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RELATIVISTIC STRING MODEL IN A SPACE-TIME OF A CONSTANT CURVATURE

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1. INTRODUCTION

Relativistic string model has rather a long history (see, e.g., refs. [1-3]). In the elementary particle physics the relativistic string was introduced as the dynamical basis of the dual resonance models. In recent years the ideas of the string model are used by the investigation of the mechanism of the quark confinement in hadrons [4,5] and by the representation of the Yang-Mills field theory in terms of the functionals defined on contours [6-9].

In all papers devoted to the relativistic string model the flat space-time was considered. We shall investigate this model in a space-time of a constant curvature (de Sitter universe). If we take the viewpoint that the gravitation may play an important role in the world of the elementary particles (see, e.g., ref. [10]), then the aim of this paper will not be perceived as the abstract pure mathematical problem.

We shall use the differential geometry methods when the world surface of the string is described by the differential quadratic forms rather than by the string coordinates [11-15]. In this approach the string dynamics is defined by a system of two nonlinear equations. The differential geometry technique enables us to construct the Lax representation for this system (more precisely, the so-called "Zero-curvature" equation [16]). For the three-dimensional de Sitter space-time the system under consideration is reduced to one nonlinear sinh-Gordon or cosh-Gordon equation.

2. MINIMAL WORLD SURFACES IN DE SITTER SPACE-TIME

The Nambu-Goto action of the relativistic string [1-3] can be easily generalized to the curved space-time [17]

$$S=-\kappa\iint d^2u\sqrt{-\det\left\|\frac{\partial x^\mu}{\partial u^i}\cdot\frac{\partial x^\nu}{\partial u^j}g_{\mu\nu}(x)\right\|}=-\kappa\iint d^2u\sqrt{-\det\|a_{ij}\|}, \qquad (1)$$

where $x^\mu(u^1=\tau,u^2=\sigma)$ is the parametric representation of the string world surface, $g_{\mu\nu}(x)$ is the metric tensor of the curved space-time, $a_{ij}=x^\mu_{,i}\,x^\nu_{,j}\,g_{\mu\nu}$ is the metric tensor on the string

world surface, $x^{\mu}_{,i} = \partial x^{\mu} / \partial u^{i}$, and κ is a constant with the dimension of the inverse squared length. The principle of least action, as applied to the functional (1), leads to the problem of determining two-dimensional minimal surface in the four-dimensional Riemannian space with metric $g_{\mu\nu}(x)$. The Euler equation has the form /18/

$$\sqrt{-a} g_{\mu\nu} (\Box x^{\nu} + \Gamma^{\nu}_{\alpha\beta}(g)\, a^{ij}\, x^{\alpha}_{,i}\, x^{\beta}_{,j}) = 0, \tag{2}$$

where $a = \det \|a_{ij}\|$, $\Box x^{\nu} = \frac{1}{\sqrt{-a}} \frac{\partial}{\partial u^{i}} (\sqrt{-a}\, a^{ik} x^{\nu}_{,k})$ is the covariant Laplace-Beltrami operator with respect to the inner metric on the string world surface a_{ij} ; $\Gamma^{\nu}_{\alpha\beta}(g)$ are the Christoffel symbols for the metric tensor $g_{\mu\nu}(x)$. If the space-time is flat, $\Gamma^{\nu}_{\alpha\beta}(g) = 0$, and we obtain the well-known result /19/ in the theory of the minimal varieties in Euclidean space $\Box x^{\nu} = 0$.

Further we suppose that the space-time has a constant curvature (de Sitter universe). In this case the metric tensor $g_{\mu\nu}(x)$ can be taken, for example, in the Riemannian form /18/

$$g_{\mu\nu}(x) = \frac{\eta_{\mu\nu}}{(1 + \frac{K}{4} x^{2})^{2}} , \tag{3}$$

where $\eta_{\mu\nu} = \mathrm{diag}\ (1,-1,-1,-1)$, $x^{2} = x^{\mu} x^{\nu} \eta_{\mu\nu}$. The constant K enters into the definition of the Riemann-Christoffel curvature tensor of the de Sitter space-time

$$R_{\mu\nu\lambda\rho} = K(g_{\mu\lambda} g_{\nu\rho} - g_{\mu\rho} g_{\nu\lambda}) . \tag{4}$$

In terms of the Weierstrass coordinates z^{μ}, $\mu = 1,2,\ldots,5$ de Sitter universe can be represented as the hyperboloid in the five-dimensional pseudo-Euclidean space /20/

$$(z^{1})^{2} - (z^{2})^{2} - (z^{3})^{2} - (z^{4})^{2} + \epsilon (z^{5})^{2} = \epsilon R^{2} , \tag{5}$$

where $\epsilon = +1$ for the de Sitter space-time of the first kind and $\epsilon = -1$ for the de Sitter space-time of the second kind. The constant K is connected with R by the formula

$$K = \frac{\epsilon}{R^{2}} . \tag{6}$$

Instead of solving equation (2) for the string coordinates $x^{\mu}(u^1,u^2)$ with a given space-time metric $g_{\mu\nu}(x)$ (see Eqs. (3), (4)) we shall use the differential geometry technique /18/ by which the string world surface can be described by its fundamental quadratic forms $a_{ij}(u^1,u^2)$, $b_{\alpha|ij}(u^1,u^2)$, and torsion vectors $\nu_{\alpha\beta|i}$ $(=-\nu_{\beta\alpha|i})$, $i,j=1,2$; $\alpha,\beta=3,4$. The theorem on the embedding of the Riemannian manifolds tells that these quantities will define a two-dimensional surface embedded in space-time of a constant curvature up to its motion as a whole if and only if the equations of Gauss

$$R_{ijk\ell} = \sum_{\sigma=3}^{4} e_{\sigma}(b_{\sigma|ik}\, b_{\sigma|j\ell} - b_{\sigma|i\ell}\, b_{\sigma|jk}) + \\ + K(a_{ik}\, a_{j\ell} - a_{i\ell}\, a_{jk}), \tag{7}$$

Codazzi

$$b_{\alpha|ij;k} - b_{\alpha|ik;j} = \sum_{\beta=3}^{4} e_{\beta}(\nu_{\beta\alpha|k} b_{\beta|ij} - \nu_{\beta\alpha|j}\, b_{\beta|ik}), \tag{8}$$

and Ricci

$$\nu_{\beta\alpha|j;k} - \nu_{\beta\alpha|k;j} + \sum_{\gamma=3}^{4} e_{\gamma}(\nu_{\gamma\beta|j}\, \nu_{\gamma\alpha|k} - \nu_{\gamma\beta|k}\, \nu_{\gamma\alpha|j}) + \\ + g^{\ell m}(b_{\beta|\ell j}\, b_{\alpha|mk} - b_{\beta|\ell k}\, b_{\alpha|mj}) = 0, \quad e_{\alpha} = \pm 1 \tag{9}$$

will be satisfied. In these equations the Latin indices take values 1,2 and the Greek indices 3,4. The left-hand side of equation (7), $R_{ijk\ell}$, is the Riemann-Christoffel curvature tensor defined by the metric tensor of the world surface of the string a_{ij}. In the case under consideration this tensor has only one essential component

$$R_{1212} = \frac{1}{g}(2a_{12,12} - a_{11,22} - a_{22,11}) + \\ + a^{\ell m}(\Gamma_{m,21}\Gamma_{\ell,12} - \Gamma_{m,22}\Gamma_{\ell,11}),$$

where $\Gamma_{\ell,\,ij}$ are the Christoffel symbols corresponding to the metric tensor a_{ij}

$$\Gamma_{i,jk}=\frac{1}{2}(a_{jk,j}+a_{ij,k}-a_{jk,i}).$$

The semicolon in Eqs.(7)-(9) means the covariant differentiation with respect to the metric tensor a_{ij}.

The metric of space-time $g_{\mu\nu}(x)$ does not enter into Eqs. (7)-(9) explicitly. The only consequence of the constant curvature of space-time is the second term in the right-hand side of the Gauss equation (7).

From the equation of motion (2) it follows that the world sheet of the string has to be a minimal surface [18]

$$a^{ij}\,b_{\alpha|ij}=0,\quad \alpha=3,4. \tag{10}$$

On the string world surface the isometric coordinates u^1, u^2 can be chosen

$$a_{11}=-a_{22}=a,\quad a_{12}=0. \tag{11}$$

In this coordinate system conditions (10) take the form

$$b_{\alpha|11}=b_{\alpha|22},\quad \alpha=3,4. \tag{12}$$

Eliminating from the Codazzi equation (8) the torsion vector $\nu_i=\{\nu_1=\nu_{43|1},\nu_2=\nu_{43|2}\}$ and taking into account (12) we obtain

$$\frac{\partial}{\partial u^{\pm}}\sum_{\alpha=3}^{4}(b_{\alpha|11}\pm b_{\alpha|12})^2=0,$$

with $u^{\pm}=u^1\pm u^2$. Therefore

$$\sum_{\alpha=3}^{4}(b_{\alpha|11}\pm b_{\alpha|12})^2=q_{\pm}(u^{\pm}),$$

where $q_{\pm}$ are two arbitrary functions. It is convenient to introduce the following variables [21]

$$b_{3|11} \pm b_{3|12} = q_{\pm}(u^{\pm}) \cos\alpha_{\pm} ,$$

$$b_{4|11} \pm b_{4|12} = q_{\pm}(u^{\pm}) \sin\alpha_{\pm} ,$$

$$\alpha_{+} - \alpha_{-} = \theta, \quad a = e^{\phi} .$$

In terms of the new variables the Gauss equation (7) and the Ricci equation (9) part from the system (7)-(9)

$$\begin{aligned} \phi_{,11} - \phi_{,22} &= 2q_{+}(u^{+}) q_{-}(u^{-}) e^{\phi} \cos\theta + 2Ke^{-\phi} , \\ \theta_{,11} - \theta_{,22} &= 2q_{+}(u^{+}) q_{-}(u^{-}) e^{\phi} \sin\theta , \end{aligned} \tag{13}$$

but the Codazzi equations (8) take the form

$$\nu_1 + \nu_2 = a_{-,1} + a_{-,2} , \quad \nu_1 - \nu_2 = a_{+,1} - a_{+,2} . \tag{14}$$

The arbitrary functions $q_{\pm}(u^{\pm})$ can be eliminated from Eq. (13) by the following change of variables

$$\tilde{u}^{\pm} = \int du^{\pm} \sqrt{2q_{\pm}(u^{\pm})} , \quad \sqrt{q_{+}(u^{+}) q_{-}(u^{-})}\, e^{\phi(u^1,u^2)} = e^{\tilde{\phi}(\tilde{u}^1,\tilde{u}^2)} ,$$

$$\theta(u^1,u^2) = \tilde{\theta}(\tilde{u}^1,\tilde{u}^2) .$$

Now Eqs. (13) become

$$\begin{aligned} \phi_{,11} - \phi_{,22} &= e^{\phi}\cos\theta + K e^{-\phi} , \\ \theta_{,11} - \theta_{,22} &= e^{\phi}\sin\theta , \end{aligned} \tag{15}$$

in which the mark $\sim$ is omitted for the notation simplicity. Thus the equations of motion which completely determine, in the given approach, the relativistic string dynamics in de Sitter space-time are Eqs. (15). These equations have to be complemented with the boundary conditions if the relativistic string is of finite extension. For example, for a closed string, $0 \le u^2 \le \pi$, we have

$$\phi(u^1, u^2=0) = \phi(u^1, u^2=\pi), \qquad \theta(u^1, u^2=0) = \theta(u^1, u^2=\pi).$$

When the curvature K tends to zero then the system (15) transforms into one nonlinear Liouville equation for the complex valued function $w = \phi + i\theta$ /21/

$$w_{,11} - w_{,22} = e^{w} . \tag{16}$$

The change of variables

$$\tilde{u}^{\pm} = |K|^{1/4} u^{\pm} , \quad e^{\phi(u^1,u^2)} = |K|^{1/2} e^{\tilde{\phi}(\tilde{u}^1,\tilde{u}^2)} ,$$

$$\theta(u^1,u^2) = \tilde{\theta}(\tilde{u}^1,\tilde{u}^2) , \quad K \neq 0$$

enables us to remove the constant K from Eqs.(15)

$$\begin{aligned} \phi_{,11} - \phi_{,22} &= e^{\phi}\cos\theta + \epsilon e^{-\phi}, \\ \theta_{,11} - \theta_{,22} &= e^{\phi}\sin\theta, \quad \epsilon = \pm 1. \end{aligned} \tag{17}$$

If we put in (17) $\theta = 0$ we obtain the equation

$$\phi_{,11} - \phi_{,22} = \begin{cases} 2\sinh\phi, & K < 0, \\ 2\cosh\phi, & K > 0, \end{cases} \tag{18}$$

which describes the metric of the minimal surface in the three-dimensional de Sitter space-time. Equation (18) is a generalization to the nondefinite metric of the known result in differential geometry concerning the embedding of the minimal surfaces into the three-dimensional space of a constant curvature /22/.

3. LAX REPRESENTATION FOR SYSTEM (17)

In differential geometry /18/ the Gauss-Codazzi-Ricci equations (7)-(9) are derived as the compatibility conditions of two systems of partial differential equations of the first order which describe the moving frame on the string world surface. Therefore, these equations can be taken as a pair of the Lax operators required for solving the nonlinear equations (17) by the inverse scattering method /23,24/ . In the de Sitter

space-time we introduce the Weierstrass coordinates (5) and as the moving frame we take two tangent vectors $z^{\mu}_{,1}$, $z^{\mu}_{,2}$, two unit normals $\eta^{\mu}_3, \eta^{\mu}_4$, and vector z^{μ}. In the theory of relativistic string /1-3/ vector $z^{\mu}_{,1}$ has to be time-like; and vectors $z^{\mu}_{,2}$, η^{μ}_3, η^{μ}_4, space-like. With this fact and the choice of signs in the quadratic form (5), we obtain the following equations /18/

$$z^{\mu}_{;ij} = -\sum_{\sigma=3}^{4} b_{\sigma|ij}\,\eta^{\mu}_{\sigma} - K a_{ij}\, z^{\mu} ,$$

$$\eta^{\mu}_{\sigma,i} = -b_{\sigma|ij}\, a^{jm}\, z^{\mu}_{,m} - \sum_{\tau} \nu_{\tau\sigma|i}\, \eta^{\mu}_{\tau} , \tag{19}$$

which describe the motion of the basis

$$z^{\mu}_{,1}, z^{\mu}_{,2}, \eta^{\mu}_3, \eta^{\mu}_4, z^{\mu} \tag{20}$$

on the string world surface. The contraction with a^{ij} of the first equation in (19), by virtue of (10), gives the known result /25/ in the theory of minimal varieties embedded in a space of constant curvature, namely, the action of the Laplace-Beltrami operator on vector z^{μ} is

$$\Box z^{\mu} = -2K z^{\mu} .$$

Instead of vectors (20) we introduce the orthonormal moving basis

$$e^{\mu}_1 = \frac{z^{\mu}_{,1}}{\sqrt{a}}, \; e^{\mu}_2 = i\frac{z^{\mu}_{,2}}{\sqrt{a}}, \; e^{\mu}_3 = i\eta^{\mu}_3, \; e^{\mu}_4 = i\eta^{\mu}_4, \; e^{\mu}_5 = \sqrt{\epsilon}\,\frac{z^{\mu}}{R}. \tag{21}$$

Using (19) we obtain the following equations describing the motion of the basis (21) on the string world surface

$$\frac{\partial e_a}{\partial u^j} = \sum_{b=1}^{5} \omega^{j}_{ab}\, e_b , \quad j = 1,2, \quad a,b = 1,2,\ldots,5, \tag{22}$$

where

where

$$\omega^{1}_{ab} = \begin{vmatrix} 0 & -i\frac{a_{,2}}{2a} & i\frac{b_{3|11}}{\sqrt{a}} & i\frac{b_{4|11}}{\sqrt{a}} & -\frac{\sqrt{\epsilon a}}{R} \\ i\frac{a_{,2}}{2a} & 0 & -\frac{b_{3|12}}{\sqrt{a}} & -\frac{b_{4|12}}{\sqrt{a}} & 0 \\ i\frac{b_{3|11}}{\sqrt{a}} & \frac{b_{3|12}}{\sqrt{a}} & 0 & -\nu_1 & 0 \\ -i\frac{b_{4|11}}{\sqrt{a}} & \frac{b_{4|12}}{\sqrt{a}} & \nu_1 & 0 & 0 \\ \frac{\sqrt{\epsilon a}}{R} & 0 & 0 & 0 & 0 \end{vmatrix} \qquad (23)$$

$$\omega^{2}_{ab} = \begin{vmatrix} 0 & -i\frac{a_{,1}}{2a} & i\frac{b_{3|12}}{\sqrt{a}} & \frac{b_{4|12}}{\sqrt{a}} & 0 \\ i\frac{a_{,1}}{2a} & 0 & -\frac{b_{3|22}}{\sqrt{a}} & -\frac{b_{4|22}}{\sqrt{a}} & i\frac{\sqrt{\epsilon a}}{R} \\ -i\frac{b_{3|12}}{\sqrt{a}} & \frac{b_{3|22}}{\sqrt{a}} & 0 & -\nu_2 & 0 \\ -i\frac{b_{4|12}}{\sqrt{a}} & \frac{b_{4|22}}{\sqrt{a}} & \nu_2 & 0 & 0 \\ 0 & -i\frac{\sqrt{\epsilon a}}{R} & 0 & 0 & 0 \end{vmatrix} \qquad (24)$$

We can reduce the dimension of the matrix equations (22) in the following way. The skewsymmetric matrices (23) and (24) describe infinitesimal rotations in the five-dimensional pseudo-Euclidean space. Some matrix elements in (23) and (24) are imaginary because the quadratic form (5) is indefinite. The matrix representation of the minimal dimension of the O(5)-group can be constructed in terms of the usual (4x4) Dirac γ-matrices [26]. In this representation the O(5)-generators are

$$I_{\mu\nu} = \frac{1}{2}\gamma_\mu\gamma_\nu , \qquad \mu \neq \nu ,$$

$$\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\delta_{\mu\nu}, \quad \gamma_\mu^+ = \gamma_\mu , \quad \mu,\nu = 1,2,...,5.$$

In what follows we shall use the representation of the γ-matrices in which $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$ is diagonal [27]

$$\gamma_k = \begin{pmatrix} 0 & i\sigma_k \\ -i\sigma_k & 0 \end{pmatrix}, \quad k = 1,2,3, \quad \gamma_4 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}, \qquad (25)$$

where σ_i are the Pauli matrices.

Now we can correlate to Eqs.(22) two systems of partial differential equations each containing only four equations

$$\frac{\partial\psi_\alpha}{\partial u^j} = \frac{1}{2}\sum_{\mu>\nu=1}^{5} \omega_{\mu\nu}^j (\gamma_\mu\gamma_\nu)_{\alpha\beta}\psi_\beta = \frac{1}{2}\Omega^j_{\alpha\beta}\psi_\beta , \quad j = 1,2, \qquad (26)$$

$$\alpha,\beta = 1,2,3,4 ,$$

where $\psi_\alpha = \psi_\alpha(u^1, u^2)$ is a four-component function and 4x4 matrices Ω^j are

$$\Omega^j = \sum_{\mu>\nu=1}^{5} \omega_{\mu\nu}^j \gamma_\mu\gamma_\nu , \qquad j = 1,2 . \qquad (27)$$

To write out these matrices explicitly we introduce the notation

$$\Omega^j = \begin{vmatrix} a_{11}^j & a_{12}^j \\ a_{22}^j & a_{22}^j \end{vmatrix} ,$$

where $a_{k\ell}^j$ are again the (2x2) matrices. Using Eqs.(23), (24), (25) and (27) we obtain the following expansions of $a_{k\ell}^j$, $j,k,\ell = 1,2$ in terms of the Pauli matrices σ_k

$$
\begin{aligned}
a^1_{11} &= -i\sigma_1 (Q_+^* - Q_-) + \sigma_2 (Q_+^* + Q_-) - \frac{1}{4}\sigma_3 (\phi_{,2} + 2i\nu_1), \\
a^1_{12} &= a^1_{21} = -i\sigma_1 \frac{\sqrt{\epsilon}}{2R} e^{-\phi/2}, \\
a^1_{22} &= -i\sigma_1 (Q_+ - Q_-^*) + \sigma_2 (Q_+ + Q_-^*) - \frac{1}{4}\sigma_3 (\phi_{,2} - 2i\nu_1), \\
a^2_{11} &= -i\sigma_1 (Q_+^* + Q_-) + \sigma_2 (Q_+^* - Q_-) - \frac{1}{4}\sigma_3 (\phi_{,1} + 2i\nu_2), \\
a^2_{12} &= a^2_{21} = -\sigma_2 \frac{\sqrt{\epsilon}}{2R} e^{-\phi/2}, \\
a^2_{22} &= -i\sigma_1 (Q_+ + Q_-^*) + \sigma_2 (Q_+ - Q_-^*) - \frac{1}{4}\sigma_3 (\phi_{,1} - 2i\nu_2),
\end{aligned}
\tag{28}
$$

in which

$$
Q_\pm = \frac{1}{4}\exp\left(\frac{\phi}{2} \pm i\alpha_\pm\right), \quad \theta = \alpha_+ - \alpha_-, \quad \epsilon = \pm 1,
$$

and ν_i, $i = 1,2$ is connected with $a_\pm$ by formulae (14).

The compatibility conditions of the linear equations (26)

$$
\Omega^1_{,2} - \Omega^2_{,1} = [\Omega^2, \Omega^1]
$$

result in Eqs. (17).

The transition to new variables $\bar{u}^1, \bar{u}^2$

$$
\bar{u}^1 + \bar{u}^2 = \lambda(u^1 + u^2), \quad u^1 - u^2 = \lambda^{-1}(u^1 - u^2)
$$

enables us to introduce the spectral parameter λ into Eq. (26). This change of variables retains the form of system (17), while the matrix elements $\Omega_{\alpha\beta}$ (28) obtain factors $(\lambda \pm 1/\lambda)$ (see, e.g., /28,29/). We shall not perform here these simple transformations.

4. CONCLUSION

The proposed model of the relativistic string in a space-time of a constant curvature gives us one more example of a system of two non-linear equations integrable by the inverse scattering technique. This system was derived in paper /30/ by investigating the world surfaces with a constant mean curvature in the four-dimensional Minkowski space-time. In the same paper the Lax representation for this system was obtained in terms of 2x2 matrices. Therefore, if would be interesting to reduce the dimension of matrices $\Omega^{j}_{\alpha\beta}$ in the linear spectral problem (26).

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