сообщения
объединенного
института
ядерных
исследований
дубна

E2-80-219

V.B.Belyaev, S.A.Rakityansky, J.Wrecionko*

THE ELASTIC SCATTERING AND CHARGE-EXCHANGE OF PIONS BY ^{3}He AT THE INTERMEDIATE ENERGIES

* On leave from Institute for Nuclear Research, Warsaw.

1980

INTRODUCTION

An adequate treatment of experimental data on the pion interaction with lightest nuclei requires to solve the corresponding multiparticle equations in the potential theory. It is, obviously, a necessary step after which the problem of meson degrees of freedom can be analysed for the considered process. The system $\pi 2N$ was carefully studied by using the Faddeev equations, e.g., in ref.[1] whereas for the system $\pi 3N$ exact four-body equations were not considered. Several approximations were proposed: the optical model with a first-order potential[2], the Glauber model[3], the fixed-scatterer model[4]. The problem appears to what extent each of them corresponds to the exact 4-particle equations.

In this paper we consider the correspondence of the fixed-scatterer model and its modifications to the exact four-particle equations. In paper[5] approximate four-body equations were proposed. The approximation implied a finite-rank-approximation of the target Hamiltonian. This approximation, an extension out of the framework of the fixed scatterer model, is well-founded mathematically at pion energies not exceeding the nearest threshold of breaking of the 3-nucleon system. On the other hand, at pion energies much larger than the mean kinetic energy of a nucleon in nucleus the use of the fixed-nucleon approximation may turn out to be justified. In this connection it is interesting to study the range of applicability of the model[5] at intermediate energies. Here two specific features of the pion-nucleon interaction, important in analysing the model applicability, are to be noted:

1) At sufficiently low energies the πN-interaction is small (i.e., scattering phases are small) in all partial waves.

2) At energies $E_{\pi} \sim 200$ MeV the P^{33}-wave has a resonance.

Due to the first feature the integral term in the equation (4) for the low-energy $\pi\,{}^3He$ elastic scattering is small. At energies much higher than the nucleus binding energy the integral term is small in virtue of its own structure. Thus, at all the considered pion energies the equation for amplitude can be restricted by the inhomogeneous term, i.e., by the amplitude of scattering on fixed centers. This description implies that in the intermediate states the nucleus propagates as a unique object. By this reason the resonance behaviour in the system $\pi 3N$ disappears and the elastic-scattering cross section sharply decreases as compared to that on the free nucleon. Since the equation for the scattering amplitude has been derived by approximating the nuclear Hamiltonian by the first-rank operator (in this case the contribution only from the bound state was taken into account in the spectral expansion of the target Hamiltonian), it is clear that an increase of the cross section requires higher terms in the spectral decomposition of nucleon Hamiltonian, i.e., the contribution from the continuous spectra of the three nucleon systems must be taken into account.

In our paper this difficulty is removed by introducing the elements of the impulse approximation; thus the agreement with experiment is achieved without using additional phenomenological parameters.

The paper is organized as follows:

Section 2 gives a short exposition of the formalism; in Sec. 3 we discuss the difficulties of the fixed scatterer model applied to the π-nucleus scattering and present the results of calculations of differential cross sections within the modified fixed-scatterer model.

2.

Let us represent the total Hamiltonian of the system in the form

$$H = h_0 + V + H_c , \tag{1}$$

where h_0 is the kinetic-energy operator of the relative motion of the "pion-nucleus", $V = \sum_{i=1}^{3} V_{\pi N_i}$, $V_{\pi N_i}$ is the potential of the pion interaction with an i-th nucleon, H_c is the nuclear Hamiltonian. It is convenient to rewrite the Lippmann-Schwinger equation for the transition operator T in the form

$$T = T^{\circ} + T^{\circ} G_0(E)\, H_c G_c(E)\, T \tag{2}$$

with

$$T^{\circ} = V - VG_0(E)\, T^{\circ}, \quad G_0(E) = (h_0 - E)^{-1}$$
$$G_c(E) = (h_0 + H_c - E)^{-1}. \tag{2'}$$

The target Hamiltonian H_c will be approximated by the first-rank operator

$$H_c \approx H^{(1)} = \epsilon |\psi\rangle\langle\psi| . \tag{3}$$

Here ϵ and $|\psi\rangle$ are, the energy and wave function of the ground state of the target nucleus, respectively.

Using the approximation (3) for the $\pi\,{}^3$He elastic scattering amplitude we obtain the simple integral equation

$$\langle\vec{k}\psi|T|\psi\vec{k}'\rangle=\langle\vec{k}\psi|T^{\circ}|\vec{k}'\psi\rangle+\epsilon\int\frac{\langle\vec{k}\psi|T^{\circ}|\psi\vec{k}''\rangle\langle\vec{k}''\psi|T|\psi\vec{k}'\rangle}{(E''-E)(E''+\epsilon-E)}\frac{d\vec{k}''}{(2\pi)^3}. \quad (4)$$

Here the inhomogeneous term and kernel are defined by the fixed-scatterer T-matrix averaged over the target ground state wave function and taken at the total energy E of the system. In the calculation of this quantity, as is seen from eq.(2'), the motion of the whole nucleus is taken into account, whereas in the traditional model of fixed scatterers the kinetic energy of the nucleus is neglected /4/ (abandoned).

As is noted in the Introduction, the integral term in eq. (4) is small at all the considered energies. If the contribution from the continuous spectrum to the spectral expansion of the target Hamiltonian is taken into account, the corresponding integrals may turn out to be not small.

Now let us discuss the construction of the amplitude T° in more detail. By definition

$$\langle\vec{k}'\psi|T^{\circ}|\vec{k}\psi\rangle=\int d\vec{r}_{12}d\vec{r}_3|\psi(\vec{r}_{12},\vec{r}_3)|\ \langle\vec{k}',\vec{r}_{12},\vec{r}_3|T^{\circ}|\vec{k},\vec{r}_{12},\vec{r}_3\rangle,$$

where $\vec{k}$, $\vec{k}'$ are momenta of the relative motion of the pion and nucleus, $\vec{r}_{12}$, $\vec{r}_3$ are the Jacobian variables of nucleons, and $\langle\vec{k}'\vec{r}_{12},\vec{r}_3|T^{\circ}|\vec{k}\vec{r}_{12}\vec{r}_3\rangle$ obeys eq.(2) which in the mixed representation has the form

$$\langle\vec{k}',\vec{r}_{12}\vec{r}_3|T^{\circ}|\vec{k},\vec{r}^{*}_{12}\vec{r}^{*}_3\rangle=\langle\vec{k}'|V(\vec{r}_{12},\vec{r}_3)|\vec{k}\rangle-\int\frac{d\vec{q}}{(2\pi)^3}\langle\vec{k}'|V(\vec{r}_{12},\vec{r}_3)|\vec{q}\rangle\times$$
$$\times\frac{\langle\vec{q},\vec{r}_{12}\vec{r}_3|T^{\circ}|\vec{k},\vec{r}_{12}\vec{r}_3\rangle}{E_q-E}. \quad (5)$$

For our purposes it is convenient to rewrite eq.(5) in the Faddeev form.

$$V=\sum_{i=1}^{3}V_i \qquad t_i=V_i-V_iG_0(E)t_i\ ,$$
$$T^{\circ}=\sum_{i=1}^{3}T_{0i} \qquad T_{0i}=t_i-t_iG_0(E)(T_{0j}+T_{0k})\,. \quad (6)$$

We stress here that the equation for the two-body operator t_i is not the equation for the free πN t-matrix since by definition (2') the Green function $G_0(E)$ contains the mass of the

^3He nucleus instead of the nucleon mass in the target kinetic energy. At the same time while S and ^{31}P wave solutions of eq.(6) for the operator t_1 practically coincide with the free πN-t -matrix, for ^{33}P -wave, where there is a resonance in the free πN-scattering, these solutions are essentially different. The difference is in that the solutions of eq.(6) for t_1 in ^{33}P wave do not manifest the resonance behaviour. A numerical experiment has been performed on calculating the pion scattering phase shift on a particle with mass m . Parameters of the pion-particle interaction were chosen so as to reproduce the ^{33}P -phase in πN scattering at $m = M_N$. In Fig.1 scattering phase shifts are drawn for different masses of the particle. Phase shifts were calculated by the formulae

$$k^3 \mathrm{ctg}\,\delta = -\mathfrak{N}(m_\pi, m, E)\, h^{-2}(k)\,[\lambda^{-1} + J(m,E)\,],$$

$$J(m,E) = \frac{1}{\pi\mu_{\pi m}} P\int_0^\infty q^2 dq\, h^2(q)\, \frac{1}{\dfrac{q^2}{2m} + \sqrt{q^2 + m_\pi^2} - m_\pi - E}, \qquad (7)$$

$$\mathfrak{N} = \frac{(m_\pi + m)\sqrt{k^2 + m_\pi^2}}{m_\pi[m + \sqrt{k^2 + m_\pi^2}\,]}, \quad E = \frac{k^2}{2m} + \sqrt{k^2 + m_\pi^2} - m_\pi . \qquad (8)$$

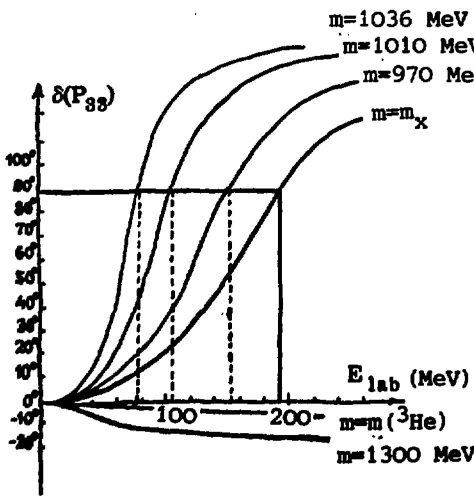

Fig.1. Dependence on mass N of the resonance p -wave phase-shift for πN -scattering.

As is seen from the figure, even a negligible increase in mass m shifts the resonance position to lower energies and at $m = M_{^3He}$ the resonance disappears at all. By this reason, the amplitude found by exactly solving the corresponding equation has no resonance behaviour. As a result, the π -nuclear scattering cross section calculated with such an amplitude becomes very small at πN resonance energies, rather smaller than the cross section of elastic scattering of pion on the free nucleon.

3.

We have established that within the approximation (3) - a natural extension of the fixed-scatterer model, using πN -potentials obtained to the corresponding phase shifts - one fails to describe the experimental data on elastic $\pi - {}^3He$ -scattering at intermediate energies. Hence it follows that the continuous spectrum of the nuclear system should necessarily be taken into account.

The accurate treatment of the contribution of the continuum spectra, implies to solve the relevant four-particle equations that is out of our purpose. Being aware of the reason for the failure of the model of Sec.2 at intermediate energies and preserving its simplicity we shall introduce its certain modification in the spirit of impulse approximation.

The partial-wave decomposition at the πN -potential is written in the form:

$$<\vec{k}'\mu | V^t | \vec{k}\mu> = \sum_{\substack{\ell JM \\ mm'}} Y_{\ell m'}(k') Y^*_{\ell m}(k) (\ell \tfrac{1}{2} m' \mu' | JM)(\ell \tfrac{1}{2} m \mu | JM) V^{tJ}_{\ell}(k,k'), \tag{9}$$

where t is the πN isospin, μ, μ' are projections of the nucleon spin, the other notation is obvious.

In the considered energy region we shall assume the πN -potential components V_0^{11}, V_0^{31}, V_1^{33}, V_1^{31} to be nonzero, as the phases in other partial waves are small. Potentials $V_\ell^{tJ}(k,k')$ were chosen in the separable form:

$$\begin{aligned} V_0^{11}(k,k') &= \lambda_S^1 h_1(k)\, h_1(k') \\ V_0^{31}(k,k') &= \lambda_S^1 h_2(k)\, h_2(k') \\ V_1^{33}(k,k') &= \lambda_p^3 h_3(k)\, h_3(k') \\ V_1^{31}(k,k') &= \lambda_p^1 h_4(k)\, h_4(k') \end{aligned} \tag{10}$$

The form factors $h_i(k)$ and parameters of potentials (10) are written down explicitly in Appendix.

For convenience we introduce the following multidimensional state vectors:

$$|\eta> = |\eta_{j\alpha m}(\vec{k})> = e^{-i\vec{k}\vec{z}_j}\, \Gamma^+_{\alpha m} h_\alpha(k),$$

$$|\omega> = |\omega_{j\alpha m}(\vec{k})> = e^{-i\vec{k}\vec{z}_j}\, \Gamma^-_{\alpha m} h_\alpha(k),$$

$$
\Gamma^{\pm}_{a\mathrm{m}} = \begin{cases} \sqrt{4\pi} & a = 1,2 \\ \sqrt{4\pi}(1\tfrac{1}{2}\mathrm{m} \pm \tfrac{1}{2} \,|\, \tfrac{3}{2}\mathrm{m} \pm \tfrac{1}{2}) Y_{1\mathrm{m}}(\hat{\mathbf{k}}) & a = 3 \\ \sqrt{4\pi}(1\tfrac{1}{2}\mathrm{m} \pm \tfrac{1}{2} \,|\, \tfrac{1}{2}\mathrm{m} \pm \tfrac{1}{2}) Y_{1\mathrm{m}}(\hat{\mathbf{k}}) & a = 4 \end{cases} \tag{11}
$$

$$
\vec{z}_1 = \frac{1}{2}\vec{r}_{12} + \frac{1}{3}\vec{r}_3 , \quad \vec{z}_2 = -\frac{1}{2}\vec{r}_{12} + \frac{1}{3}\vec{r}_3 , \quad \vec{z}_3 = -\frac{2}{3}\vec{r}_3 .
$$

In the notation (11) the matrix elements of the two-body operators t_1 (see formula (6)), over $\pi 3N$ wave functions with a given value of the total isispin T can be rewritten as

$$
\langle \Psi^{TS=1/2}_{M_S=1/2} | t \; | \Psi^{TS=1/2}_{M_S=1/2} \rangle \equiv \langle \eta | t_1 | \eta \rangle = \langle \eta_1(\vec{k}) \, | \Lambda | \eta_1(\vec{k}') \rangle \tag{12}
$$

for the nonspin-flip amplitude, where matrix $\Lambda_{aa'}$:

$$
\Lambda_{aa'} = \delta_{aa'} \Lambda_a = \frac{\delta_{aa'} C^T_a}{\lambda_a^{-1} + I_a(E)} \tag{13}
$$

and

$$
\lambda_1 \equiv \lambda^1_S , \quad \lambda_2 \equiv \lambda^3_S , \quad \lambda_3 \equiv \lambda^3_P , \quad \lambda_4 \equiv \lambda^1_P .
$$

Constants C^T_a and integrals $I_a(E)$ are given in Appendix. The spin-flip amplitude will be expressed as follows:

$$
\langle \Psi^{TS=1/2}_{M_S=1/2} | t_1 | \Psi^{TS=1/2}_{M_S=-1/2} \rangle = \langle \eta_1(\vec{k}) \, | \bar{\Lambda} | \omega_1(\vec{k}') \rangle , \tag{14}
$$

where

$$
\bar{\Lambda}_{aa'} = \delta_{aa'} \bar{\Lambda}_a ; \quad \bar{\Lambda}_a = \begin{cases} 0 & a = 1,2 \\ r^T \Lambda_a & a = 3,4 \end{cases}
$$

$$
r^T = \begin{cases} 1 & \text{for } T = \frac{1}{2} \\ \frac{1}{7} & \text{for } T = \frac{3}{2} \end{cases}
$$

With the matrix elements of the two-body operators t_i we obtain the system of two matrix equations for the spin-flip and nonspin-flip amplitude T°:

$$\begin{aligned}
&<\eta|T_{0i}|\eta> = <\eta|t_i|\eta> - <\eta|t_i|\eta> G_0(E) <\eta|T_{0j} + T_{0k}|\eta> - \\
&\qquad - <\eta|t_i|\omega> G_0(E) <\omega|T_{0j} + T_{0k}|\eta>, \\
&<\omega|T_{0i}|\eta> = <\omega|t_i|\eta> - <\omega|t_i|\eta> G_0(E) <\eta|T_{0j} + T_{0k}|\eta> - \\
&\qquad - <\omega|t_i|\omega> G_0(E) <\omega|T_{0j} + T_{0k}|\eta>.
\end{aligned} \tag{15}$$

The solutions of the system for a state with definite isospin are of the form

$$\begin{aligned}
&<\eta|T_0^T|\eta> \equiv <\eta|r_{\eta\eta}^T|\eta>, \quad r_{\eta\eta}^T = [1 - \hat{X}^{-1}\hat{R}_T <\omega|G_0|\omega> \times \\
&\times \hat{Y}^{-1}\hat{R}_T <\eta|G_0|\eta>]^{-1} \hat{X}^{-1}[1 - \hat{R}_T <\omega|G_0|\omega> \hat{Y}^{-1}\hat{R}_T], \\
&r_{\omega\eta}^T = \hat{Y}^{-1}\hat{R}_T[1 - \eta|G_0|\eta> r_{\eta\eta}^T],
\end{aligned} \tag{16}$$

where matrices $\hat{X}$ and $\hat{Y}$ are defined as follows:

$$\hat{X}^{-1} = \Lambda^{-1} + |\eta> G_0(E) <\eta|; \quad \hat{Y}^{-1} = \bar{\Lambda}^{-1} + |\omega> G_0(E) <\omega|$$

and matrix $\hat{R}_T$ has a form

$$\hat{R}_T = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & r^T & 0 \\ 0 & 0 & 0 & r^T \end{pmatrix}.$$

The exact form of matrix elements of the matrices $\hat{X}$ and $\hat{Y}$ is given in Appendix.

Differential cross sections are presented in terms of the above amplitudes in the following manner.

$$\frac{d\sigma_{\pi^+\,^3He}}{d\theta} = |f^{T=3/2}|^2 + |g^{T=3/2}|^2,$$

$$\frac{d\sigma_{\pi^-\,^3He}}{d\theta} = |\frac{1}{3}f^{3/2} + \frac{2}{3}f^{1/2}|^2 + |\frac{1}{3}g^{3/2} + \frac{2}{3}g^{1/2}|^2,$$

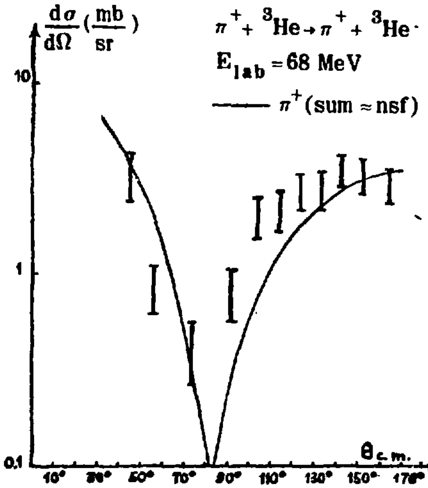

Fig.2. Differential cross section for the elastic π^+ ^3He - scattering at E_π=68 MeV, nsf is non-spin-flip cross section.

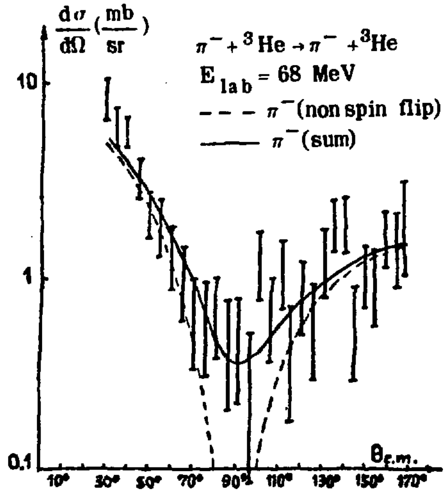

Fig.3. - - - -is non-spin-flip cross section for elastic π^- ^3He scattering at E_π=68 MeV, ——— sum spin-flip and non-spin-flip cross-section.

$$\frac{d\sigma_{\pi^- \, ^3He \to \pi^0 \, ^3H}}{d\theta} = \frac{2}{9}\{|f^{3/2} - f^{1/2}|^2 + |g^{3/2} - g^{1/2}|^2\},$$

where

$$f^T = -4\pi^2 \mu_{\pi ^3He} \int \Psi^2_{^3He}(\vec{r}_{12}, \vec{r}_3) \langle \eta | T^{\circ\, T} | \eta \rangle d\vec{r}_{12} d\vec{r}_3 \; ,$$

$$g^T = -4\pi^2 \mu_{\pi ^3He} \int \Psi^2_{^3He}(\vec{r}_{12}, \vec{r}_3) \langle \eta | T^{\circ\, T} | \omega \rangle d\vec{r}_{12} d\vec{r}_3 \; .$$

The modification of the model consists in the change, in integrals $I_a(E)$ in (13), of the π-nuclear Green function by the free π-nucleon Green function. What approximation in the eq.(6) this change corresponds to, will be considered below.

Besides the πN-phase shifts the "input" information contains also the eigenfunction and eigenvalue of the target Hamiltonian. The calculations used the experimental value of the ^3He binding energy and an approximate function, containing the symmetric S-component with the Irving radial dependence /6/. Under these assumptions we have calculated the differential

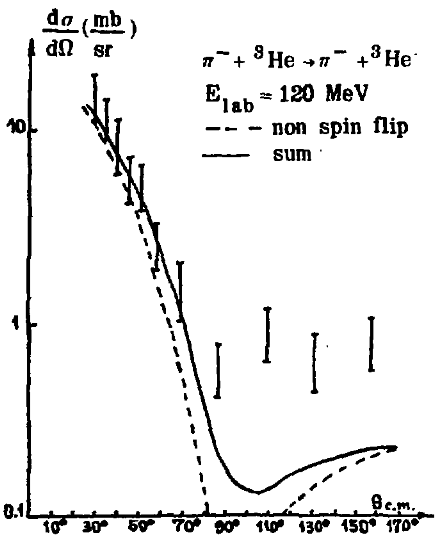

Fig.4. The same as in Fig.3, but at $E_\pi = 120$ MeV.

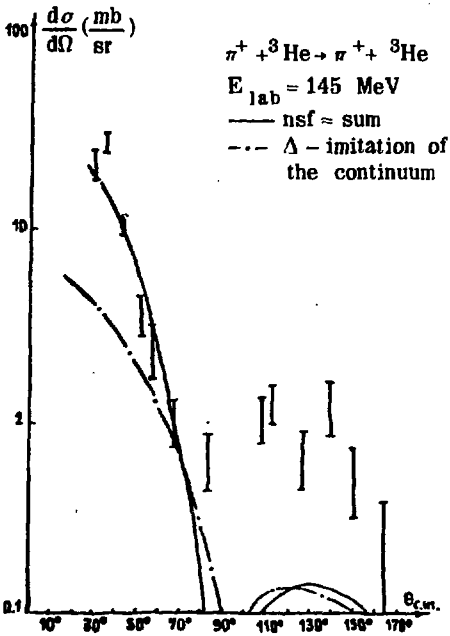

Fig.5. —— is sum cross section for the elastic $\pi^+\,{}^3$He scattering at $E_\pi = 145$ MeV, -·-· imitation for the contribution of the continuum spectrum of the nuclear subsystem by introducing an additional parameter Λ .

cross sections of elastic $\pi^\pm\,{}^3$He *scattering and charge* exchange reactions at various energies. Results of the calculation are shown in Figs.2-7; experimental data are taken from refs. /7-9/ . As is seen from the figures our predictions are in good agreement with the experimental data on elastic scattering over the pion energy region before and after the π-nuclear resonance (see, e.g., the data at E_π = 68 MeV and 208 MeV), which is observed at an energy of 150 MeV.

At other pion energies the agreement with experiment is achieved only in the forward hemisphere of the scattering angles. This relation between theory and experiment may be understood if one has in mind the smallness of the πN-amplitude far from the resonance. This smallness will, obviously, justify the use of impulse approximation like in our calculations.

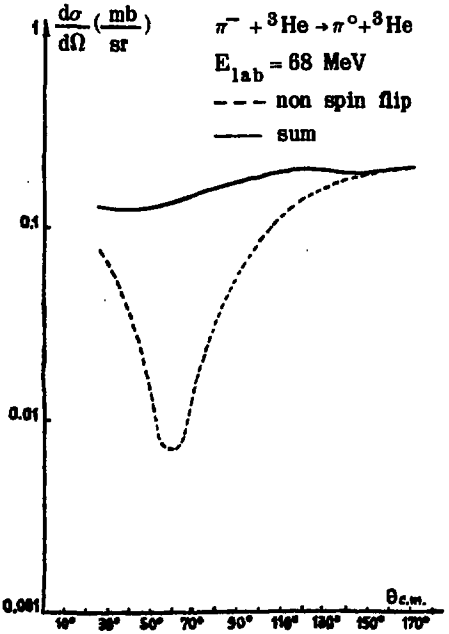

Fig.6. --- nsf cross section to the single charge-exchange π^- by ${}^3He$ at $E_\pi = 68$ MeV, ——— sum sf and nsf.

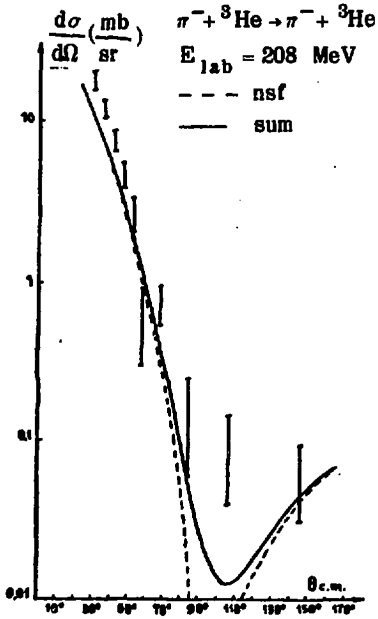

Fig.7. The same as in fig.3, but at $E_\pi = 208$ MeV.

And vice versa, in the resonance region the πN amplitude has a magnitude comparable with the mean distance between nucleons and the applicability of impulse approximation is rather doubtful in that region.

Like in earlier attempts of the description of elastic $\pi\ {}^3He$-scattering and charge exchange, our approach also reveals a significance of spin effects in the state with total isospin of the system $\pi + {}^3He$ $T = 1/2$, i.e., in elastic $\pi^-\ {}^3He$-scattering and charge exchange (see Fig.6). Based on the above results, we should conclude that the magnitude of the spin-flip amplitude in the state with $T = 1/2$ depends slightly on the dynamics and character of approximations used. In view of the good agreement of our calculations with experimental data at $E_\pi = 68$ MeV, the same can be expected at lower energies. In this region there are no experimental data, nevertheless we report the calculated cross sections at energy $E_\pi = 24$ MeV. (see Fig.8).

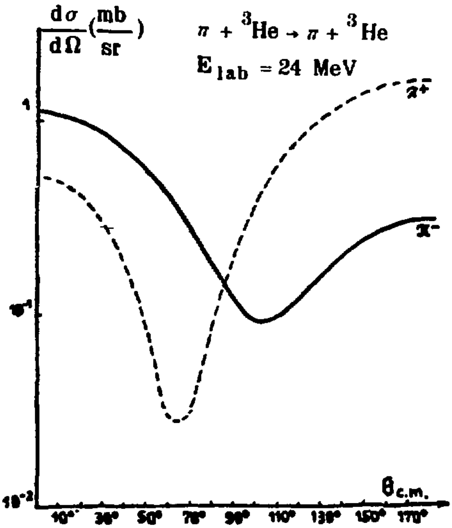

Fig.8. Cross section for the elastic $\pi\ {}^3He$ scattering at $E_\pi = 24$ MeV, - - - is π^+-mesons, —— is π^--mesons.

As was indicated above, the modification of initial equations consists in the change, in eqs.(6) for T_{0i}, of the two-body operator t_i by the free πN t-matrix. Obviously, this means that the elementary act of πN-scattering occurs on the free nucleon, i.e., on that one which is in a state of the continuous spectrum, and the wave function, over which the operator is averaged, only holds the nucleons inside the nucleus. Thus, the procedure described simulates the contribution from states of the continuous spectrum of the nucleon system. To what extent this simulation is, from a theoretical point of view, adequate to the exact consideration of the continuous spectrum in four-particle equations, is a difficult task at present, however, the agreement between calculated and experimental cross sections indicates that operations of this type are sufficient.

One more simple procedure can be mentioned, which simulates the contribution of the continuous spectrum of nucleons. As has been shown above (see Fig.1), the πN-resonance position shifts to lower energies with increasing mass of the scatterer, therefore the resonance position may naturally be reconstructed by simply adding a constant to the denominator of the Green function (2'), and the two-body operator t_i, eq.(6), being used. This addition of a constant is equivalent to the change of the contribution of the continuum to the spectral expansion of the target Hamiltonian by some constant Λ. Figure 5 shows the pion angular distribution at $E_\pi = 145$ MeV, calculated by introducing Λ. As is seen, the choice of Λ may provide the agreement of the theoretical curve with experimental data.

APPENDIX

$$C_1^T = C_4^T = \frac{1}{3}\left[1 + 2(-1)^{T+1/2}\left\{\begin{matrix} 1 & 1 & 1 \\ \frac{1}{2} & T & \frac{1}{2}\end{matrix}\right\}\right],$$

$$C_2^T = C_3^T = \frac{2}{3}\left[1 - 2(-1)^{T+1/2}\left\{\begin{matrix} 1 & 1 & 1 \\ \frac{1}{2} & T & \frac{1}{2}\end{matrix}\right\}\right],$$

$$X^{ij}_{\alpha\beta mm'}(E) = \Lambda_\alpha^{-1}\delta_{\alpha\beta}\delta_{ij}\,\delta_{mm'} + \delta_{mm'}\,{}^{+}S_m^{\alpha\beta}\,H_{\alpha\beta}(E, |\vec{z}_i - \vec{z}_j|)$$

$\hat{Y}$ is given by the same formula with ${}^{+}S_m^{\alpha\beta} \to {}^{-}S_m^{\alpha\beta}$, ${}^{+}S_m^{\alpha\beta} = 1$ for $\alpha, \beta = 1, 2$.

$${}^{\pm}S_m^{33} = (1\tfrac{1}{2}m \pm \tfrac{1}{2}\,|\,\tfrac{3}{2}m \pm \tfrac{1}{2})^2 \;,$$

$${}^{\pm}S_m^{44} = (1\tfrac{1}{2}m \pm \tfrac{1}{2}\,|\,\tfrac{1}{2}m \pm \tfrac{1}{2})^2 \;,$$

$${}^{\pm}S_m^{34} = {}^{\pm}S_m^{43} = (1\tfrac{1}{2}m \pm \tfrac{1}{2}\,|\,\tfrac{3}{2}m \pm \tfrac{1}{2})(1\tfrac{1}{2}m \pm \tfrac{1}{2}\,|\,\tfrac{1}{2}m \pm \tfrac{1}{2}).$$

$$H_{\alpha\beta}(E, z) = \frac{1}{\pi\mu_{\pi\,{}^3He}} \int_0^\infty \frac{q^2 dq\, h_\alpha(q)\, h_\beta(q)\, j_0(qz)}{\dfrac{q^2}{2m_{{}^3He}} + \sqrt{q^2 + m_\pi^2} - m_\pi - E + i\eta},$$

where $\mu_{\pi\,{}^3He}$ is the reduced mass of the $\pi\,{}^3He$ system, $m_{{}^3He}$ is the mass of ${}^3He$.

$$I_\alpha(E) = H_{\alpha\alpha}(E, 0)\,[\,m_{{}^3He} \to m_N\,].$$

Form factors

$$S_{11}:\ h_1(k)=\frac{1}{k^2+\alpha_1^2}+\frac{S_1}{k^2+\beta_1^2}$$

$$S_{13}:\ h_2(k)=\frac{1}{k^2+\alpha_2^2}+\frac{S_2}{k^2+\beta_2^2}$$

$$P_{33}:\ h_3(k)=\frac{k}{(k^2+\alpha_3^2)^2}+\frac{S_3 k^2}{(k^2+\beta_3^2)^2}$$

$$P_{31}:\ h_4(k)=\frac{k}{(k^2+\alpha_4^2)^2}$$

Parameters

$\alpha_1=3.188\ \mathrm{fm}^{-1}$, $\alpha_2=3.382\ \mathrm{fm}^{-1}$,

$\alpha_3=1.366\ \mathrm{fm}^{-1}$, $\alpha_4=1.756\ \mathrm{fm}^{-1}$,

$\beta_1=0.823\ \mathrm{fm}^{-1}$, $\beta_2=1.107\ \mathrm{fm}^{-1}$,

$\beta_3=5.270\ \mathrm{fm}^{-1}$,

$S_1=0.0502$, $S_2=-0.0273$,

$S_3=30.846\ \mathrm{fm}$,

$\lambda^1_S=-6.137\ \mathrm{fm}^{-5}$, $\lambda^3_S=71.408\ \mathrm{fm}^{-5}$,

$\lambda^3_P=-0.5907\ \mathrm{fm}^{-7}$, $\lambda^1_P=12.353\ \mathrm{fm}^{-7}$.

REFERENCES

1. Petrov N.M., Peresypkin V.V. Nucl.Phys., 1974, A220, p.147; Thomas A.W. Nucl.Phys., 1976, A258, p.417.
2. Mach R., Nichitiu F., Shcherbakov Yu.A. Phys.Lett., 1974, 53B, p.133; Landau R.H. Phys.Rev., 1977, C15, p.2127.
3. Sparrow D.A. Phys.Lett., 1975, 58B, p.309; Eisenberg J., Mandelzweig V. Phys.Lett., 1975, 53B, p.405; Mach R., Sapozhnikov M.G., Shcherbakov Yu.A. Czech.Journ. of Phys., 1976, B26, p.1248; Lohs K., Mandelzweig V. Z.Phys., 1977, A283, p.51.
4. Gibson B.F., Hess A.T. Phys.Rev., 1976, C13, p.749.
5. Belyaev V.B., Wrzecionko J. J.Nucl.Phys. (USSR), 1978, 28, p.78; JINR, E2-10668, Dubna, 1977.
6. Irving I. Phil.Mag., 1952, 42, p.338.
7. Shcherbakov Yu.A. et al. N.C., 1976, 31A, p.262.
8. Nefkens B. Lecture Notes in Physics, 1978, 87, p.189.
9. Källne J. et al. Phys.Rev.Lett., 1979, 42, p.159.

Received by Publishing Department
on March 18 1980.