2701/2-80

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E2-80-197

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ON ORIGIN OF NONLOCAL CONSERVED CURRENTS FOR THE SUPERSYMMETRIC NONLINEAR SIGMA MODELS

Submitted to Письма в ЖЭТФ.

1980

1. In paper [1] it was shown that the classical nonlocal conserved currents (NCC) for the nonlinear sigma models introduced first in ref. [2] are of Noether type, i.e., they can be derived from the generalized Noether theorem [3]. In the present article, the results of ref. [1] for the case of two-dimensional supersymmetric nonlinear sigma models (SNSM) are generalized. The corresponding NCC for such models in papers [4-6] were found.

2. Consider the action for the generalized (G) SNSM

$$S = \frac{1}{2} \int d^2 x\, d^2 \Theta\, \mathrm{tr}\{D^{\alpha} G^{-1}(x;\Theta) D_{\alpha} G(x;\Theta)\}, \tag{1}$$

where $G \in gl(N)$ ($U(N)$ or $O(N)$), i.e., the Lie algebra of the general linear (unitary or orthogonal) global gauge group in N-dimensional flavour space and

$$D_{\alpha} = i\frac{\partial}{\partial\Theta^{\alpha}} + (\bar{\Theta}\not{\partial})_{\alpha}, \qquad (\alpha = 1, 2)$$

are supercovariant derivatives. From (1) we derive the following equation of motion

$$D^{\alpha} A_{\alpha}(x;\Theta) = 0 , \tag{2}$$

where we denote

$$A_{\alpha}(x;\Theta) = G^{-1}(x;\Theta) D_{\alpha} G(x;\Theta). \tag{3}$$

Consider an arbitrary infinitesimal gauge transformation

$$G'(x;\Theta) = G(x;\Theta) + \delta G(x;\Theta) = U G(x;\Theta) U^{-1}, \tag{4}$$

where

$$U(x;\Theta) = I + \Omega(x;\Theta) \tag{5}$$

and consequently

$$\delta G(x;\Theta) = [\Omega(x;\Theta), G(x;\Theta)]. \tag{6}$$

Here $\delta G(x;\Theta) = \overline{\delta G}(x;\Theta)$ is the variation of the form and $\Omega(x;\Theta)$

ТЕКА

is an arbitrary infinitesimal matrix function for the present moment. Assume that

$$\Omega(x;\Theta) = [\, X(x;\Theta),\ \epsilon\,], \tag{7}$$

where ϵ is an infinitesimal constant matrix parameter. Then the variation of action (1) under the transformation (4) gets the form

$$\begin{aligned} \delta S &= \int d^2x\, d^2\Theta\, \mathrm{tr}\{A^\alpha(x;\Theta)\,\delta A_\alpha(x;\Theta)\} = \\ &= 2\int d^2x\, d^2\Theta\, \mathrm{tr}\{D^\alpha A_\alpha[X,\epsilon] - D^\alpha([A_\alpha, X])\epsilon\}. \end{aligned} \tag{8}$$

Here

$$\delta A_\alpha(x;\Theta) = G^{-1}[\, D_\alpha X, \epsilon\,]G - \{V_\alpha, [X, \epsilon]\},$$

where

$$V_\alpha = D_\alpha + A_\alpha(x;\Theta).$$

Suppose that there exists a matrix function $Y(x;\Theta)$ connected with $X(x;\Theta)$ by the differential equation

$$D_\alpha X(x;\Theta) = [(\gamma_5 V)_\alpha, Y(x;\Theta)] \quad (\alpha = 1,2). \tag{9}$$

Then substituting eq. (9) in eq. (8) we have

$$\delta S = -2\int d^2x\, d^2\Theta\, \mathrm{tr}\{D^\alpha[(\gamma_5 A)_\alpha, Y]\epsilon\}, \tag{10}$$

where the following identity is used

$$\{V_1, V_2\} = 2(C\gamma^\mu)_{1,2}\,(\partial_\mu + G^{-1}\partial_\mu G) = 0,$$

which follows from $(C\gamma^\mu)_{1,2} = 0$ (see ref. [6]).

From eqs. (8) and (10) it follows that the generalized Noether's theorem can be applied [3], i.e., the identity

$$J_\alpha^{(X,Y)}(x;\Theta) = [\, A_\alpha(x;\Theta), X] + [(\gamma_5 A)_\alpha, Y] \tag{11}$$

is conserved (in a weak sense, i.e., when $D^\alpha A_\alpha = 0$) supercurrent if $X(x;\Theta)$ and $Y(x;\Theta)$ are coupled with (9). Therefore, functions $X(x;\Theta)$ and $Y(x;\Theta)$ satisfying eq. (9) generate infinite number of NCC. It can be checked, without any difficulties, that eq. (9) is the necessary and sufficient condition for the conservation of (11) if the equation of motion (2) is satisfied.

3. Three N^2-parametric infinite sequences of functions satisfying eq. (9) were found in ref. [6]. Substituting these functions in (11) we get the corresponding NCC. One of these series is generated from the following matrix functions

$$X^{(k)}(x;\Theta) = \chi^{(k)}(x) + \bar\Theta\kappa^{(k)}(x) + \frac{1}{2}\bar\Theta\Theta\zeta^{(k)}(x), \quad (k = 1,2,\ldots,),$$

where $\chi^{(k)}(x)$, $\kappa^{(k)}(x)$, and $\zeta^{(k)}(x)$ are given by [6]

$$\begin{aligned} \chi^{(k)}(x) &= -\int^{x_1} dy_1\, [\, v_0(x_0, y_1), \chi^{(k-1)}(x_0, y_1)] + C_k, \\ \kappa^{(k)}(x) &= \gamma_5(\kappa^{(k-1)} - i\,a(x)\chi^{(k-1)}), \\ \zeta^{(k)}(x) &= i\,r(x)\chi^{(k-1)}(x) - \frac{i}{2}\bar a(x)\gamma_5\kappa^{(k-1)}(x). \end{aligned} \tag{12}$$

Here $r(x)$, $a(x)$ and $v_\mu(x)$ are pseudoscalar, spinor and vector components of $A_\alpha(x;\Theta)$, respectively [6].

4. It follows from eqs. (4-7) that functions $X(x;\Theta)$, generating the NCC (11), are generator functions of global gauge transformations as well. For these generators we have, in general, nonlinear and nonlocal realizations (12). If we restrict ourselves to the local representations we have only the first Noether's current.

There exist two possibilities: First, the generator functions $X^{(k)}(x;\Theta)$ $(k = 0,1,\ldots)$ are considered to be nonlocal and nonlinear realizations of $GL(N)$ ($U(N)$ or $O(N)$) group, and second, these functions are considered as realization of an infinite-parameter non-Abelian group. The second possibility results from the corresponding Lie algebra given by the Poisson brackets for $X^{(k)}(x;\Theta)$. However, because of the nonlocality of $X^{(k)}(x;\Theta)$ this statement is nonunique.

Acknowledgements are due to E.A.Ivanov for valuable discussions.

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Received by Publishing Department
on March 7 1980.