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E2-80-121

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INTERACTION DYNAMICS AND NONTOPOLOGICAL SOLITON STABILITY IN ESSENTIALLY NONLINEAR MODEL OF COMPLEX SCALAR FIELD

Submitted to **"Physics Letters A".**

1980

In recent years there has been observed wide interest in particle-like solutions (PLS) to nonlinear equations arising in plasma physics, solid state theory and nonlinear optics [1,2]. It is hoped that the PLS are probably connected with constructing consistent elementary particle theory [3]. In two-dimensional space-time there are many models bearing PLS. Furthermore, some of them are integrable [4]. There are many interesting nonlinear equations with Lagrangians for which one knows nothing about solitary wave collision nature. The only reliable method to distinguish whether a solitary wave is the true soliton or not remains computer experiment. A significant number of qualitative effects has been observed in numerical calculations. These phenomena include the Fermi-Pasta-Ulam effect, the "soliton" character of PLS interactions for the sine-Gordon and other equations, the self-induced transparency and the exponential decrease of mode energy with the wave number [5]. The theoretical importance of synergetic use of computers was mentioned by Zabusky [6].

The interest to such computations is restricted by the possibility of constructing stable solutions in a nonlinear model for which nonlinearity depends essentially on fields and their derivatives.

Consider the model [7]

$$\mathcal{L} = |\partial_\mu \phi|^2 - m^2 |\phi|^2 + g_1 |\phi|^4 + g_2 J_\mu J^\mu , \tag{1}$$

where

$$J_\mu = \frac{i}{2}(\phi^* \partial_\mu \phi - \phi \partial_\mu \phi^*), \quad \mu = 0,1 .$$

We study the following properties of this model:

1) stability of PLS in numerical experiments (the transverse and longitudinal stabilities have been considered in linear theory of perturbations [8]);

2) the stable PLS interaction dynamics.

We find the PLS stability regions versus the parameters g_1, g_2, m, ω. This regions are in good agreement with the results [8]. The PLS interaction dynamics and its features are described.

The equation of motion for the model (1) is

$$(\Box + m^2)\phi = 2g_1 \phi |\phi|^2 + g_2 \frac{\delta (J_\mu J^\mu)}{\delta \phi^*} , \tag{2}$$

where

$$\frac{\delta(J_\mu J^\mu)}{\delta\phi^*} = \phi|\partial_\mu\phi|^2 - \phi^*(\partial_\mu\phi)^2 + \frac{1}{2}\phi^2\partial_\mu\partial^\mu\phi^* - \frac{1}{2}|\phi|^2\partial_\mu\partial^\mu\phi .$$

Take the solution to (2) in the form

$$\phi(x,t) = \chi(x)\exp(i\omega t) .$$

At $\omega=\text{const}$ it is extremum for the energy functional

$$H = \int_{-\infty}^{\infty} dx\{|\phi_t|^2 + |\phi_x|^2 + m^2|\phi|^2 - g_1|\phi|^4 + \frac{1}{2}g_2[|\phi|^2(|\phi_t|^2 + |\phi_x|^2) - \frac{1}{2}\phi^{*2}(\phi_t^2 + \phi_x^2) - \frac{1}{2}\phi^2(\phi_t^{*2} + \phi_x^{*2})]\} . \quad (3)$$

Instead of (2) we have a boundary value problem

$$\chi_{xx} - (m^2-\omega^2)\chi + 2(g_1 + g_2\omega^2)\chi^3 = 0 , \qquad \chi(\pm\infty) = 0, \quad \chi_x(\pm\infty) = 0 . \quad (4)$$

It has the PLS

$$\chi(x) = \sqrt{\frac{m^2-\omega^2}{g_1+g_2\omega^2}}\;\frac{1}{\cosh(x\sqrt{m^2-\omega^2})} . \quad (5)$$

Taking into account the Lorentz invariance of model (1) as well as ratio (5), solution of eq. (2) has the form

$$\phi(x,t) = \sqrt{\frac{m^2-\omega^2}{g_1+g_2\omega^2}}\;\frac{\exp[iD\omega\gamma(t - vx + \delta)]}{\cosh[\gamma(x - vt + x_0)\sqrt{m^2-\omega^2}]} , \quad (6)$$

where $\gamma = 1/(1-v^2)^{1/2}$, v is the PLS velocity, $D=\pm1$. The functionals of energy (3), momentum

$$P = \int_{-\infty}^{\infty} dx\{\phi_t\phi_x^* + \phi_t^*\phi_x + \frac{1}{2}g_2[|\phi|^2(\phi_t\phi_x^* + \phi_t^*\phi_x) - \phi^2\phi_t^*\phi_x^* - \phi^{*2}\phi_t\phi_x]\}$$

and charge

$$Q = i\int_{-\infty}^{\infty} dx(\phi_t^*\phi - \phi^*\phi_t)(1 + g_2|\phi|^2)$$

on the solution (6) are

$$E = M\gamma, \qquad P = Mv\gamma, \quad (7)$$

where

$$Q = \frac{4\omega\sqrt{m^2-\omega^2}}{3(g_1+g_2\omega^2)^2}(g_2\omega^2 + 3g_1 + 2g_2m^2) ,$$

$$M = \frac{4\sqrt{m^2-\omega^2}}{3(g_1+g_2\omega^2)^2}[\omega^2(2g_1 + 3g_2m^2) + g_1m^2] .$$

One may construct the conserved current $j_\mu = \epsilon_{\mu\nu}\partial^\nu\phi(x,t)$, where $\epsilon_{\mu\nu}$ is the totally antisymmetric tensor ($\epsilon_{01}=1$). Then define the topological charge

$$Q_T = \int_{-\infty}^{\infty} j_0 dx = \int_{-\infty}^{\infty}\phi_x dx = [\phi(+\infty) - \phi(-\infty)] = 0 .$$

Thus the solutions (6) for the model (1) have a trivial topology (nontopological solitons).

In the following, the mass and charge (7) are measured in units of $m^2=1$. From the general necessary conditions of stability [8] and behaviour of $M(g_1,g_2,\omega)$ and $Q(g_1,g_2,\omega)$ (see fig.1) we assume that the solutions (6) have stable states in the region $\omega_i > \omega^\ell_{CR}$ and unstable ones at $\omega_k \le \omega^\ell_{CR}$, where ω_{CR} corresponds to

$$\frac{dM}{d\omega}\Big|_{g_2/g_1=\text{const}} = 0 , \qquad \frac{dQ}{d\omega}\Big|_{g_2/g_1=\text{const}} = 0 . \quad (8)$$

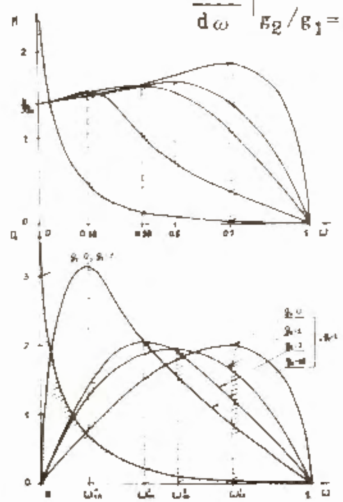

Fig.1. The functions $M(\omega)$ and $Q(\omega)$ for different values of g_2 at $g_1=1$. The initial part of dotted line defined the boundary of stability region of solutions (6) for corresponding ω^ℓ_{CR}.

Table

Computer experiment runs	Amplitude		Velocity	Sign of charge		Phase difference	Remarks
	A_1	A_2	$v_1=-v_2=v$	D_1	D_2	$\Delta\delta$	
	0.5	0.5	0.9	+	-	0 $\pi/2, \pi$	At $D_1=+1$, $D_2=-1$ we have inelastic interaction of solitons
I	0.7 0.9	0.7 0.9	0.9 0.9	+ +	+ +	0 $\pi/2, \pi$	At $D_1=D_2=+1$ we have weakly inelastic interaction of solitons
	0.9	0.9	0.07 0.2 0.4 0.6 0.9	+ +	+ -	0	1. At $D_1=D_2=+1$ we have production of third soliton. 2. At $D_1=+1, D_2=-1$ we have weakly inelastic interaction of solitons.
	A_1	A_2	v_1 v_2	D_1	D_2	$\Delta\delta$	Remarks
II	0.6	0.9	0.9 0	+	-	0 $\pi/2$ π	"Repulsion" of solitons
				+	+	0 $\pi/2$ π	"Transmission" of solitons

Our computations confirm this assumption. The considered values of $\omega_{i,k}$ satisfy the condition $\omega_k < \omega^{\ell}_{CR} < \omega_i$, where ω^{ℓ}_{CR} are determined from equation (8) (see fig. 1).

We study the interaction dynamics of stable PLS in numerical experiments (the results are presented in the Table).

Consider the case $g_1=0$, $g_2=1$ (the pure current $\times$ current interaction).

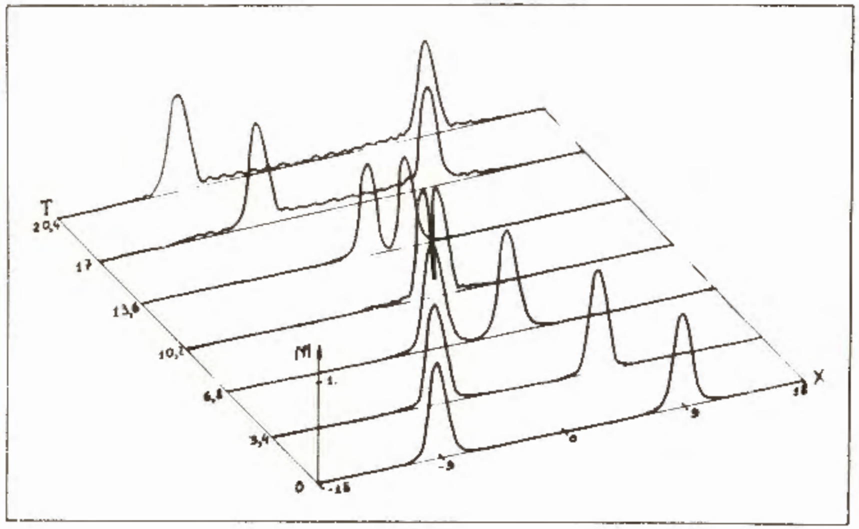

Fig.2. Weak inelastic interaction of solitons with equal charges $(D_1=D_2=+1)$: $A_1=A_2=0.7$, $v_1=-v_2=0.9$, $\Delta\delta=|\delta_1-\delta_2|=0$.

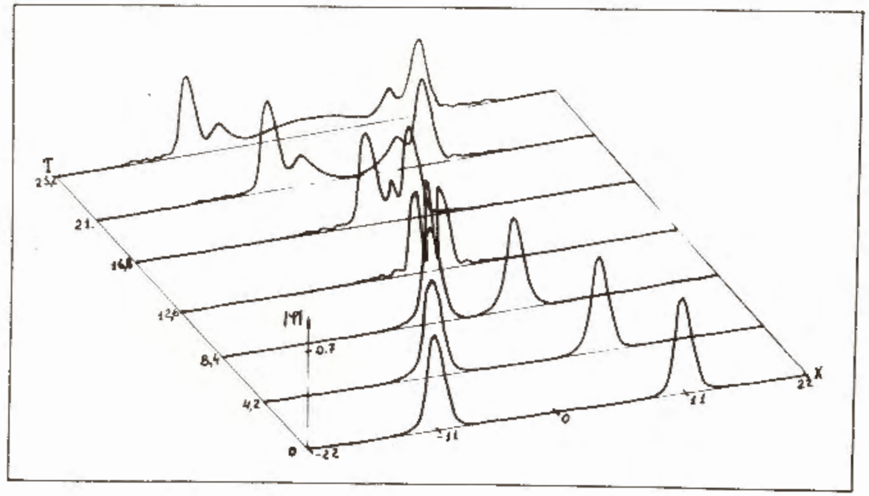

Fig.3. Inelastic interaction of solitons with opposite charges $(D_1=+1, D_2=-1)$: $A_1=A_2=0.5$, $v_1=-v_2=0.9$, $\Delta\delta=|\delta_1-\delta_2|=0$.

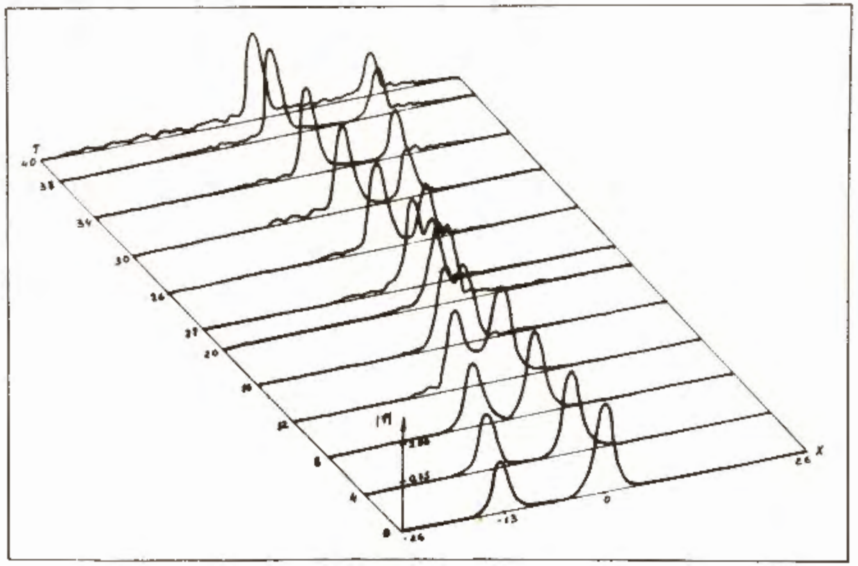

Fig.4. Interaction of test solitons with resting heavy target soliton for equal charges ($D_1 = D_2 = +1$)("transmission"): $A_1 = 0.6$, $A_2 = 0.9$, $v_1 = 0.9$, $v_2 = 0$, $\Delta\delta = \pi/2$.

Our computations reveal a rich spectrum of interactions versus the PLS masses, charge signs and velocities:

1) weak inelastic interaction (fig.2);
2) inelastic interaction (fig.3);
3) production of an additional soliton (fig.6).

The first interaction type is observes in PLS collisions with equal charges and presented in fig. 2. Note that at the moment of collision of solitons they are not overlapped.

The second interaction type takes place in PLS collisions with opposite charges (fig.3).In this case $\Delta\delta = \pi$ too and the solitons are not overlapped also.

As a result of the collisions of a light soliton with heavy one, either "transmission" (fig.4) or "repulsion" (fig.5) of the solitons was observed depending on the sign of the ratio of their charges.

The most interesting phenomenon is the third interaction type since the production of an additional soliton has been observed early only for interactions of very specific objects, gaussons [8]. Our computer experiment reveals that the third so-

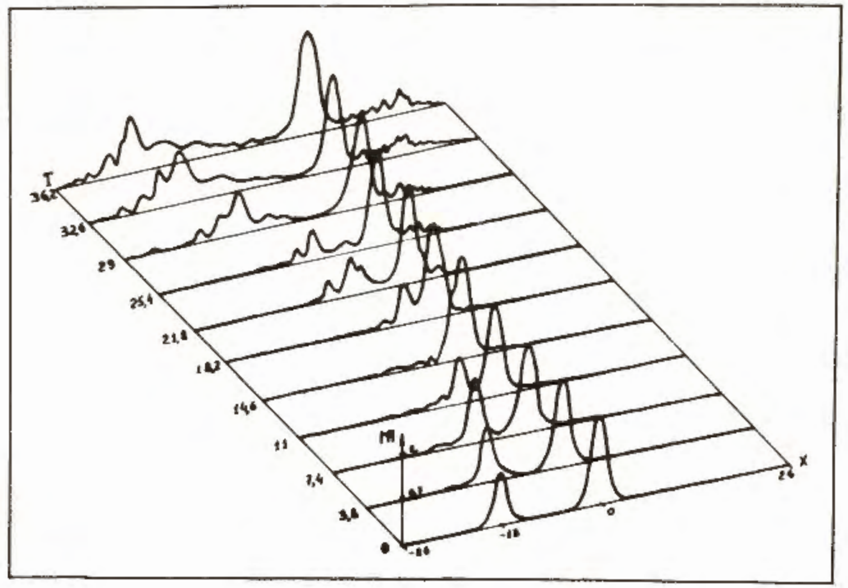

Fig.5. Interaction of test soliton with resting heavy target soliton for opposite charges ($D_1 = +1$, $D_2 = -1$) ("repulsion") $A_1 = 0.6$, $A_2 = 0.9$, $v_1 = 0.9$, $v_2 = 0$, $\Delta\delta = \pi/2$.

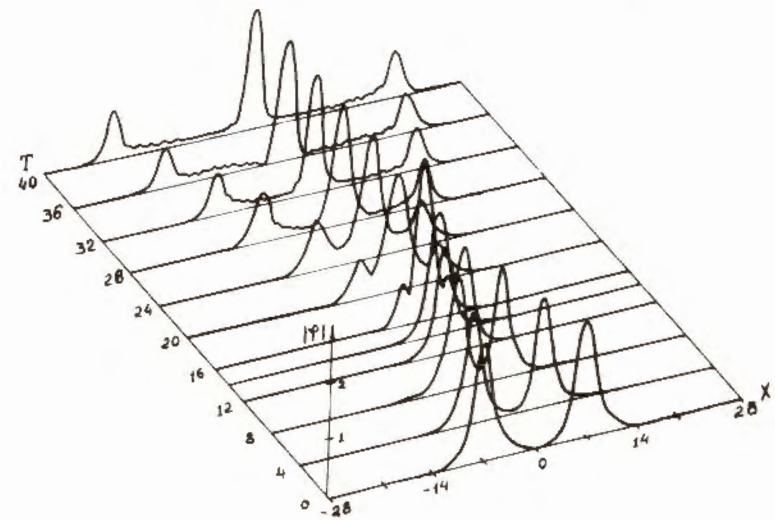

Fig.6. Production of the third soliton at $D_1 = D_2 = +1$: $A_1 = A_2 = 0.9$, $v_1 = -v_2 = 0.6$, $\Delta\delta = 0$.

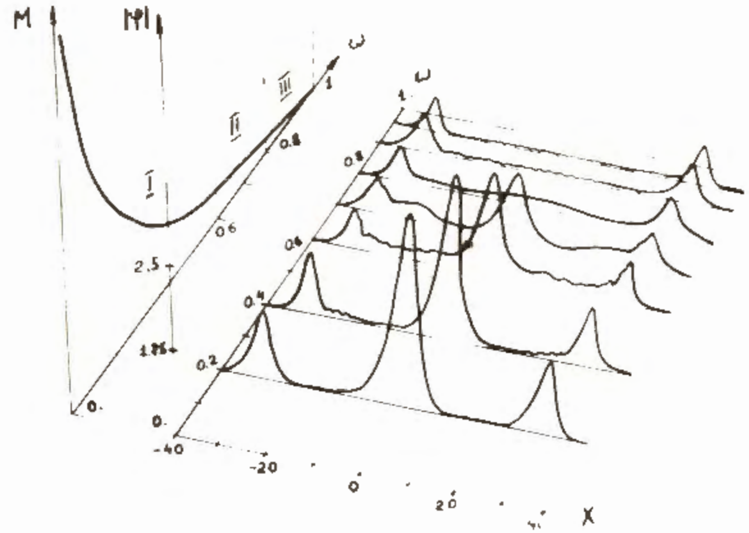

Fig.7. Prodiction and decay of the third soliton versus the frequency ω (final stages after the collision) I - region of the third PLS production, II - region of decay of the third PLS, III - region of weakly inelastic interaction.

liton production takes place in collisions of two solitons with equal charges $D_1 = D_2 = +1$ and sufficiently large masses ($\omega \leq 0.6$). The regions of the additional soliton production and decay are presented in fig. 7. On the left picture one can see the function $M(\omega)$ having characteristic zones I-III.

Concluding, note that peculiarities of the PLS (repulsion, attraction, overlapping and so on) depend essentially on the energy E, ratio of charge signs ($d = D_1/D_2 = \pm 1$) and initial phase difference $\Delta\delta$. That means that for relevant description of real physical phenomena by such nonlinear models and numerical experiments it is necessary to have an adequate procedure of averaging in the phase difference of colliding objects. More detailed dependence of interactions on E, d and $\Delta\delta$ is to be studied.

One of the most interesting results of our investigation is the discovery of the third PLS production in the interaction of two solitons. Until now similar phenomena have been observed only for nonrelativistic equations /9-11/.

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Received by Publishing Department
on February 14 1980.