\tilde{\psi}(x) = 0 . \tag{2.8}$$

Here the labelling

$$\tilde{L} = \begin{bmatrix} L_1 & & & & \\ & L_2 & & 0 & \\ & & \ddots & & \\ & 0 & & L_n & \\ & & & & (iz\partial\nabla_{w})^{2(j_1-j_2)}\square^c \end{bmatrix}, \quad \tilde{\kappa} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & & 0 & 1 \\ 0 & 0 & 0 & & 0 & 0 \end{bmatrix}, \tilde{\psi} = \begin{bmatrix} \psi_0 \\ \psi_1 \\ \vdots \\ \psi_n \end{bmatrix} \tag{2.9}$$

are used, where $L_k (k=1,\dots,n)$ are second order differential operators defined by (2.6).

It can be pointed out that from definition (2.7), it follows that fields $\psi_k(x;z,w)(k=0,\dots,2j_2)$ are transformed according to the direct product of two IR of $SL(2,C)$ $\{j_1-k, j_2-k\} \otimes \{k,k\}$. The last representation is reducible if $k \neq 0$ or $k \neq 2j_2$. Here the numbers $j_1-k\,(j_2-k)$ and $k(k)$ give degree of homogeneity of $\psi_k(x;z,w)$ with respect to $z(\bar{z})$ and $w(\bar{w})$, respectively. We have equations with the lowest order, for which the assumptions (i)-(iv) are satisfied, if $c = 1$.

First we consider the case of tensor fields, i.e., $2j_1=2j_2=n$. Then matrices (2.9) have the form

$$\tilde{L} = \begin{bmatrix} L_1 & & & \\ & L_2 & 0 & \\ & & \ddots & \\ & 0 & & L_n \\ & & & & \square \end{bmatrix}, \qquad \tilde{\kappa} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ & & & & 0 & 1 \\ & & & & 0 & -m^2 \end{bmatrix} \tag{2.10}$$

where the nonvanishing mass term is introduced in $\tilde{\kappa}$, and L_k are second-order differential operators defined by (2.6).

Introducing matrix

$$\beta = \begin{bmatrix} 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & \dots & 1 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 1 & \dots & 0 & 0 \\ 1 & 0 & \dots & 0 & 0 \end{bmatrix} \tag{2.11}$$

and denoting that

$$\bar{\Phi} = \tilde{\Phi}^{+} (x; z)\ \beta, \tag{2.12}$$

where $\tilde{\Phi}^{+}$ is a Hermitean conjugate and, consequently, $\bar{\Phi}$ is an analog of the Dirac conjugation, then action (2.1) can be written in terms of the fields (2.7)

$$S_k = \frac{1}{(n!)^2} \int d^4x\, (\bar{\Phi}(x), (\tilde{L} - \tilde{\kappa}, \Phi(x))). \tag{2.13}$$

As an example we consider some particular cases of the tensor fields with the lowest tensor ranks.

a) For the scalar fields, i.e., $j_1 = j_2 = 0$, we get the Klein-Gordon equation from eq. (2.8).

$$(\Box + m^2)\ \phi(x) = 0. \tag{2.14}$$

b) For the four-vector field, i.e., $2j_1 = 2j_2 = n = 1$ we have

$$(4\partial_\mu \partial_\nu - 3g_{\mu\nu} \Box) A^\nu (x) = (\beta^{\lambda r})_{\mu\nu}\ \partial_\lambda \partial_r A^\nu = B_\mu (x),$$

$$(\Box + m^2) B_\mu(x) = 0. \tag{2.15}$$

c) For the second rank tensor fields, i.e., $2j_1 = 2j_2 = n = 2$, we have

$$L_1\Phi(x,\eta)=\frac{1}{2}[(1-a_1)(\xi\partial)(\nabla_\eta\partial)-\Box(\xi\nabla_\eta)]\Phi(x;\eta)=$$

$$=\frac{1}{2}(\beta^{(1)}_{\lambda\tau})_{\mu\nu}\,\partial^\lambda\partial^\tau\xi^\mu\nabla^\nu_\eta\Phi(x;\eta)=\Phi_1(x;\xi,\eta),$$

$$L_2\Phi_1(x;\xi,\eta)=[(1-a_2)(\xi\partial)-\Box(\xi\nabla_\eta)]\Phi_1(x;\xi,\eta)= \qquad (2.16)$$

$$=(\beta^{(2)}_{\lambda\tau})_{\mu\nu}\,\partial^\lambda\partial^\tau\xi^\mu\nabla^\nu_\eta\Phi_1(x;\xi,\eta)=\Phi_2(x;\xi),$$

$$(\Box+m^2)\Phi_2(x;\xi)=0,$$

where $\xi_\mu=z\sigma_\mu\bar{z}$, $\eta_\mu=w\sigma_\mu\bar{w}$ and $a_{1,2}=-1\pm\sqrt{21}/4$ are the zeros of Jacobi polynomial $P^{(1,0)}_2(x)$. It can be checked that $\beta^{1,2}_{00}$ are nondegenerate matrices, therefore the Cauchy problem for equations (2.16) is correct. It can be proved that whenever $a_k\neq 0, L_k$ are nonsingular matrices. As can be seen from (2.16), $\Phi_1(x;\xi,\eta)=\Phi^{\mu\nu}_1(x)\,\xi_\mu\eta_\nu$ is the general second rank tensor, i.e., $\Phi^1_{\mu\nu}\neq\Phi$ and $g^{\mu\nu}\Phi_{1\,\mu\nu}\neq 0$, and $\Phi_2(x;\xi)=\Phi^{\mu\nu}_2(k)\,\xi_\mu\xi_\nu$ where $\Phi^{\mu\nu}_2$ is a symmetric traceless tensor.

For fields with half-integer spin, the invariance under the space-reflection requires the fields $\psi(x;z)$ to be transformed with respect to the $SL(2,C)$ group by the representations $\{j_1,j_2\}\oplus\{j_2,j_1\}$. Here we restrict our considerations to the $j_1=j_2+½=j$ case only. Then using the ordinary representation of bispinor field

$$\psi(x;z)=\begin{pmatrix}\phi(x;z)\\ \chi(x;z)\end{pmatrix}, \qquad (2.17)$$

where the fields $\phi(x;z)$ and $\chi(x;z)$ are transformed according to the IR $\{j,j-½\}$ and $\{j-½,j\}$ respectively. Then the equality

$$\Box=\underset{\sim}{\sigma}_\mu\tilde{\sigma}_\nu\partial^\mu\partial^\nu,$$

where $\sigma=(1,\underset{\sim}{\sigma})$ and $\tilde{\sigma}=(1,-\underset{\sim}{\sigma})$ allow us to introduce new fields in (2.1)

$$\psi' = \begin{bmatrix} \frac{1}{(2j)!} z\underset{\sim}{\partial}\epsilon \nabla_z \phi \\ \\ \frac{1}{(2j-1)!} \bar{z}\epsilon \underset{\sim}{\partial}\nabla_{\tilde{z}} \chi \end{bmatrix} \tag{2.18}$$

With respect to the field ψ', we have the following equations of motion

$$(\tilde{\mathfrak{L}} - \tilde{\kappa})\tilde{\psi}(x) = 0, \tag{2.19}$$

where

$$\tilde{\mathfrak{L}} = \begin{bmatrix} L_1 & & & & \\ & L_2 & & 0 & \\ & & \ddots & & \\ & & & L_{2j_2-1} & \\ 0 & & & & \\ & & & & i(z\underset{\sim}{\partial}\nabla_{\bar{w}} + \nabla_w \partial \bar{z}) \end{bmatrix} \qquad \tilde{\kappa} = \begin{bmatrix} 0 & 1 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & \dots & 0 & -m(z) \end{bmatrix}$$

$$\tilde{\psi} = \begin{bmatrix} \psi' \\ \psi_1 \\ \vdots \\ \psi_{2j-1} \end{bmatrix} \tag{2.20}$$

where $m(z, w) = m(z\epsilon\nabla_w + \bar{z}\epsilon\nabla_{\bar{w}})$ and operators L (k=1,...) are given by the formula (2.6).

For the Dirac fields, i.e., $j = 1/2$ from (2.19) we have the Dirac equation

$$(i\not{\partial} - m)\psi(x) = 0. \tag{2.21}$$

For the fields with spin 3/2, i.e., $j = 1$, we have

$$L_1\psi'(x;w) = [2(z\,\partial\,\bar{z})(\nabla_w\,\partial\,\nabla_{\bar{w}}) - 3(z\epsilon\nabla_w)(\bar{z}\epsilon\nabla_{\bar{w}})\,\Box\,]\psi(x;w) = \psi_1(x;z,w),$$

$$[i(z\partial\nabla_{\bar{w}} + \nabla_w\partial\bar{z}) - m(z\epsilon\nabla_w + \bar{z}\epsilon\nabla_{\bar{w}})]\psi_1(x,z,w) = 0. \qquad (2.22)$$

Similar to the case of tensor fields, action (2.1) can be written in terms of the fields ψ_k (2.7) or (2.20). Introducing the "Dirac" conjugated spinor

$$\bar{\psi} = \psi^+\beta', \qquad (2.23)$$

where

$$\beta' = \begin{bmatrix} 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & \dots & 1 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \beta_0 & 0 & & 0 & 0 \end{bmatrix}$$

and β_0 is 2x2 matrix $\beta_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, (2.1) has the form

$$S = \int d^4x\,(\bar{\tilde{\psi}}, (\tilde{L} - \tilde{\kappa}, \tilde{\psi}))(x). \qquad (2.24)$$

Equations (2.8) and (2.19) can formally be derived from (2.13) and (2.24). However, in this case, because of (2.7) the components of $\tilde{\psi}$ are not independent.

Equations (2.8), (2.15), (2.16), (2.19) and (2.22) describe only massless particles in the case when $m = 0$ and massive and massless particles, when $m \neq 0$.

3. GREEN FUNCTIONS FOR FIELDS WITH ARBITRARY SPIN

From eq. (2.3) it follows that $F_{\chi_1\chi_2}$ is the Green function for eq. (2.5). In tensor case, the causal Green function is given by

$$D^{(n)}(p;\xi,\eta) = \frac{(p^2-m^2-i\epsilon)^{-1}}{(p^2-i\epsilon)^n}\left(\frac{p\xi p\eta}{p^2}\right)^n P_n^{(-1-2n,0)}\left(1-\frac{p^2\xi\eta}{p\xi p\eta}\right) =$$

$$= \frac{(p^2-m^2-i\epsilon)^{-1}}{(p^2-i\epsilon)^n}\sum\frac{(s-n)_{n-s}}{(n+s+2)_{n-s}}\Pi_s^{(n)}(p;\xi,\eta), \tag{3.1}$$

where $\Pi_s^{(n)}$ are the projection operators on the subspace with spin s. The transition to the ordinary used tensor indices can be done by means of n-fold differentiation of eq. (3.1) with respect to ξ and η with the subsequent symmetrization and subtraction of the traces. The other Green functions can be found from eq. (3.1) with the corresponding going round of the poles.

In the case of scalar field, i.e., $n = 0$ we receive from eq. (3.1) the ordinary propagator for the scalar fields. For the four-vector ((n = 1) field we get from eq. (3.1)

$$D^{(1)}(p;\xi,\eta) = D_{\mu\nu}^{(1)}(p)\xi^\mu\eta^\nu = \frac{(p\xi)(p\eta)}{p^2(p^2-m^2-i\epsilon)}P_1^{(-3,0)}\left(1-\frac{p^2\xi\eta}{p\xi p\eta}\right) =$$

$$= \left(\xi\eta - 4\frac{(p\xi)(p\eta)}{p^2}\right)\frac{(p^2-m^2-i\epsilon)^{-1}}{p^2-i\epsilon} = \tag{3.2}$$

$$= \frac{\Pi_1^{(1)} - 3\Pi_0^{(1)}}{(p^2-i\epsilon)(p^2-m^2-i\epsilon)} = \frac{(g_{\mu\nu}-p_\mu p_\nu/p^2) - 3p_\mu p_\nu/p^2}{(p^2-m^2-i\epsilon)(p^2-i\epsilon)}\xi^\mu\eta^\nu,$$

where $\Pi_0^{(2)}$ and $\Pi_1^{(1)}$ are the projection operators on the states with spin 0 and 1, respectively.

For the symmetric tracelles tensor fields of second rank, i.e., $n = 2$, we get from eq. (3.1)

$$D^{(2)}(p;\xi,\eta) = D^{(2)}_{\mu_1,\mu_2;\nu_1\nu_2}(p)\,\xi^{\mu_1}\xi^{\mu_2}\eta^{\mu_1}\eta^{\mu_2} =$$

$$= \frac{(p^2-m^2-i\epsilon)^{-1}}{(p^2-i\epsilon)^2}\left(\frac{p\xi p\eta}{p^2}\right)^2 P_2^{(-5,0)}\left(1-\frac{p^2\xi\eta}{p\xi p\eta}\right) =$$

$$= \frac{(p^2-m^2-i\epsilon)^{-1}}{(p^2-i\epsilon)^2}\left(6\Pi_0^{(2)} - 5\Pi_1^{(2)} + \Pi_2^{(2)}\right). \tag{3.3}$$

Here $\Pi_0^{(2)}$, $\Pi_1^{(2)}$ and $\Pi_2^{(2)}$ are spin projection operators on the subspaces with spin $s = 0,1$ and 2, respectively.

For the case of spinor fields the Green function of eq. (2.19) is given by

$$S(p;z,w) = \frac{z\underset{\sim}{p}\bar{w} + w\underset{\sim}{p}\bar{z} + m(z\epsilon w + \bar{z}\epsilon\bar{w})}{(p^2 - m^2 - i\epsilon)(p^2 - i\epsilon)^{2j-1}} \times$$

$$\times \left(\frac{(z\underset{\sim}{p}\bar{z})(w\underset{\sim}{p}\bar{w})}{p^2}\right)^{2j-1} P_{2j-1}^{(-2j,1)}\left(1 - 2\frac{p^2 z\epsilon w\bar{z}\epsilon\bar{w}}{z\underset{\sim}{p}\bar{z}w\underset{\sim}{p}\bar{w}}\right) = \tag{3.4}$$

$$= \frac{zp\bar{w} + wp\bar{z} + m(z,w)}{(p^2 - i\epsilon)^{2j-1}(p^2 - m^2 - i\epsilon)} \sum_{s=1/2}^{2j-1} \frac{(s-2j+1)_{2j-s-1}}{(s+2j+3)_{2j-s-1}} \Pi_s^{(j)}(p;z,w),$$

where Π_s are the spin projection operators. The transition to the spinor indices can be done by differentiation with respect to $z, \bar{z}$ and $w, \bar{w}$ with subsequent symmetrization over any group of indices.

The explicit form of propagators (3,1)-(3,4) shows that the here considered theory is ultraviolet renormalizable, in contrast with the ordinary massive theories for the higher spin fields based on the first or second order equations. However, here the metric in space states is indefinite.

Acknowledgements are due to Prof. I.T.Todorov, Dr. D.Tz.Stoyanov and Dr. L.B.Nikolova for valuable discussions and remarks.

APPENDIX

To make reading of the paper easier, some results of refs.[12-14] about decomposition of interwining operator (2.4) and Green function (3.1) in terms of spin projection operators are given. In the tensor case [12,13] we have

$$F(p;\xi,\eta) = (p^2)^c \left(\frac{p\xi p\eta}{p^2}\right)^n P_n^{(c,0)}\left(1 - p^2\frac{\xi\eta}{p\xi p\eta}\right) =$$

$$= 2^{-c}(p^2)^c \sum_{s=0}^{n} \frac{(s-c-n+1)_{n-s}}{(c+s+n+1)_{n-s}} \Pi_s^{(n)}(p;\xi,\eta), \tag{A.1}$$

where

$$(a)_k = a(a+1) \ldots (a+k-1) = \frac{\Gamma(a+k)}{\Gamma(a)}$$

and $\Pi_s^{(n)}(p;\xi,\eta)$ are the projection operators on the subspace with spin $s\,(s=0,1,\ldots,n)$. . These operators satisfy the following conditions

$$\Pi_s^{(n)}\Pi_{s'}^{(n)} = \delta_{ss'}\,\Pi_s^{(n)} ,$$

$$\sum_{s=0}^{n} \Pi_s^{(n)} = 1 = (\xi\eta)^n , \qquad (A.2)$$

when $(\xi\eta)^n$ is the identity operator in the space of homogeneous polynomials with degree n, i.e., symmetric traceless tensor of rank n. Then it can be checked out that

$$\Pi_s^{(n)} = (-1)^s \frac{(2s+1)\,n!\,(n-1)!}{(n-s)!\,(n+s)!}\left(\frac{2p\xi p\eta}{p^2}\right)^n P_s\left(1 - \frac{p^2\xi\eta}{p\xi p\eta}\right) \qquad (A.3)$$

are eigenfunctions of the spin operator squared with eigenvalue s /11/, and satisfy the normalization conditions (A.2).

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Received by Publishing Department
on March 31 1980.