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TORSION AND CURVATURE IN TERMS OF THE AXIAL GRAVITATIONAL SUPERFIELD

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1. Introduction

The success of the component approach to supergravity (the so-called "tensor calculus") is explained by the existence of a manifestly covariant superfield approach to this theory. Supergravity can be formulated as a theory of the curved superspace and the powerful methods of differential geometry can be applied, as it is done in Einstein's general relativity. In such a framework the basic objects are the supervierbeins $E_A{}^M$ and the connections $\omega_{AB}{}^C$. They allow one to define covariant derivatives

$$\mathcal{D}_A \varphi_B = E_A{}^M \partial_M \varphi_B + \omega_{AB}{}^C \varphi_C \quad . \tag{1.1}$$

The (anti) commutators of these derivatives give covariant quantities, the torsion $T_{AB}{}^{\mathcal{D}}$ and curvature $R_{AB}{}^{ef}$ tensors

$$[\mathcal{D}_A, \mathcal{D}_B\} \varphi_C = - T_{AB}{}^{\mathcal{D}} \mathcal{D}_{\mathcal{D}} \varphi_C - R_{AB,C}{}^{\mathcal{D}} \varphi_{\mathcal{D}} \, , \tag{1.2}$$

where

$$R_{AB,C}{}^{\mathcal{D}} = R_{AB}{}^{ef} (\Lambda_{ef})_C{}^{\mathcal{D}}$$

and Λ_{ef} are the Lorentz-group generators.

One of the first geometric approaches[1,2] is based on the general coordinate transformation group in superspace $\{(x^m, \theta^\mu, \bar\theta^{\dot\mu})\}$. There the supervierbeins $E_A{}^M$ are the primary potentials. This theory cannot be made a theory of supergravity by purely geometric means. Additional covariant constraints on the torsion components have to be postulated in order to eliminate a large number of the field-components of $E_A{}^M(x, \theta, \bar\theta)$. The form of these constraints has successfully been guessed by Wess and Zumino[1/*)].

In Ref.[5] it was shown that supergravity can be constructed on a more transparent, purely geometric ground. To this end one starts with the general coordinate transformation group in 4+2-

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*) In Ref.[3] the classification and possible algorithms in searching for those constraints have been discussed. In some recent papers[4] they have been guessed for $N=2$ -supergravity too.

dimensional complex superspace $\{(x^m_L, \theta^\mu_L)\}$. The imaginary part of x^m_L is identified with the axial superfield $\mathcal{H}^m(x,\theta,\bar\theta)$ that is the primary potential in the theory. All geometric objects are expressed in terms of the derivatives of $\mathcal{H}^m$. The corresponding formulae for $E_A{}^M$ and $\omega_{AB}{}^C$ were given in Ref. /6/.

In the present paper the components of torsion $T_{AB}{}^C$ and curvature $R_{ABC}{}^D$ are written down in terms of several basic superfields $R, \bar R, G, W, \bar W$. Explicit expressions in terms of $\mathcal{H}^m$ and identities for these basic superfields are derived. All covariant constraints guessed in Refs. /1,2/ are automatically fulfilled here. Thus, our geometric approach is shown to be equivalent to the approach of Ref. /1/ *), although involving less postulates and superfluous field variables.

Instead of the Bianchi identities we shall intentionally use the normal gauge (NG below) /8/ in order to show its effectiveness and constructiveness. In this gauge at a given superspace point $z_0^N = (x_0^n, \theta_0^\nu, \bar\theta_0^{\dot\nu})$ the following derivatives of $\mathcal{H}^m$ do not vanish (their dimensions and types of Lorentz-group representations are listed also):

$$\partial_\nu \bar\partial_{\dot\nu} \mathcal{H}^m|_0 = -(\sigma^m)_{\nu\dot\nu} \qquad (cm^0) \qquad (1.3)$$

$$\partial^\nu \partial_\nu \partial_m \mathcal{H}^m|_0 = \frac{3i}{2} \bar R|_0 \text{ and h.c.} \qquad (cm^{-1}, (0,0)) \qquad (1.4)$$

$$\partial^\nu \partial_\nu \bar\partial_{\dot\nu} \bar\partial^{\dot\nu} \mathcal{H}^m|_0 = -3G^m|_0 \qquad (cm^{-1}, (\tfrac{1}{2}, \tfrac{1}{2})) \qquad (1.5)$$

$$\underset{(\alpha\beta\gamma)}{\mathrm{Sym}} \bar\partial_{\dot\nu} \bar\partial^{\dot\nu} \partial_\alpha \partial_{\beta\dot\beta} \mathcal{H}^{\dot\beta}_\gamma|_0 = -4i W_{\alpha\beta\gamma}|_0 \text{ and h.c.} \quad (cm^{-\frac{3}{2}}, (\tfrac{3}{2}, 0)) \qquad (1.6)$$

$$\underset{(\dot\alpha\dot\beta)}{\mathrm{Sym}} \bar\partial_{\dot\nu} \bar\partial^{\dot\nu} \partial^\beta \partial_{\beta\dot\alpha} \mathcal{H}_{\alpha\dot\beta}|_0 = -\frac{3i}{2} \underset{(\dot\alpha\dot\beta)}{\mathrm{Sym}} \bar{\mathcal{D}}_{\dot\beta} G_{\alpha\dot\alpha}|_0 \text{ and h.c.} \quad (cm^{-\frac{3}{2}}, (\tfrac{1}{2}, 1)) \qquad (1.7)$$

$$\bar\partial_{\dot\nu} \bar\partial^{\dot\nu} \partial_\alpha \partial_m \mathcal{H}^m|_0 = \frac{3i}{2} \mathcal{D}_\alpha R|_0 \text{ and h.c.} \qquad (cm^{-\frac{3}{2}}, (\tfrac{1}{2}, 0)). \qquad (1.8)$$

Here $\partial_m = \partial/\partial x^m$, etc.; $\partial_{\nu\dot\nu} = \sigma^m_{\nu\dot\nu} \partial_m$, $\mathcal{H}_{\nu\dot\nu} = \sigma^m_{\nu\dot\nu} \mathcal{H}_m$; the world and Lorentz indices are not distinguishable in NG; h.c. means Hermitian conjugated terms. The remaining derivatives of $\mathcal{H}^m$ with

*) It is equivalent to the approach of Siegel and Gates /7/ also.

dimension from $cm^{+\frac{1}{2}}$ to $cm^{-\frac{3}{2}}$, as well as $\mathcal{H}^m$ itself, vanish in NG.

In the right-hand sides of Eqs.(1.3)-(1.8) there are basic tensor superfields that reduce just to these derivatives of $\mathcal{H}^m$ in NG.

NG is analogous to the normal coordinate frames in general relativity. It simplifies significantly and makes transparent a number of calculations and derivations. For instance, Eqs.(1.3)-(1.8) immediately yield the absence of gauge-independent (i.e., tensor) quantities with dimension $cm^{-\frac{1}{2}}$ (of course, without using the coupling constant $\varkappa$ as a dimension-regulating factor). This simply means that the torsion components with dimension $cm^{-\frac{1}{2}}$ should vanish. We hope that similar gauges will be possible and constructive in extended supergravity too.

For convenience of the reader the paper is planned as follows: In Section 2 a complete list of the results is given, and their derivation is demonstrated in the subsequent sections. The notation used is explained in the Appendices. They also contain some necessary information about the operators Δ, the spinor connections, etc., as well as some identities following from the main text.

2. Summary of the Results

Here the components of the torsion tensor $T_{AB}{}^C$ are listed and their dimensions are indicated.

$$T_{\alpha\dot\beta}{}^c = 2i(\sigma^c)_{\alpha\dot\beta} \ (a), \quad T_{\alpha\beta}{}^c = T_{\dot\alpha\dot\beta}{}^c = 0 \ (b); \qquad (cm^0) \qquad (2.1)$$

$$T_{\alpha\beta}{}^\gamma = T_{\alpha\dot\beta}{}^\gamma = T_{\dot\alpha\dot\beta}{}^\gamma = 0 \text{ and h.c.}; \qquad (cm^{-\frac{1}{2}}) \qquad (2.2)$$

$$T_{\alpha b}{}^c = 0 \text{ and h.c.}; \qquad (cm^{-\frac{1}{2}}) \qquad (2.3)$$

$$T_{\alpha,\beta\dot\beta}{}^\gamma = -\frac{i}{4} \varepsilon_{\alpha\beta} G^\gamma_{\dot\beta} \text{ and h.c.} \qquad (cm^{-1}) \qquad (2.4)$$

(throughout the paper the vector indices are often replaced by pairs of spinor ones, e.g., $T_{\alpha,\beta\dot\beta}{}^\gamma = (\sigma^b)_{\beta\dot\beta} T_{\alpha b}{}^\gamma$; see Appendix A);

$$T_{\alpha,\beta\dot\beta,\dot\gamma} = \frac{i}{4} \varepsilon_{\alpha\beta} \varepsilon_{\dot\beta\dot\gamma} \bar R \text{ and h.c.}; \qquad (cm^{-1}) \qquad (2.5)$$

$$T_{ab}{}^{c} = -\frac{1}{8}\eta^{cd}\varepsilon_{abde}G^{e}; \qquad (cm^{-1}) \quad (2.6)$$

$$T_{\alpha\dot\alpha,\beta\dot\beta,\gamma} = \frac{1}{2}\varepsilon_{\dot\alpha\dot\beta}W_{\alpha\beta\gamma} + \frac{1}{16}\varepsilon_{\alpha\beta}(\bar{\mathcal{D}}_{\dot\alpha}G_{\gamma\dot\beta} + \bar{\mathcal{D}}_{\dot\beta}G_{\gamma\dot\alpha}) - $$
$$- \frac{1}{16}\varepsilon_{\dot\alpha\dot\beta}(\varepsilon_{\beta\gamma}\mathcal{D}_{\alpha}R + \varepsilon_{\alpha\gamma}\mathcal{D}_{\beta}R) \text{ and h.c.} \quad (cm^{-\frac{3}{2}}) \quad (2.7)$$

The components of the curvature are related to the torsion components[9], so, we shall give here the simplest ones only, that will be discussed below.

$$R_{\alpha\beta,\gamma\delta} = -\frac{1}{2}(\varepsilon_{\alpha\gamma}\varepsilon_{\beta\delta} + \varepsilon_{\alpha\delta}\varepsilon_{\beta\gamma})\bar{R} \text{ and h.c.}; \qquad (cm^{-1}) \quad (2.8a)$$

$$R_{\alpha\beta,\dot\gamma\dot\delta} = 0 \text{ and h.c.}; \qquad (cm^{-1}) \quad (2.8b)$$

$$R_{\alpha\dot\beta,CD} = 0. \qquad (cm^{-1}) \quad (2.9)$$

As it is seen from Eqs.(2.1)-(2.9) the components of torsion and curvature are all expressed in terms of the basic superfields $R, \bar{R}, G_{\alpha\dot\alpha}, W_{\alpha\beta\gamma}, \bar{W}_{\dot\alpha\dot\beta\dot\gamma}$ [2]. The latter have the following properties. R, $\bar{R}$, W, $\bar{W}$ are chiral:

$$\bar{\mathcal{D}}_{\dot\alpha}R = \mathcal{D}_{\alpha}\bar{R} = 0 \ (a), \quad \bar{\mathcal{D}}_{\dot\delta}W_{\alpha\beta\gamma} = \mathcal{D}_{\delta}\bar{W}_{\dot\alpha\dot\beta\dot\gamma} = 0 \ (b). \quad (2.10)$$

The superfield $G_{\alpha\dot\alpha}$ is Hermitian

$$G^{+}_{\alpha\dot\alpha} = G_{\alpha\dot\alpha}. \quad (2.11)$$

The superfields $W_{\alpha\beta\gamma}$, $\bar{W}_{\dot\alpha\dot\beta\dot\gamma}$ are totally symmetric in their indices, i.e., they realize Lorentz-group representations $(\frac{3}{2}, 0)$, $(0, \frac{3}{2})$, respectively. Finally, the following identities hold

$$\mathcal{D}^{\alpha}G_{\alpha\dot\alpha} = -\bar{\mathcal{D}}_{\dot\alpha}\bar{R} \ (a), \quad \bar{\mathcal{D}}_{\dot\alpha}G^{\dot\alpha}_{\alpha} = -\mathcal{D}_{\alpha}R \ (b); \quad (2.12)$$

$$\mathcal{D}^{\alpha}W_{\alpha\beta\gamma} = -\frac{1}{8}\bar{\mathcal{D}}_{\dot\alpha}(\mathcal{D}_{\gamma}G^{\dot\alpha}_{\delta} + \mathcal{D}_{\delta}G^{\dot\alpha}_{\gamma}) \text{ and h.c.} \quad (2.13)$$

The basic superfields can be expressed in terms of derivatives of $\mathcal{H}^{m}$:

$$R = \bar{\Delta}_{\dot\alpha}\bar{\Delta}^{\dot\alpha}(\bar{F}^{2}) \ (a), \quad \bar{R} = \Delta^{\alpha}\Delta_{\alpha}(F^{2}) \ (b); \quad (2.14)$$

$$G_{\alpha\dot\alpha} = 2\Big(F\Delta^{\beta}\bar{\omega}_{\dot\alpha\alpha\beta} + 4\bar{\omega}_{\dot\alpha\alpha}{}^{\beta}\Delta_{\beta}F + \frac{\bar{F}}{F}\omega_{\alpha\dot\alpha\dot\beta}\bar{\Delta}^{\dot\beta}F +$$
$$+ \omega^{\beta}{}_{\dot\beta\dot\alpha}\bar{\omega}^{\dot\beta}{}_{\beta\alpha} + 2\bar{F}\bar{\Delta}_{\dot\alpha}\Delta_{\alpha}F + \bar{F}\Delta_{\alpha}\bar{\Delta}_{\dot\alpha}F + \Delta_{\alpha}\bar{F}.\bar{\Delta}_{\dot\alpha}F + \frac{\bar{F}}{F}\Delta_{\alpha}F\bar{\Delta}_{\dot\alpha}\bar{F}\Big) \quad (2.15)$$

$$W_{\alpha\beta\gamma} = \frac{1}{3}(V_{\alpha\beta\gamma} + V_{\beta\gamma\alpha} + V_{\gamma\alpha\beta}), \quad \bar{W}_{\dot\alpha\dot\beta\dot\gamma} = (W_{\alpha\beta\gamma})^{+}$$

where

$$2V_{\alpha\beta\gamma} = -F\bar{F}\{\Delta_{\alpha}, \bar{\Delta}_{\dot\alpha}\}\bar{\omega}^{\dot\alpha}{}_{\beta\gamma} - F\bar{\omega}_{\dot\alpha\alpha}{}^{\delta}(\Delta_{\delta}\bar{\omega}^{\dot\alpha}{}_{\beta\gamma} +$$
$$+ \frac{1}{2}\Delta_{\beta}\bar{\omega}^{\dot\alpha}{}_{\delta\gamma} + \frac{1}{2}\Delta_{\gamma}\bar{\omega}^{\dot\alpha}{}_{\delta\beta}) + \Big(\frac{F}{\bar{F}}\Delta^{\delta}\bar{F}.\bar{\omega}_{\dot\alpha\alpha\delta} -$$
$$-F[\Delta_{\alpha}, \bar{\Delta}_{\dot\alpha}]\bar{F} - \bar{F}\Delta_{\alpha}\bar{\Delta}_{\dot\alpha}F + 2\frac{F}{\bar{F}}\Delta_{\alpha}\bar{F}.\bar{\Delta}_{\dot\alpha}\bar{F} + \frac{\bar{F}}{F}\Delta_{\alpha}F.\bar{\Delta}_{\dot\alpha}F\Big)\bar{\omega}^{\dot\alpha}{}_{\beta\gamma}. \quad (2.16)$$

The explicit form[6] of the operators Δ_{α}, $\bar{\Delta}_{\dot\alpha}$, spinor connections $\omega_{\alpha\dot\beta\dot\gamma}$, $\bar{\omega}_{\dot\alpha\beta\gamma}$ and factors F, $\bar{F}$, all in terms of $\mathcal{H}^{m}$, is given in Appendix B.

The NG-values of the basic superfields and of some of their covariant derivatives have already been given in Section 1, Eqs. (1.3)-(1.8).

Now, let us discuss the proofs.

3. Torsion Components with Dimension cm^{0} and $cm^{-\frac{1}{2}}$

In Ref.[6] we adopted the following definition of the vector covariant derivative

$$\mathcal{D}_{a} = \frac{i}{4}(\tilde{\sigma}_{a})^{\dot\alpha\alpha}\{\mathcal{D}_{\alpha}, \bar{\mathcal{D}}_{\dot\alpha}\}. \quad (3.1)$$

Comparing Eq.(3.1) with Eq.(1.2) one immediately gets Eq.(2.1a). Moreover, Eq.(2.1a) is unique, because in NG the only nonvanishing dimensionless derivative of $\mathcal{H}^{m}$ is the invariant tensor $\sigma^{a}_{\alpha\dot\alpha}$ (see Eq.(1.3)). For the same reason the dimensionless torsion components $T^{\ c}_{\alpha\beta}$ and $T^{\ c}_{\dot\alpha\dot\beta}$ vanish, as stated in Eq.(2.1b). Analogously, the absence of nonvanishing derivatives of $\mathcal{H}^{m}$ with dimension $cm^{-1/2}$ in NG proves the correctness and uniqueness of Eqs.(2.2), (2.3). The same arguments show the uniqueness of our definition of the spinor connections /6/ (see Eqs.(B.4), (B.5)). Two possible connections would have to differ by a tensor with dimension $cm^{-1/2}$.

Note that Eqs.(2.1)-(2.3) are part of the kinematic constraints of Wess and Zumino /1,2/. In our case they are automatically fulfilled. The last of the kinematic constraints, $T_{ab}{}^c = 0$ (dimension cm^{-1}) is not unique: It can be modified by redefining the vector connection (see Sect.6).

Finally, Eqs.(3.1) and (1.2) yield Eq.(2.9) too. Eq.(2.8b) means that in NG the tensor $R_{\alpha\beta,\mu\delta}$, symmetric in α,β and μ,δ cannot be constructed out of the nonvanishing derivatives with dimension cm^{-1} (Eqs.(1.4), (1.5)). Eq.(2.8b) leads to identity (C.1) when written out in detail.

4. Explicit Form of the Basic Superfields $R, \bar{R}$

To obtain an expression for $\bar{R}$ consider the anticommutator

$$\{\mathcal{D}_\alpha, \mathcal{D}_\beta\}\varphi_\mu = \mathcal{D}_\alpha(F\Delta_\beta\varphi_\mu + \omega_{\beta\mu}{}^\delta\varphi_\delta) + (\alpha \leftrightarrow \beta) =$$
$$= -\frac{1}{2}\Delta^\delta\Delta_\delta(F^2)(\varepsilon_{\alpha\mu}\varphi_\beta + \varepsilon_{\beta\mu}\varphi_\alpha). \tag{4.1}$$

Here Eqs.(B.1), (B.5), (B.9) were used. From Eq.(4.1) statements (2.8a), (2.14b) follow. The chirality of $\bar{R}$ (statement (2.10a) is obvious (see Eq.(B.9)). Note that Eq.(2.1b) has once again been derived.

Let us find the NG-value of $\bar{R}$. From the definition (B.3) of factor F follows

$$\Delta_\alpha F = F\left(-\frac{1}{3}\overset{1}{\tau}_m{}^a\Delta_\alpha\overset{1}{\tau}_a{}^m + \frac{1}{6}\overset{1}{\ell}_m{}^a\Delta_\alpha\overset{1}{\ell}_a{}^m\right). \tag{4.2}$$

Further, in NG (see Eq.(1.3))

$$\overset{1}{\tau}_a{}^m|_0 = -\overset{1}{\ell}_a{}^m|_0 = 2\delta_a{}^m, \quad \overset{1}{\tau}_m{}^a|_0 = -\overset{1}{\ell}_m{}^a|_0 = \frac{1}{2}\delta_m{}^a, \quad F|_0 = 1; \tag{4.3}$$

$$\Delta_\alpha\overset{1}{\tau}_a{}^m|_0 = \Delta_\alpha\Delta\sigma_a\bar{\Delta}\mathcal{H}^m|_0 = (\tilde{\sigma}_a)^{\beta\dot\beta}[\partial_\alpha\partial_\beta(-\bar\partial_{\dot\beta} - i\bar\partial_{\dot\beta}\mathcal{H}^n(1+i\mathcal{H})^{-1}{}_n{}^k\partial_k)\mathcal{H}^m]_0 =$$
$$= (\tilde{\sigma}_a)^{\beta\dot\beta}\partial_\alpha\partial_\beta\bar\partial_{\dot\beta}\mathcal{H}^m|_0 = 0; \tag{4.4}$$

similarly, $\Delta_\alpha\overset{1}{\ell}_a{}^m|_0 = 0$, i.e.,

$$\Delta_\alpha F|_0 = 0. \tag{4.5}$$

Then we find

$$\bar{R}|_0 = -\frac{1}{3}\Delta^\alpha\Delta_\alpha\overset{1}{\tau}_a{}^a|_0 - \frac{1}{6}\Delta^\alpha\Delta_\alpha\overset{1}{\ell}_a{}^a|_0 =$$
$$= -\frac{1}{3}\Delta\Delta.\Delta\sigma_a\bar{\Delta}\mathcal{H}^a|_0 - \frac{1}{6}\Delta\Delta.\bar{\Delta}\tilde{\sigma}_a\Delta\mathcal{H}^a|_0. \tag{4.6}$$

The first term contains three Δ's and vanishes according to Eq.(B.9). In the second term $\bar{\Delta}$ and Δ change their places with the help of Eq.(B.10). Finally, taking into account that the lower-order derivatives vanish, one gets

$$\bar{R}|_0 = \frac{i}{6}(\tilde{\sigma}_a)^{\dot\alpha\alpha}\left[\Delta^\beta\Delta_\beta([\Delta_\alpha,\bar{\Delta}_{\dot\alpha}]\mathcal{H}^m\partial_m\mathcal{H}^a)\right]_0 = -\frac{2i}{3}\partial^a\partial_a\partial_m\mathcal{H}^m|_0$$

which is just Eq.(1.4). The expression (1.8) for $\bar{\mathcal{D}}_{\dot\alpha}\bar{R}|_0$ is easily obtained in the same manner.

We have carried out the above calculations in such detail for illustration. In what follows many straightforward steps of these NG-calculations will be omitted.

5. Torsion Components $T_{\alpha,\beta\dot\beta}{}^C$ and Basic Superfield $G_{\alpha\dot\alpha}$

Consider the commutator

$$[\mathcal{D}_\alpha, \mathcal{D}_{\beta\dot\beta}]\varphi = \frac{i}{2}[\mathcal{D}_\alpha, \{\mathcal{D}_\beta, \bar{\mathcal{D}}_{\dot\beta}\}]\varphi = -T_{\alpha,\beta\dot\beta}{}^C\mathcal{D}_C\varphi. \tag{5.1}$$

This quantity is antisymmetric in α and β. Indeed,

$$[\mathcal{D}_\alpha, \{\mathcal{D}_\beta, \bar{\mathcal{D}}_{\dot\beta}\}]\varphi = \mathcal{D}_\alpha\mathcal{D}_\beta\bar{\mathcal{D}}_{\dot\beta}\varphi - \bar{\mathcal{D}}_{\dot\beta}\mathcal{D}_\beta\mathcal{D}_\alpha\varphi +$$
$$+ (\mathcal{D}_\alpha\bar{\mathcal{D}}_{\dot\beta}\mathcal{D}_\beta - \mathcal{D}_\beta\bar{\mathcal{D}}_{\dot\beta}\mathcal{D}_\alpha)\varphi. \tag{5.2}$$

The antisymmetry of the first two terms is due to Eqs.(1.2),(2.1), (2.8b), and the last two are obviously antisymmetric. Then,

$$T_{\alpha,\beta\dot\beta}{}^{c} = \frac{1}{2}\varepsilon_{\alpha\beta}T^{\varepsilon}{}_{,\varepsilon\dot\beta}{}^{c}, \tag{5.3}$$

and the indices α and β can be contracted when evaluating Eq.(5.1). Putting the explicit expressions (B.1) for $\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot\alpha}$ and (B.5) for $\omega_{\alpha\beta\dot\beta}$ into Eq.(5.1), and making some straightforward rearrangements, one gets

$$\begin{aligned}
[\mathcal{D}^\alpha,\{\mathcal{D}_\alpha,\bar{\mathcal{D}}_{\dot\alpha}\}]\varphi &= F\bar F\{F[\Delta^\alpha,\{\Delta_\alpha,\bar\Delta_{\dot\alpha}\}]+(2F\Delta^\alpha\ln\bar F+4\Delta^\alpha F)\{\Delta_\alpha,\bar\Delta_{\dot\alpha}\}-\\
&-2\omega_{\alpha\alpha\dot\beta}\{\Delta^\alpha,\bar\Delta^{\dot\beta}\}\}\varphi+\Big\{\frac{F^2}{\bar F}\Delta\Delta\bar F\varepsilon_{\dot\alpha\dot\beta}+F\Delta^\alpha\omega_{\alpha\dot\alpha\dot\beta}+\\
&+3\Delta^\alpha F.\omega_{\alpha\dot\alpha\dot\beta}+\frac{2F}{\bar F}\Delta^\alpha\bar F.\omega_{\alpha\dot\alpha\dot\beta}+\omega^\alpha{}_{\dot\alpha\dot\rho}\omega_\alpha{}^{\dot\rho}{}_{\dot\beta}+\\
&+\frac{4F}{\bar F}\Delta^\alpha F.\Delta_\alpha F\varepsilon_{\dot\alpha\dot\beta}\Big\}\bar{\mathcal{D}}^{\dot\beta}\varphi+\\
&+2\Big\{F\Delta^\alpha\bar\omega_{\dot\alpha\alpha}{}^{\beta}+4\bar\omega_{\dot\alpha}{}^{\beta\alpha}\Delta_\alpha F+\frac{\bar F}{F}\omega^\beta{}_{\dot\alpha\dot\beta}\bar\Delta^{\dot\beta}F+\\
&+\omega^\delta{}_{\dot\beta\dot\alpha}\bar\omega^{\dot\beta}{}_\delta{}^\beta+2\bar F\bar\Delta_{\dot\alpha}\Delta^\beta F+\bar F\Delta^\beta\bar\Delta_{\dot\alpha}F+\\
&+\Delta^\beta\bar F.\bar\Delta_{\dot\alpha}F+\frac{\bar F}{F}\Delta^\beta F.\bar\Delta_{\dot\alpha}\bar F\Big\}\mathcal{D}_\beta\varphi.
\end{aligned} \tag{5.4}$$

With the help of Eqs.(B.10), (B.11) one can easily show that all the terms in the first braces in Eq.(5.4) are proportional to the spatial derivative. So, according to Eqs.(1.2), (B.1), (B.10) these terms form the torsion component $T^\alpha{}_{,\alpha\dot\beta}{}^{c}$. However, the latter has already been shown to vanish (Eq.(2.3)).

The second braces in Eq.(5.4) determine the component $T^\alpha{}_{,\alpha\dot\beta,\dot\beta}$. Let us find its NG-value. One of the higher-order-derivative terms (the lower-order ones vanish), $F\Delta^\alpha\omega_{\alpha\dot\beta\dot\beta}$, equals zero in NG (see Eq.(B.6a)). The second one gives

$$\Big[\frac{F^2}{\bar F}\Delta\Delta\bar F\Big]_0\varepsilon_{\dot\beta\dot\beta} = \frac{2i}{3}\partial^\alpha\partial_\alpha\partial_m H^m\big|_0\,\varepsilon_{\dot\beta\dot\beta}. \tag{5.5}$$

Comparing Eq.(5.5) with Eq.(1.4) and recalling the tensor character of the quantities considered one proves statement (2.5). Identity (C.2) is simultaneously obtained also.

The third braces in Eq.(5.4) determine $T^\alpha{}_{,\alpha\beta,}{}^{\beta}$ so, that statements (2.4) and (2.15) are confirmed. The NG-value of $G_{\alpha\dot\alpha}$ (1.5) is obtained considering only the terms $\Delta^\alpha\bar\omega_{\dot\alpha\alpha\beta}$, $\bar\Delta_{\dot\alpha}\Delta_\beta F$ and $\Delta_\beta\bar\Delta_{\dot\alpha}F$, the remaining ones vanish in NG. Eq.(1.5) clearly shows that $G_{\alpha\dot\alpha}$ is Hermitian, i.e., that Eq.(2.11) holds. The vanishing of the antihermitian part of $G_{\alpha\dot\alpha}$ (2.15) leads to identity (C.3).

Consider $\mathcal{D}^\alpha G_{\alpha\dot\alpha}$ in NG:

$$\begin{aligned}
\mathcal{D}^\alpha G_{\alpha\dot\alpha}\big|_0 &= 2\big[\Delta^\alpha\Delta^\beta\bar\omega_{\dot\alpha\beta\alpha}+2\Delta^\alpha\bar\Delta_{\dot\alpha}\Delta_\alpha F+\Delta^\alpha\Delta_\alpha\bar\Delta_{\dot\alpha}F\big]_0 = \\
&= -\frac{2i}{3}\partial^\alpha\partial_\alpha\bar\partial_{\dot\alpha}\partial_m H^m\big|_0 = -\bar{\mathcal{D}}_{\dot\alpha}\bar R\big|_0
\end{aligned} \tag{5.6}$$

according to Eq.(1.8). Thus, statement (2.12a) is also proved. Finally, Eq.(1.7) can easily ba verified.

6. Torsion Components $T_{ab}{}^{c}$

From Eqs.(1.1), (1.2) one finds

$$T_{ab}{}^{c} = E_a{}^M\partial_M E_b{}^N E_N{}^c + \omega_{ab}{}^c - (a\leftrightarrow b). \tag{6.1}$$

In NG the supervierbeins $E_A{}^M$ turn into $\delta_A{}^M$ /8/, hence the first term in Eq.(6.1) includes the spatial derivative ∂_a. However, according to Eqs.(1.4), (1.5) in NG there are no appropriate derivatives with dimension cm^{-1} containing ∂_a. Therefore, (see Eqs.(B.7), (1.5))

$$T_{ab}{}^c\big|_0 = [\omega_{ab}{}^c - \omega_{ba}{}^c]_0 = -\frac{1}{8}\eta^{cd}\varepsilon_{abde}G^e\big|_0\,. \tag{6.2}$$

$T_{ab}{}^c$ and G^e are tensors, so Eq.(6.2), i.e. Eq.(2.6) holds in an arbitrary gauge.

Stress that the vector connection ω_{abc} is not defined uniquely. There is a tensor, $G_{\alpha\dot\alpha}$, with the same dimension cm^{-1} that can be added to ω_{abc}. For instance, if one chooses

$$\omega'_{abc} = \omega_{abc} + \frac{1}{4}\varepsilon_{abcd}G^d \tag{6.3}$$

one obtains $T'_{ab}{}^c = 0$, i.e., just the last kinematic constraint in Refs./1,2/. So, one sees that this constraint, being correct, is not unique. In our case, to obtain $T_{ab}{}^{c}=0$ we have to adopt the vector covariant derivative

$$\mathcal{D}_a = \frac{1}{4}(\tilde{\sigma}_a)^{\dot{\alpha}\alpha}\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot{\alpha}}\} + \frac{1}{4}\varepsilon_{abcd}G^d \Lambda^{bc} \tag{6.4}$$

instead of the natural definition (3.1).

7. Torsion Component $T_{ab}{}^{\mu}$ and Basic Superfield $W_{\alpha\beta\mu}$

Now, from Eqs.(1.1), (1.2) follows

$$T_{ab}{}^{\mu} = E_a{}^M \partial_M E_b{}^N E_N{}^{\mu} - (a \leftrightarrow b) \tag{7.1}$$

since $\omega_{ab}{}^{\mu}$ (or $\omega_{ba}{}^{\mu}$) does not exist. In NG

$$T_{ab}{}^{\mu}|_0 = \partial_a E_b{}^{\mu}|_0 - \partial_b E_a{}^{\mu}|_0 . \tag{7.2}$$

The explicit form of $E_a{}^{\mu}$ was given in Ref.[6] (it can easily be obtained from Eqs.(3.1), (B.1)). Putting it into Eq.(7.2) one finds (in spinor notation)

$$T_{\alpha\dot{\alpha},\beta\dot{\beta},\mu}|_0 = (\sigma^a)_{\alpha\dot{\alpha}}(\sigma^b)_{\beta\dot{\beta}}\varepsilon_{\mu\delta}T_{ab}{}^{\delta}|_0 =$$
$$= \frac{i}{2}\partial_{\alpha\dot{\alpha}}(\varepsilon_{\beta\mu}\bar{\partial}_{\dot{\beta}}F + \bar{\omega}_{\dot{\beta}\beta\mu})|_0 - (\alpha\dot{\alpha} \leftrightarrow \beta\dot{\beta}) =$$
$$= \frac{i}{8}\varepsilon_{\dot{\alpha}\dot{\beta}}\underset{(\alpha\beta\mu)}{\mathrm{Sym}}\bar{\partial}_{\dot{\nu}}\bar{\partial}^{\dot{\nu}}\partial_\alpha\partial_{\beta\dot{\beta}}\mathcal{H}^{\dot{\mu}}_{\mu}|_0 - \frac{i}{24}\varepsilon_{\alpha\beta}\underset{(\dot{\alpha}\dot{\beta})}{\mathrm{Sym}}\bar{\partial}_{\dot{\nu}}\bar{\partial}^{\dot{\nu}}\partial_{\dot{\alpha}\nu}\partial^{\nu}\mathcal{H}_{\mu\dot{\beta}}|_0 - \tag{7.3}$$
$$-\frac{i}{24}\varepsilon_{\dot{\alpha}\dot{\beta}}(\varepsilon_{\beta\mu}\bar{\partial}_{\dot{\nu}}\bar{\partial}^{\dot{\nu}}\partial_\alpha\partial_m\mathcal{H}^m|_0 + \varepsilon_{\alpha\mu}\bar{\partial}_{\dot{\nu}}\bar{\partial}^{\dot{\nu}}\partial_\beta\partial_m\mathcal{H}^m|_0).$$

The second term in the last equation is identified with $\underset{(\dot{\alpha}\dot{\beta})}{\mathrm{Sym}}\bar{\mathcal{D}}_{\dot{\alpha}}G_{\mu\dot{\beta}}|_0$ (see Eq.(1.7)), and the third one with $\underset{(\alpha\beta)}{\mathrm{Sym}}\varepsilon_{\beta\mu}\mathcal{D}_\alpha R|_0$ (see Eq.(1.8)). So, Eq.(7.3) establishes the coefficients of these superfields in Eq.(2.7). The first term in Eq.(7.3) determines a new quantity, $W_{\alpha\beta\mu}$, which is to be examined now. Eq.(7.3) shows that $W_{\alpha\beta\mu}$ is totally symmetric in α, β, μ. To obtain an explicit expression for $W_{\alpha\beta\mu}$ one has to evaluate the commutator (see Eqs.(1.2), (3.1))

$$\frac{1}{4}[\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot{\alpha}}\}, \{\mathcal{D}_\beta, \bar{\mathcal{D}}_{\dot{\beta}}\}]\varphi = T_{\alpha\dot{\alpha},\beta\dot{\beta}}{}^{C}\mathcal{D}_C\varphi . \tag{7.4}$$

When doing this, one can keep the terms proportional to $\mathcal{D}_\mu\varphi = F\Delta_\mu\varphi$ only (do not forget that $\Delta_\mu\varphi$ takes part in the derivative $\mathcal{D}_c\varphi$ too). Further, one can also contract the indices $\dot{\alpha}$ and $\dot{\beta}$ at the beginning, and omit all the terms antisymmetric in any pair of the indices α, β, μ. After all that one obtains $W_{\alpha\beta\mu}$ as in Eq.(2.16). Note that $W_{\alpha\beta\mu}$ can also be found by evaluating the curvature component

$$R_{\dot{\alpha},\beta\dot{\beta},\alpha\mu} = -i\varepsilon_{\dot{\alpha}\dot{\beta}}W_{\alpha\beta\mu} . \tag{7.5}$$

The chirality of $W_{\alpha\beta\mu}$ (2.10b) is easily proved in NG. Indeed, in NG the spinor connections and terms such as $\Delta_\alpha F$ vanish, and so does the term

$$\bar{\Delta}_{\dot{\mu}}\bar{\omega}_{\dot{\alpha}\beta\mu}|_0 = -\frac{1}{8}\bar{\Delta}_{\dot{\mu}}\bar{\Delta}\bar{\Delta}\Delta_\beta\mathcal{H}_{\mu\dot{\alpha}}|_0 + (\beta \leftrightarrow \mu) = 0 \tag{7.6}$$

due to Eq.(B.6b). So, verifying Eq.(2.10b) one has only to look at the term

$$\bar{\Delta}_{\dot{\mu}}\{\Delta_\alpha, \bar{\Delta}_{\dot{\alpha}}\}\bar{\omega}^{\dot{\alpha}}{}_{\beta\mu}|_0 = -2i\,\partial_{\alpha\dot{\alpha}}\bar{\Delta}_{\dot{\mu}}\bar{\omega}^{\dot{\alpha}}{}_{\beta\mu}|_0 =$$
$$= \frac{i}{4}\partial_{\alpha\dot{\alpha}}\bar{\Delta}_{\dot{\mu}}\bar{\Delta}\bar{\Delta}\Delta_\beta\mathcal{H}_{\mu\dot{\alpha}}|_0 + (\beta \leftrightarrow \mu) = 0 . \tag{7.7}$$

Here Eqs.(B.10), (B.11), (B.6b) were used.

Finally, identity (2.13) can also be derived in NG.

8. Conclusion

So, the statements made in Section 2 have been proved. The normal gauge has frequently and effectively been used in the proof. It has simplified the considerations and underlined the geometric nature of the axial gravitational superfield $\mathcal{H}^m$. The main aim of the paper has been to demonstrate the abilities of our geometric approach in deriving the explicit form of all the components of torsion and curvature and establishing identities between them. The results obtained agree with those of Grimm, Wess and Zumino/2/. Starting from a more adequate group in superspace we have derived the constraints they had guessed. One of those constraints, $T_{ab}{}^{c}=0$, has turned out not to be unique (see Sect.6).

The results of the present paper concern the case of $N=1$ - supergravity. We hope that in extended supergravity the search for an adequate group as a framework for a minimal geometric approach will succeed. Then the methods described above could be generalized.

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Appendix A. Notation

The following notation is used in the paper. The world vector $(m, n, \dots)$ and spinor $(\mu, \nu, \dots; \dot\mu, \dot\nu, \dots)$ indices are combined into $M, N, \dots$, and the Lorentz ones $(a, \alpha, \dot\alpha; b, \beta, \dot\beta, \dots)$ into $A, B, \dots$. The Lorentz vector indices are raised and lowered by $\eta^{ab}=\mathrm{diag}(+---)$, the spinor ones by $\varepsilon^{\alpha\beta}, \varepsilon^{\dot\alpha\dot\beta}$ $(\varepsilon^{12}=-\varepsilon^{21}=-\varepsilon_{12}=\varepsilon^{\dot1\dot2}=1)$. The contraction rule is: $\varphi^A\psi_A=\varphi^a\psi_a+\varphi^\alpha\psi_\alpha+\varphi_{\dot\alpha}\psi^{\dot\alpha}$.

The vector indices are often represented as a pair of spinor ones

$$\varphi_{\alpha\dot\alpha}=(\sigma^a)_{\alpha\dot\alpha}\varphi_a,\quad \varphi^{\dot\alpha\alpha}=(\tilde\sigma^a)^{\dot\alpha\alpha}\varphi_a,\quad \varphi^{\alpha\dot\alpha}\varphi_{\alpha\dot\alpha}=2\varphi^a\varphi_a$$

using 2×2 matrices $\sigma^a=(1,\vec\sigma)$, $\tilde\sigma^a=(1,-\vec\sigma)$, $\vec\sigma$ being the Pauli matrices. Further,

$$(\sigma_{ab})_{\alpha\beta}=\frac{i}{2}[\sigma_a,\tilde\sigma_b]_{\alpha\beta},\quad (\tilde\sigma_{ab})^{\dot\alpha\dot\beta}=\frac{i}{2}[\tilde\sigma_a,\sigma_b]^{\dot\alpha\dot\beta};$$

$$(\sigma_a\tilde\sigma_b)_{\alpha\beta}=-\eta_{ab}\varepsilon_{\alpha\beta}-i(\sigma_{ab})_{\alpha\beta};\quad (\sigma_{ab})_{\alpha\beta}=(\sigma_{ab})_{\beta\alpha};$$

$$(\sigma^a)_{\alpha\dot\alpha}(\sigma_a)_{\beta\dot\beta}=2\varepsilon_{\alpha\beta}\varepsilon_{\dot\alpha\dot\beta},\quad (\sigma_{ab})_\alpha{}^\beta(\tilde\sigma^{ab})_{\dot\gamma}{}^{\dot\delta}=0,$$

$$(\sigma_{ab})_\alpha{}^\beta(\sigma^{ab})_\gamma{}^\delta=4(2\delta_\alpha^\delta\delta_\gamma^\beta-\delta_\alpha^\beta\delta_\gamma^\delta).$$

The derivatives at a point $z^N=(x^n,\theta^\nu,\bar\theta^{\dot\nu})$ are abbreviated by

$$\partial_N=\frac{\partial}{\partial z^N},\quad \partial_n=\frac{\partial}{\partial x^n},\quad \partial_{\nu\dot\nu}=(\sigma^n)_{\nu\dot\nu}\frac{\partial}{\partial x^n},$$

$$\partial_\nu=\frac{\partial}{\partial\theta^\nu},\quad \bar\partial_{\dot\nu}=\frac{\partial}{\partial\bar\theta^{\dot\nu}},\quad \partial^\nu=\frac{\partial}{\partial\theta_\nu}=-\varepsilon^{\nu\mu}\partial_\mu,\quad \bar\partial^{\dot\nu}=-\varepsilon^{\dot\nu\dot\mu}\bar\partial_{\dot\mu}.$$

The symbol Sym means symmetrization over the indices indicated below, e.g.,

$$\mathop{\mathrm{Sym}}_{(\alpha\beta)}A_{\alpha\beta}=\frac{1}{2}(A_{\alpha\beta}+A_{\beta\alpha})$$

$$\mathop{\mathrm{Sym}}_{(\alpha\beta\gamma)}A_{\alpha\beta\gamma}=\frac{1}{6}(A_{\alpha\beta\gamma}+A_{\alpha\gamma\beta}+A_{\beta\alpha\gamma}+A_{\beta\gamma\alpha}+A_{\gamma\beta\alpha}+A_{\gamma\alpha\beta}).$$

Appendix B. Connections and Operators $\Delta, \bar\Delta$

Here we list some of the results of Ref./6/. The spinor covariant derivatives look like

$$\mathcal{D}_\alpha\varphi_B=F\Delta_\alpha\varphi_B+\omega_{\alpha B}{}^C\varphi_C,$$
$$\bar{\mathcal{D}}_{\dot\alpha}\varphi_B=\bar F\bar\Delta_{\dot\alpha}\varphi_B+\bar\omega_{\dot\alpha B}{}^C\varphi_C, \tag{B.1}$$

where

$$\Delta_\alpha=\partial_\alpha+i\partial_\alpha\mathcal{H}^n(1-i\mathcal{H})^{-1}{}_n{}^m\partial_m=\partial_\alpha+i\Delta_\alpha\mathcal{H}^m\partial_m,$$

$$\bar\Delta_{\dot\alpha}=-\bar\partial_{\dot\alpha}+i\bar\partial_{\dot\alpha}\mathcal{H}^n(1+i\mathcal{H})^{-1}{}_n{}^m\partial_m=-\bar\partial_{\dot\alpha}-i\bar\Delta_{\dot\alpha}\mathcal{H}^m\partial_m. \tag{B.2}$$

$$\mathcal{H}_n{}^m=\partial_n\mathcal{H}^m.$$

The factors $F, \bar F$ are*)

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*) There was an error in Ref./6/. The factors $2^{2/3}$ in the definitions of F and $\bar F$ were omitted.

$$F = 2^{\frac{2}{3}} \det{}^{-\frac{1}{3}}(\hat{\tau}_a{}^m) . \det{}^{\frac{1}{6}}(\hat{\ell}_a{}^m) ,$$
$$\bar{F} = 2^{\frac{2}{3}} \det{}^{-\frac{1}{3}}(\hat{\ell}_a{}^m) . \det{}^{\frac{1}{6}}(\hat{\tau}_a{}^m) . \qquad \text{(B.3)}$$

where

$$\hat{\tau}_a{}^m = \Delta \sigma_a \bar{\Delta} \mathcal{H}^m , \quad \hat{\ell}_a{}^m = \bar{\Delta} \tilde{\sigma}_a \Delta \mathcal{H}^m . \qquad \text{(B.4)}$$

Spinor connections:

$$\omega_{\alpha\beta\gamma} = \varepsilon_{\alpha\beta} \Delta_\gamma F + \varepsilon_{\alpha\gamma} \Delta_\beta F ,$$
$$\bar{\omega}_{\dot{\alpha}\dot{\beta}\dot{\gamma}} = -\varepsilon_{\dot{\alpha}\dot{\beta}} \bar{\Delta}_{\dot{\gamma}} \bar{F} - \varepsilon_{\dot{\alpha}\dot{\gamma}} \bar{\Delta}_{\dot{\beta}} \bar{F} ; \qquad \text{(B.5)}$$

$$\omega_{\alpha\dot{\beta}\dot{\gamma}} = \frac{i}{2} F (\Delta_\alpha \hat{\tau}_a{}^m . \hat{\tau}_{mb}) (\tilde{\sigma}^{ab})_{\dot{\beta}\dot{\gamma}} =$$
$$= \frac{1}{4} F \Delta\Delta \bar{\Delta}_{\dot{\beta}} \mathcal{H}^m . \hat{\tau}_{m,\alpha\dot{\gamma}} + (\dot{\beta} \leftrightarrow \dot{\gamma}) , \qquad \text{(B.6a)}$$

$$\bar{\omega}_{\dot{\alpha}\beta\gamma} = \frac{1}{4} \bar{F} \bar{\Delta}\bar{\Delta} \Delta_\beta \mathcal{H}^m . \hat{\ell}_{m,\gamma\dot{\alpha}} + (\beta \leftrightarrow \gamma) . \qquad \text{(B.6b)}$$

where

$$\hat{\tau}_a{}^m \hat{\tau}_m{}^b = \delta_a{}^b , \quad \hat{\ell}_a{}^m \hat{\ell}_m{}^b = \delta_a{}^b .$$

Vector connection:

$$\omega_{a\,BC} = \frac{i}{4} (\tilde{\sigma}_a)^{\dot{\alpha}\alpha} \left[F \Delta_\alpha \bar{\omega}_{\dot{\alpha}BC} + \bar{F} \bar{\Delta}_{\dot{\alpha}} \omega_{\alpha BC} + \right.$$
$$\left. + \omega_{\alpha\dot{\alpha}\dot{\delta}} \bar{\omega}^{\dot{\delta}}{}_{BC} + \bar{\omega}_{\dot{\alpha}\alpha}{}^{\delta} \omega_{\delta BC} \right] . \qquad \text{(B.7)}$$

Conjugation rules:

$$(\Delta_\alpha A . \bar{\Delta}_{\dot{\beta}} B)^+ = \Delta_\beta B^+ . \bar{\Delta}_{\dot{\alpha}} A^+ , \quad (\Delta_\alpha \bar{\Delta}_{\dot{\beta}} A)^+ = -\bar{\Delta}_{\dot{\alpha}} \Delta_\beta A^+ , \qquad \text{(B.8)}$$
$$(\omega_{\alpha\beta\gamma})^+ = -\bar{\omega}_{\dot{\alpha}\dot{\beta}\dot{\gamma}} \quad , \text{etc.}$$

Identities for the operators $\Delta , \bar{\Delta}$:

$$\{\Delta_\alpha , \Delta_\beta\} = \{\bar{\Delta}_{\dot{\alpha}} , \bar{\Delta}_{\dot{\beta}}\} = 0 , \quad \Delta_\alpha \Delta_\beta \Delta_\gamma = \bar{\Delta}_{\dot{\alpha}} \bar{\Delta}_{\dot{\beta}} \bar{\Delta}_{\dot{\gamma}} = 0 ; \qquad \text{(B.9)}$$

$$\{\Delta_\alpha , \bar{\Delta}_{\dot{\beta}}\} = -i [\Delta_\alpha , \bar{\Delta}_{\dot{\beta}}] \mathcal{H}^m \partial_m =$$
$$= -2i \Delta_\alpha \bar{\Delta}_{\dot{\beta}} \mathcal{H}^n (1 - i\mathcal{H})^{-1}{}_n{}^m \partial_m = 2i \bar{\Delta}_{\dot{\beta}} \Delta_\alpha \mathcal{H}^n (1 + i\mathcal{H})^{-1}{}_n{}^m \partial_m ; \qquad \text{(B.10)}$$

$$[\partial_m , \Delta_\alpha] = i \partial_m \Delta_\alpha \mathcal{H}^n \partial_n = i \Delta_\alpha \partial_m \mathcal{H}^k (1 - i\mathcal{H})^{-1}{}_k{}^n \partial_n , \qquad \text{(B.11)}$$
$$[\partial_m , \bar{\Delta}_{\dot{\alpha}}] = -i \partial_m \bar{\Delta}_{\dot{\alpha}} \mathcal{H}^n \partial_n = -i \bar{\Delta}_{\dot{\alpha}} \partial_m \mathcal{H}^k (1 + i\mathcal{H})^{-1}{}_k{}^n \partial_n .$$

Appendix C. Identities for ω_α and ΔF

The vanishing of the curvature component $R_{\alpha\beta,\dot{\gamma}\dot{\delta}}$ (Eq.(2.8)) yields the identity

$$F^2 \Delta_\alpha \left(\frac{1}{F} \omega_{\beta\dot{\gamma}\dot{\delta}} \right) + \omega_{\alpha\dot{\gamma}\dot{\rho}} \omega_\beta{}^{\dot{\rho}}{}_{\dot{\delta}} + (\alpha \leftrightarrow \beta) = 0 . \qquad \text{(C.1)}$$

Similarly, the derivation of Eq.(2.5) gives, by the way, the identity

$$\Delta^\alpha \omega_{\alpha\dot{\gamma}\dot{\delta}} + \Delta^2 \ln F^3 \bar{F}^2 . \omega_{\alpha\dot{\gamma}\dot{\delta}} = 0 . \qquad \text{(C.2)}$$

Finally, the hermiticity of $G_{\alpha\dot{\alpha}}$ (Eqs.(2.11), (2.15)) means that

$$F \Delta^\beta \bar{\omega}_{\dot{\alpha}\alpha\beta} + \bar{F} \bar{\Delta}_{\dot{\beta}} \omega_{\alpha\dot{\alpha}}{}^{\dot{\beta}} + 4 \bar{\omega}_{\dot{\alpha}\alpha}{}^{\beta} \Delta_\beta F + 4 \omega_{\alpha\dot{\alpha}\dot{\beta}} \bar{\Delta}^{\dot{\beta}} \bar{F} +$$
$$+ \frac{\bar{F}}{F} \omega_{\alpha\dot{\alpha}\dot{\beta}} \bar{\Delta}^{\dot{\beta}} F + \frac{F}{\bar{F}} \bar{\omega}_{\dot{\alpha}\alpha}{}^{\beta} \Delta_\beta \bar{F} + 2 \bar{F} \bar{\Delta}_{\dot{\alpha}} \Delta_\alpha F + 2 F \Delta_\alpha \bar{\Delta}_{\dot{\alpha}} \bar{F} +$$
$$+ \bar{F} \Delta_\alpha \bar{\Delta}_{\dot{\alpha}} F + F \bar{\Delta}_{\dot{\alpha}} \Delta_\alpha \bar{F} + \frac{\bar{F}}{F} \Delta_\alpha F . \bar{\Delta}_{\dot{\alpha}} \bar{F} + \frac{F}{\bar{F}} \bar{\Delta}_{\dot{\alpha}} \bar{F} . \Delta_\alpha F = 0 . \qquad \text{(C.3)}$$

All these identities can also be proved directly without using NG.

Added Notes

1. Recently[10] the nonuniqueness of the constraint $T_{ab}{}^c = 0$ (discussed in Sect.6 of the present paper) was demonstrated on the component-field level too. Our choice (Eq.(2.6)) was shown

to be related to the introduction of an "improved" ordinary-field connection $\hat{\omega}(e,\psi)$.

2. In a recent paper [11] the constraints for conformal supergravity have been discussed. We would like to point out that they can easily be derived in the framework described in this paper. As shown in Ref. [5], the conformal supergravity group is the general coordinate transformation group in the left and right chiral superspaces. The Einstein's case [12] is obtained when invariance of the supervolume is required in addition. As mentioned in Ref. [8], dropping this restriction one can fix the normal gauge so, that the derivatives (1.4), (1.5), (1.7), (1.8) vanish at point z_0 too. Then, following the arguments of this paper one concludes that the only nonvanishing torsion component is

$$\underset{(\alpha\beta\mu)}{\mathrm{Sym}}\, T_{\alpha}{}^{\dot{\alpha}}{}_{,\beta\dot{\alpha},\mu} = W_{\alpha\beta\mu} .$$

This fact explains why higher-order equations of motion are unavoidable in conformal supergravity.

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