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NORMAL GAUGE IN SUPERGRAVITY

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In the theory of gravitation local geodesic coordinate frames are well known and widely used. In such a frame (or a gauge, in the field-theory language) at a given arbitrary point with coordinates x_0^m the following equations take place

$$g_{mn}(x_0) = \eta_{mn} \, , \quad \left(\frac{\partial g_{mn}}{\partial x^k}\right)_{x_0} = 0 . \tag{0.1}$$

Here $\eta_{mn} = diag(+---)$ is the flat-space metric. The second equation means that the connection coefficients $\Gamma_{mn}^{\ \ k}(x_0)$ vanish. One of the advantages of this gauge is that various tensor (i.e., gauge-independent) relations can easily be proved. The derivation of the Einstein equation from the action principle given in the Landau and Lifshitz textbook/1/, is a good example.

The gauge (0.1) can further be specified. There exists a coordinate frame in which the expansion of the metric tensor in a vicinity of the point x_0 has the form

$$\begin{aligned} g_{mn}(x) = \eta_{mn} - \tfrac{1}{3} R_{mknl}(x_0) \cdot (x-x_0)^k (x-x_0)^l - \\ - \tfrac{1}{3!} R_{mknl,p}(x_0) \cdot (x-x_0)^k (x-x_0)^l (x-x_0)^p + \ldots \; . \end{aligned} \tag{0.2}$$

The subsequent terms in Eq.(0.2) can also be expressed in terms of the Riemann tensor and its covariant derivatives/2/. Such coordinates are called normal at point x_0 . They are used in the analysis of divergences in gravity, etc.

It is quite natural to suppose that analogous gauges exist in supergravity too. They are a matter of considerable interest in the framework of our approach[3] where all the quantities (supervierbeins, connections, torsion, curvature) are expressed in terms of derivatives of a single axial superfield $\mathcal{H}^m(x,\theta,\bar\theta)$. In this approach, on the one hand, the group structure is simple and transparent, so different gauges can easily and fully be investigated. On the other hand, the explicit expressions in terms of $\mathcal{H}^m$ for such quantities as the torsion tensor $T_{AB}{}^C$ are rather complicated, so a gauge of the type (0.1), (0.2) considerably simplifies the analysis of a number of tensor relations.

In the present paper a gauge is proposed that we shall call "normal" (by analogy with Eq.(0.2)). We are going to use it effectively many times in our future papers*). It provides a simple way to express the components of torsion in terms of $\mathcal{H}^m$ and to prove the relations between them without using the Bianchi identities. The variational procedure for the supergravity action is carried out quite easily with its help. Perhaps similar gauge will prove to be highly constructive in extended supergravity too.

Now we are going to formulate the results and then to prove them.

Starting from the transformation laws for the superspace coordinates and the gravitational superfield $\mathcal{H}^m(x,\theta,\bar\theta)$ (see Eq.(0.5) below) one can fix the gauge in such a way that the expansion of $\mathcal{H}^m$ in a vicinity of a given point $z_0^M=(x_0^m,\theta_0^\mu,\bar\theta_0^{\dot\mu})$ becomes

$$\mathcal{H}^m(x,\theta,\bar\theta)=\frac{1}{2}(\theta-\theta_0)^\nu(\sigma^m)_{\nu\dot\nu}(\bar\theta-\bar\theta_0)^{\dot\nu}+$$

$$+\frac{3i}{32}(\theta-\theta_0)^\nu(\theta-\theta_0)_\nu(x-x_0)^m\bar R-$$

$$-\frac{3}{32}(\theta-\theta_0)^\nu(\theta-\theta_0)_\nu(\bar\theta-\bar\theta_0)_{\dot\nu}(\bar\theta-\bar\theta_0)^{\dot\nu}G^m-$$

*) As Prof. A.S.Schwarz informed us, he had partially used our gauge to derive the invariant action for supergravity avoiding the differential geometry formalism[4].

$$-\frac{1}{8}(\bar\theta-\bar\theta_0)_{\dot\nu}(\bar\theta-\bar\theta_0)^{\dot\nu}(\theta-\theta_0)^\nu(x-x_0)^\kappa(\sigma_\kappa{}^m)^{\lambda\rho}W_{\nu\lambda\rho}- \quad (0.3)$$

$$-\frac{3i}{32}(\bar\theta-\bar\theta_0)_{\dot\nu}(\bar\theta-\bar\theta_0)^{\dot\nu}(\theta-\theta_0)^\nu(x-x_0)^\kappa(\sigma^n\tilde\sigma^m\sigma_\kappa)_{\nu\dot\mu}\bar{\mathcal{D}}^{\dot\mu}G_n+$$

$$+h.c.+\ldots$$

(h.c. means hermitian conjugated terms). The meaning of the quantities R, $\bar R$, G, W, $\bar W$ will be cleared up in a forthcoming paper. They will turn out to be the point-z_0-values of those basic superfields in terms of which all the components of torsion and curvature tensors are expressed[5]. The further terms in Eq. (0.3) can also be written down as covariant derivatives and products of those quantities.

The gauge (0.3) can be rewritten in a form, more convenient for applications

$$(\mathcal{H}^m)_0=0, \quad (0.4.1)$$

$$(\partial_N\mathcal{H}^m)_0=0, \quad (0.4.2)$$

$$(\partial_\nu\bar\partial_{\dot\varkappa}\mathcal{H}^m)_0=-(\sigma^m)_{\nu\dot\varkappa}\,, \quad (0.4.3)$$

$$(\partial_\nu\partial_\varkappa\mathcal{H}^m)_0=0, \quad (0.4.4)$$

$$(\partial_N\partial_\kappa\mathcal{H}^m)_0=0,\quad (\partial_\nu\partial_\varkappa\bar\partial_{\dot\lambda}\mathcal{H}^m)_0=0, \quad (0.4.5)$$

$$(\partial_\nu\partial_\varkappa\partial_\ell\mathcal{H}^m)_0=0 \qquad \text{except}\ (\partial_\nu\partial_\varkappa\partial_m\mathcal{H}^m)_0=-\frac{3i}{4}\varepsilon_{\nu\varkappa}\bar R, \quad (0.4.6)$$

$$(\partial_\nu\bar\partial_{\dot\varkappa}\partial_\ell\mathcal{H}^m)_0=0, \quad (0.4.7)$$

$$(\partial_\nu\partial_\varkappa\bar\partial_{\dot\lambda}\bar\partial_{\dot\rho}\mathcal{H}^m)_0=\frac{3}{4}\varepsilon_{\nu\varkappa}\varepsilon_{\dot\lambda\dot\rho}G^m, \quad (0.4.8)$$

$$(\partial_\nu \partial_\kappa \partial_\varrho \mathcal{H}^m)_0 = 0, \qquad (0.4.9)$$

$$(\bar\partial_{\dot\rho} \bar\partial_{\dot\lambda} \partial_\nu \partial_\kappa \mathcal{H}^m)_0 = 0 \quad \text{except} \qquad (0.4.10)$$

$$\underset{(\nu\lambda\mu)}{\mathrm{Sym}} (\bar\partial_{\dot\rho} \bar\partial^{\dot\rho} \partial_\nu \partial_\kappa \mathcal{H}^m)_0 (\sigma^\kappa \tilde\sigma_m)_{\lambda\mu} = -4i W_{\nu\lambda\mu} ,$$

$$\underset{(\dot\nu\dot\mu)}{\mathrm{Sym}} (\bar\partial_{\dot\rho} \bar\partial^{\dot\rho} \partial_\nu \partial_\kappa \mathcal{H}^m)_0 (\sigma^\kappa)^\nu_{\dot\nu} (\sigma^m)_{\mu\dot\mu} = -\frac{3i}{2} \underset{(\dot\nu\dot\mu)}{\mathrm{Sym}} \bar{\mathcal{D}}_{\dot\nu} G^n (\sigma_n)_{\mu\dot\mu} ,$$

$$(\bar\partial_{\dot\rho} \bar\partial^{\dot\rho} \partial_\nu \partial_m \mathcal{H}^m)_0 = -\frac{3i}{2} \bar{\mathcal{D}}_{\dot\nu} G^n (\sigma_n)^{\dot\nu}_\nu .$$

The conjugated equations are implied. Here

$$\partial_N = (\partial_n, \partial_\nu, \bar\partial_{\dot\nu}) \rightleftarrows \frac{\partial}{\partial z^N} = \left(\frac{\partial}{\partial x^n}, \frac{\partial}{\partial \theta^\nu}, \frac{\partial}{\partial \bar\theta^{\dot\nu}}\right);$$

the index "o" means that the quantities are considered at point z_0; Sym means symmetrization over the corresponding indices.

In normal gauge the supervierbeins and connections (their definitions and explicit expressions in terms of $\mathcal{H}^m$ see in Ref.[3]) are the following:

$$(E_A{}^M)_0 = \delta_A^M , \quad (\omega_{\alpha a b})_0 = 0, \quad (\bar\omega_{\dot\alpha a b})_0 = 0,$$

$$(\omega_{a\beta\mu})_0 = \frac{1}{16} (\sigma_{ab})_{\beta\mu} (G^b)_0 .$$

So, only the values of $E_\alpha^m (E_{\dot\alpha}^m)$ and $\omega_{a\beta\mu}$ differ from the expected flat-superspace ones

$$(E_\alpha^m)_0 = i(\sigma^m \bar\theta_0)_\alpha , \quad (\omega_{a\beta\mu})_0 = 0 .$$

The first of these equalities can be achieved by a slight change of the gauge (0.4.2) (see Step 9 below). The second one can be obtained either by modifying the gauge (however, Eq.(0.4.7) will not be valid then), or by redefining ω_a with the help of the covariant tensor G_a (which is always possible). Thus, full correspondence with the flat limit can be achieved, if necessary.

Before going to the proof of our statements, we ought to remind the transformations of the gauge group in consideration[3]:

$$x'^m = x^m + \frac{1}{2} \lambda^m (x_L^n, \theta^\nu) + \frac{1}{2} \bar\lambda^m (x_R^n, \bar\theta^{\dot\nu}), \qquad (0.5.1)$$

$$\theta'^\mu = \theta^\mu + \lambda^\mu (x_L^n, \theta^\nu), \qquad (0.5.2)$$

$$\bar\theta'^{\dot\mu} = \bar\theta^{\dot\mu} + \bar\lambda^{\dot\mu} (x_R^n, \bar\theta^{\dot\nu}), \qquad (0.5.3)$$

$$\mathcal{H}'^m (x', \theta', \bar\theta') = \mathcal{H}^m (x, \theta, \bar\theta) - \frac{i}{2} \lambda^m (x_L^n, \theta^\nu) + \frac{i}{2} \bar\lambda^m (x_R^n, \bar\theta^{\dot\nu}). \qquad (0.5.4)$$

Here

$$\bar\lambda^m = (\lambda^m)^+ , \quad \bar\lambda^{\dot\mu} = (\lambda^\mu)^+ \quad , \text{ and}$$

$$x_L^n = x^n + i\mathcal{H}^n (x, \theta, \bar\theta), \quad x_R^n = x^n - i\mathcal{H}^n (x, \theta, \bar\theta) \qquad (0.6)$$

are the bosonic coordinates of the left and right complex superspaces. The chiral superfunctions-parameters in Eq.(0.5) are constrained by the conditions of supervolume preservation in left and right superspaces (here x_L^m, θ^μ and $x_R^m, \bar\theta^{\dot\mu}$ are pairs of independent variables):

$$\mathrm{Ber} \left\| \frac{\partial z'_L}{\partial z_L} \right\| = \mathrm{Ber} \left\| \frac{\partial z'_R}{\partial z_R} \right\| = 1 \quad , \text{ i.e.,}$$

$$\det \left[\frac{\partial x'^m_L}{\partial x^n_L} - \frac{\partial \theta'^\mu}{\partial x^n_L} \left(\frac{\partial \theta'^\mu}{\partial \theta^\nu} \right)^{-1} \frac{\partial x'^m_L}{\partial \theta^\nu} \right] = \det \left(\frac{\partial \theta'^\mu}{\partial \theta^\nu} \right) \qquad (0.7)$$

as well as the conjugated equation.

Note that these constraints are very important. The same group (0.5) but without condition (0.7) corresponds to conformal supergravity[6]. There, the quantities $R, \bar R, G^a$ can also be eliminated, so only W, $\overline{W}$ remain and the equations of motion contain higher-order derivatives.

The proof of the statements (0.3), (0.4) goes as follows: The parameters $\lambda(\bar{\lambda})$ are expanded in the powers of $x_L - x_{oL}$, $\theta - \theta_o$ $(x_R - x_{oR}, \bar{\theta} - \bar{\theta}_o)$. The coefficients in this expansion are successively used for eliminating the Z_o -values of the derivatives of $\mathcal{H}^m$ having the proper dimension. For illustration, recall the proof of the second part of Eq.(0.1). The metric tensor transforms according to the law

$$g'_{mn}(x') = \frac{\partial x^z}{\partial x'^m} g_{zs} \frac{\partial x^s}{\partial x'^n}$$

and its derivative

$$(\partial_k g_{mn})' = \frac{\partial x^t}{\partial x'^k} \partial_t \left(\frac{\partial x^z}{\partial x'^m} g_{zs} \frac{\partial x^s}{\partial x'^n} \right).$$

Choose the coordinate transformation to be

$$x'^z = x^z + \frac{1}{2} a^z_{uv} (x - x_o)^u (x - x_o)^v$$

(the parameter a^z_{uv} has the same dimension cm^{-1} as the eliminated quantity $\partial_k g_{mn}$). Then at point $x = x_o$ one has

$$(\partial_k g_{mn})'_0 = (\partial_k g_{mn})_0 - a^z_{mk} (g_{zn})_0 - (g_{ms})_0 a^s_{nk}$$

and one can always determine a^n_{mk} so, that the left hand side vanishes.

We shall do the same with $\mathcal{H}^m$ and its derivatives in a number of successive steps. It is convenient to put $x_o = 0$, $\theta_o = 0$, $\bar{\theta}_o = 0$ which does not cause misunderstandings (except for Step 9).

<u>Step 1.</u> First we shall transform $\mathcal{H}^m$ to zero at point Z_o. For this purpose let us choose the parameters

$$\lambda^m = i a^m, \quad \lambda^\mu = 0 \tag{1.1}$$

(here and in what follows the parameters λ^m, λ^μ without their conjugates will be given; all bosonic coefficients will be real). Evidently, condition (0.7) is satisfied. From Eq.(0.5.4) one gets

$$(\mathcal{H}^m)'_0 = (\mathcal{H}^m)_0 - a^m$$

which allows one to find a^m and obtain Eq.(0.4.1).

It could seem strange that in normal gauge $\mathcal{H}^m$ equals zero instead of its flat-superspace value $\theta_o \sigma^m \bar{\theta}_o$ (as the analogy with Eq.(0.1) requires). Here we stress that the latter is also possible (see Step 9) but we prefer the gauge (0.4.1) for our purposes.

<u>Step 2.</u> Now, preserving Eq.(0.4.1) we shall further specify the gauge in order to obtain Eq.(0.4.2)

$$\lambda^m = i x^n_L a^m_n + i \theta^\nu \varphi^m_\nu, \qquad \lambda^\mu = (b + ic)\theta^\mu . \tag{2.1}$$

The parameters b and c are determined by the condition (0.7)

$$\det(\delta^m_n + i a^m_n) = (1 + b + ic)^2. \tag{2.2}$$

Then, the derivatives $\partial_N \mathcal{H}^m$ transform as follows (at the point Z_o)

$$\left(\frac{\partial \mathcal{H}^m}{\partial Z^N}\right)'_0 = \left(\frac{\partial Z^K}{\partial Z'^N}\right)_0 \left[\partial_K\left(\mathcal{H}^m - \frac{i}{2}\lambda^m + \frac{i}{2}\bar{\lambda}\right)\right]_0 . \tag{2.3}$$

The terms in the brackets vanish if the parameters a and φ satisfy

$$\left[\partial_k\left(\mathcal{H}^m - \frac{i}{2}\lambda^m + \frac{i}{2}\bar{\lambda}^m\right)\right]_0 = (\partial_k \mathcal{H}^m)_0 + a^m_k = 0, \tag{2.4}$$

$$\left[\partial_\alpha\left(\mathcal{H}^m - \frac{i}{2}\lambda^m + \frac{i}{2}\bar{\lambda}^m\right)\right]_0 = (\partial_\alpha \mathcal{H}^m)_0 + \frac{1}{2}\varphi^m_\alpha = 0.$$

Putting Eq.(2.4) into Eq.(2.3) one obtains Eq.(0.4.2).

Clearly, the new coordinate transformation with parameters (2.1) does not alter the already fixed value $(\mathcal{H}^m)_o$(0.4.1) because at point Z_o $\lambda^m = 0$ (Eq.(0.4.1) leads to $(x^n_L)_o = (x^n + i\mathcal{H}^n)_o = 0)$.

Step 3. Next, the second derivatives of $\mathcal{H}^m$ are to be considered. Let us begin with $\partial_\nu \bar{\partial}_{\dot{\varkappa}} \mathcal{H}^m$. Choose

$$\lambda^m = x^n_L a^m_n, \quad \lambda^\mu = (b+ic)\,\theta^\mu. \tag{3.1}$$

The condition (0.7) now gives

$$(1+b+ic)^2 = (1+b-ic)^2 = \det(\delta^m_n + a^m_n). \tag{3.2}$$

The transformation law of the second derivatives is

$$\left(\frac{\partial^2 \mathcal{H}^m}{\partial Z^N \partial Z^K}\right)'_o = \left\{\frac{\partial Z^R}{\partial Z'^N}\partial_R\left[\frac{\partial Z^S}{\partial Z'^K}\partial_S\left(\mathcal{H}^m - \frac{i}{2}\lambda^m + \frac{i}{2}\bar{\lambda}^m\right)\right]\right\}_o =$$

$$= \left(\frac{\partial Z^R}{\partial Z'^N}\right)_o \left(\partial_R \frac{\partial Z^S}{\partial Z'^K}\right)_o \left[\partial_S\left(\mathcal{H}^m - \frac{i}{2}\lambda^m + \frac{i}{2}\bar{\lambda}^m\right)\right]_o +$$

$$+(-1)^{R(S+K)}\left(\frac{\partial Z^R}{\partial Z'^N}\right)_o \left(\frac{\partial Z^S}{\partial Z'^K}\right)_o \left[\partial_R \partial_S\left(\mathcal{H}^m - \frac{i}{2}\lambda^m + \frac{i}{2}\bar{\lambda}^m\right)\right]_o. \tag{3.3}$$

Using Eq.(0.4.2) it is not hard to check that

$$\left[\partial_S\left(\mathcal{H}^m - \frac{i}{2}\lambda^m + \frac{i}{2}\bar{\lambda}^m\right)\right]_o = (\partial_S \mathcal{H}^n)_o(\delta^m_n + a^m_n) = 0. \tag{3.4}$$

Further, from Eqs.(0.5.1), (3.1) one finds

$$\left(\frac{\partial x^n}{\partial x'^m}\right)_o = (\delta^m_n + a^m_n)^{-1}, \quad \left(\frac{\partial \theta^\nu}{\partial \theta'^\mu}\right)_o = \frac{1}{1+b+ic}\,\delta^\nu_\mu,$$

$$\left(\frac{\partial \bar{\theta}^{\dot{\nu}}}{\partial \bar{\theta}'^{\dot{\mu}}}\right)_o = \frac{1}{1+b-ic}\,\delta^{\dot{\nu}}_{\dot{\mu}}, \quad \text{the rest} = 0. \tag{3.5}$$

Putting Eqs. (3.1), (3.4), (3.5) into Eq. (3.3) one obtains

$$\left(\frac{\partial^2 \mathcal{H}^m}{\partial \theta^\nu \partial \bar{\theta}^{\dot{\varkappa}}}\right)'_o = \frac{1}{(1+b)^2 + c^2}\,\partial_\nu \bar{\partial}_{\dot{\varkappa}} \mathcal{H}^n(\delta^m_n + a^m_n).$$

Introducing the notation

$$h^m_k = 2(\sigma_k)^{\nu\dot{\varkappa}}\,\partial_\nu \bar{\partial}_{\dot{\varkappa}} \mathcal{H}^m$$

one finds that Eq.(0.4.3) takes place if (see Eq.(3.2))

$$a^m_n = -\delta^m_n - \sqrt[3]{\det h}\cdot h^{-1\,m}_{\ \ n},$$

$$\begin{cases} \det h > 0: & c = 0,\ (1+b)^2 = \sqrt[3]{\det h} \\ \det h < 0: & b = -1,\ c^2 = -\sqrt[3]{\det h}. \end{cases}$$

Stress that due to Eq.(3.4) the transformation (3.1) does not change the values of $\partial_N \mathcal{H}^m$ (0.4.2) and $\mathcal{H}^m$ (0.4.1).

Note that the parameters

$$\lambda^m = a^m, \quad \lambda^\mu = \varepsilon^\mu + \theta^\nu \omega_\nu{}^\mu \quad (\omega_{\nu\mu} = \omega_{\mu\nu})$$

have not been put to use. They generate global translations (a^m), supertranslations (ε^μ) and Lorentz rotations $(\omega_{\mu\nu})$. These transformations remain unfixed in normal gauge.

Step 4. The choice of parameters

$$\lambda^m = i\theta\theta(a^m + ib^m), \quad \lambda^\mu = 0$$

leads to Eq.(0.4.4) if

$$(\partial_\nu \partial_\varkappa \mathcal{H}^m)_o - \varepsilon_{\nu\varkappa}(a^m + ib^m) = 0$$

(Eq.(0.7) is automatically fulfilled). The values earlier fixed do not change.

Step 5. The aim of this step is to obtain all equations (0.4.5) simultaneously.

$$\lambda^m = i x_L^n x_L^k a_{nk}^m + i x_L^n \theta^\nu \eta_{n\nu}^m ,$$

$$\lambda^\mu = x_L^n \varphi_n^{\;\mu} + \theta\theta \psi^\mu + x_L^n \theta^\mu b_n . \tag{5.1}$$

When evaluating the Berezinian one can restrict oneself with terms linear in x_L and θ. Indeed, one can compensate for the higher terms adding to Eq.(5.1) three- and more linear terms. For instance, if the $\theta\theta$ -terms in the Berezinian are considered, one can add a $\theta\theta x_L$ -term to λ^m. In $(\partial x_L' / \partial x_L)$ and then in the Berezinian this term will appear as a $\theta\theta$ -term and will help to fulfil the corresponding condition. Such a procedure is possible in any order. Therefore, Eq.(0.7) reduces to two constraints only

$$i(2a_{mk}^m - \varphi_m^{\;\mu} \eta_{k\mu}^m) = 2 b_k , \tag{5.2}$$

$$i \eta_{m\nu}^m = -2 \psi_\nu .$$

In Eq.(0.4.5) there are third-order derivatives of $\mathcal{H}^m$, so one needs their transformation law (the terms with $[\partial_T(\mathcal{H}^m - \frac{i}{2}\lambda^m + \frac{i}{2}\bar\lambda^m)]_0$ are omitted due to Eqs.(0.4.2), (5.1))

$$\left(\frac{\partial^3 \mathcal{H}^m}{\partial z^N \partial z^K \partial z^L}\right)_0' = \left(\frac{\partial z^R}{\partial z'^N}\right)_0 \Big\{ \left(\frac{\partial z^S}{\partial z'^K}\right)_0 \Big[(-1)^{R(K+T+L)} \times$$
$$\times \left(\partial_S \frac{\partial z^T}{\partial z'^L}\right)_0 \left[\partial_R \partial_T (\mathcal{H}^m - \tfrac{i}{2}\lambda^m + \tfrac{i}{2}\bar\lambda^m)\right]_0 + (-1)^{S(T+L)+R(K+S)} (R \leftrightarrow S) \Big] +$$
$$+ (-1)^{S(T+L)} \left(\partial_R \frac{\partial z^S}{\partial z'^K}\right)_0 \left(\frac{\partial z^T}{\partial z'^L}\right)_0 \left[\partial_S \partial_T (\mathcal{H}^m - \tfrac{i}{2}\lambda^m + \tfrac{i}{2}\bar\lambda^m)\right]_0 +$$
$$+ (-1)^{(R+S)(T+L)+R(S+K)} \left(\frac{\partial z^S}{\partial z'^K}\right)_0 \left(\frac{\partial z^T}{\partial z'^L}\right)_0 \left[\partial_R \partial_S \partial_T (\mathcal{H}^m - \tfrac{i}{2}\lambda^m + \tfrac{i}{2}\bar\lambda^m)\right]_0 \Big\} \tag{5.3}$$

Having in mind the already established Eqs.(0.4.1)-(0.4.4) one finds from Eqs.(3.3), (5.3), (5.1):

$$\left(\frac{\partial^2 \mathcal{H}^m}{\partial \theta^\nu \partial x^k}\right)_0' = (\partial_\nu \partial_k \mathcal{H}^m)_0 + \frac{1}{2}\eta_{k\nu}^m - \bar\varphi_k^{\dot\nu} (\sigma^m)_{\nu\dot\nu} = 0, \tag{0.4.5}$$

$$\left(\frac{\partial^2 \mathcal{H}^m}{\partial x^n \partial x^k}\right)_0' = (\partial_n \partial_k \mathcal{H}^m)_0 + 2a_{nk}^m - \varphi_n \sigma^m \bar\varphi_k - \varphi_k \sigma^m \bar\varphi_n = 0, \tag{0.4.5}$$

$$\left(\frac{\partial^3 \mathcal{H}^m}{\partial \theta^\nu \partial \theta^\varkappa \partial \bar\theta^{\dot\lambda}}\right)_0' = \frac{1}{2}\varepsilon_{\nu\varkappa} \Big[-(\partial^\rho \partial_\rho \bar\partial_{\dot\lambda} \mathcal{H}^m)_0 - 2\psi^\rho (\sigma^m)_{\rho\dot\lambda} -$$
$$- i(\sigma^m)_{\rho\dot\lambda} \eta_n^{m\rho} + 2i(\tilde\sigma^n \sigma^m)_{\dot\lambda\dot\tau} \bar\varphi_n^{\dot\tau} \Big] = 0. \tag{0.4.5}$$

These three equations together with Eq.(5.2) allow one to determine all the coefficients.

This time the parameters (5.1) cannot affect Eqs.(0.4.1) - (0.4.4) for dimensional reasons: They have higher dimension than the quantities present in Eqs.(0.4.1)-(0.4.4).

<u>Step 6</u>. The next set of parameters is

$$\lambda^m = \theta\theta x_L^n (a_n^m + i b_n^m), \quad \lambda^\mu = 0 . \tag{6.1}$$

Condition (0.7) reads

$$a_m^m = 0, \quad b_m^m = 0 . \tag{6.2}$$

Using once more Eq.(5.3) we obtain

$$\left(\frac{\partial^3 \mathcal{H}^m}{\partial \theta^\nu \partial \theta^\varkappa \partial x^\ell}\right)_0' = (\partial_\nu \partial_\varkappa \partial_\ell \mathcal{H}^m)_0 + i\varepsilon_{\nu\varkappa}(a_\ell^m + i b_\ell^m). \tag{6.3}$$

From Eqs.(6.3), (6.2) it follows that everything can be eliminated except $(\partial_\nu \partial_\varkappa \partial_m \mathcal{H}^m)_0'$. This quantity will be denoted by $-\frac{3i}{4}\varepsilon_{\nu\varkappa}\bar R$ foreseeing its geometric meaning.

Dimensional and tensor arguments prove that the parameters (6.1) do not change the values (0.4.1)-(0.4.5).

Step 7. Take the parameters

$$\lambda^m = x_L^n x_L^k a_{nk}^m \,, \tag{7.1}$$

$$\lambda^\mu = x_L^n \theta^\nu b_{n\nu}{}^{\mu}, \quad \bar\lambda^{\dot\mu} = x_R^n \bar\theta_{\dot\nu} \bar b_n{}^{\dot\nu\dot\mu}$$

(b is complex). The terms in Eq.(0.7) linear in x_L and x_R give

$$2 a_{mk}^m = b_{k\mu}{}^{\mu} = -\bar b_{k\dot\mu}{}^{\dot\mu} \,. \tag{7.2}$$

From Eq.(5.3) it follows

$$\left(\frac{\partial^3 \mathcal{H}^m}{\partial\theta^\nu \partial\bar\theta^{\dot x} \partial x^\ell}\right)'_0 = \left(\partial_\nu \bar\partial_{\dot x} \partial_\ell \mathcal{H}^m\right)_0 - 2(\sigma^k)_{\nu\dot x}\, a_{\ell k}^m + b_{\ell\nu}{}^{\tau}(\sigma^m)_{\tau\dot x} - \bar b_{\ell\dot x}{}^{\dot\tau}(\sigma^m)_{\nu\dot\tau} = 0. \tag{0.4.7}$$

The quantity $(\partial_\nu\bar\partial_{\dot x}\partial_\ell \mathcal{H}^m)_0$ is a third-rank tensor, $a_{\ell k}^m$ is symmetric in two indices, and b and $\bar b$ form an antisymmetric in two indices tensor (see Eq.(7.2)). A more detailed decomposition of these quantities into irreducible parts shows that Eqs.(7.2), (0.4.7) can be solved.

The parameters (7.1) could change only $(\partial_n\partial_k \mathcal{H}^m)_0$ but explicit checks rule out this possibility.

The realtion (0.4.8) expresses the fact that the quantity G^m (see its definition and explicit form in a forthcoming paper) reduces to the derivative $(\partial^2\bar\partial^2 \mathcal{H}^m)_0$ in the normal gauge.

Step 8. Our last aim are the derivatives with dimension $cm^{-\frac{3}{2}}$. Let us write down the corresponding parameters

$$\lambda^m = x_L^n x_L^k \theta^\nu \varphi_{\nu nk}^m \,,$$

$$\lambda^\mu = x_L^n x_L^k \psi_{nk}^\mu + x_L^n \theta\theta\, \eta_n^\mu \,. \tag{8.1}$$

The condition (0.7) (its $x_L\theta$ - terms) gives

$$\varphi_{\nu k m}{}^{m} = -\eta_{\nu k} \,. \tag{8.2}$$

From Eq.(5.3) one gets

$$\left(\frac{\partial^3 \mathcal{H}^m}{\partial\theta^\nu \partial x^k \partial x^\ell}\right)'_0 = \left(\partial_\nu\partial_k\partial_\ell \mathcal{H}^m\right)_0 - 2\bar\psi_{k\ell}{}^{\dot\tau}(\sigma^m)_{\nu\dot\tau} - i\,\varphi_{\nu k\ell}{}^{m} \,. \tag{8.3}$$

Similar calculations lead to

$$\left(\frac{\partial^4 \mathcal{H}^m}{\partial\bar\theta^{\dot\lambda}\partial\bar\theta_{\dot\lambda}\partial\theta^\nu\partial x^k}\right)'_0 = \left(\bar\partial^2\partial_\nu\partial_k \mathcal{H}^m\right)_0 - 4\bar\eta_k{}^{\dot\tau}(\sigma^m)_{\nu\dot\tau} - 2(\sigma^n)_{\nu\dot\lambda}\bar\varphi_{\ell k}{}^{\dot\lambda m} - 4i(\sigma^n\tilde\sigma^m)_{\nu\rho}\psi_{nk}^{\rho} \,. \tag{8.4}$$

The parameter η is fixed by Eq.(8.2). Eq.(8.3) allows one to obtain Eq.(0.4.9) using completely the parameter $\varphi_{\nu k\ell}^m$. The last free parameter in Eq.(8.4) is ψ_{nk}^ρ . Splitting the left-hand side of Eq.(8.4) and ψ_{nk}^ρ (taking into account the symmetry of ψ in n and k) into irreducible parts one confirms the gauge (0.4.10).

Stress once more that the quantities $W, \bar\partial G$ appearing in Eq.(0.4.10) are just notations. Their geometric meaning will be revealed in a paper on the derivation of the torsion and curvature components.

Step 9. As was pointed out earlier, in normal gauge $\mathcal{H}^m$ and E_α^m do not coincide with their expected flat-superspace values. If one wishes, this can be achieved by a slight modification of the gauge (0.4). The following parameters (here the manifest notation $z_0 = (x_0, \theta_0, \bar\theta_0)$ is necessary) have to be introduced

$$\lambda^m = i\,\theta_0\sigma^m\bar\theta_0 + 2i(\theta-\theta_0)^\nu(\sigma^m\bar\theta_0)_\nu \,, \quad \lambda^\mu = 0.$$

Then

$$(\mathcal{H}^m)'_0 = \theta_0\sigma^m\bar\theta_0 \,, \quad (\partial_\nu \mathcal{H}^m)'_0 = (\sigma^m\bar\theta_0)_\nu \,.$$

The rest quantities do not change.

So, all the parameters in the expansion of λ^m, λ^μ having dimensions from cm^{+1} to $cm^{-\frac{3}{2}}$ have been investigated. The corresponding transformation has caused the vanishing of $\mathcal{H}^m$

and a great number of its derivatives at the point z_0. The quantities $R, \bar{R}, G, W, \bar{W}$ that are not eliminated by the gauge fixing are the z_0 -values of those basic superfields in terms of which the components of torsion and curvature can be expressed (more details elsewhere).

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