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ON NONLOCAL CONSERVED CURRENTS IN THE SUPERSYMMETRIC GENERALIZED NONLINEAR SIGMA MODELS

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1. INTRODUCTION

An infinite number of nonlocal conserved currents for the two-dimensional O(N) nonlinear sigma model on the classical level [1] and on the quantum level [2] was introduced. A simple constructive proof of the existence of one infinite set of classical nonlocal conserved currents was given in paper [3] for the generalized nonlinear sigma models [4-6]. The supersymmetric extension of nonlinear sigma models in [7] for the O(N) model and in [8] for generalized sigma models was considered. With the method given in [3] an infinite set of conserved currents was constructed for the supersymmetric O(N) sigma model, in paper [9] and for supersymmetric generalized sigma models in [10]. For the self-dual sector of Yang-Mills theory method given in [3] was applied in paper [11]. More general consideration was given in [12], where from the Yang-Mills action by using the variational principle the conserved currents depending on some arbitrary functions were derived.

In the present paper, the method given in [12] for derivation of nonlocal conserved currents is applied. In such a way three linear independent infinite series of nonlocal conserved currents are found. The problem of deriving of all conserved currents is reduced to solving of one supersymmetric linear equation of second order.

First, in the second section the supersymmetric two-dimensional generalized nonlinear sigma model is introduced. This model contains the principal chiral and the O(N) chiral, CP^{N-1} and the fields defined on the Grassmannian manifolds.

In the third section the existence of the infinite number (with the power of continuum) of classical conserved supercurrents for the supersymmetric generalized sigma models is proved.

Three linear independent N^2-parameter infinite series of conserved currents are constructed in the fourth section. The explicit form of first nontrivial terms of any of these series is given.

Our consideration is made in the Minkowski space. However, it can be extended also to the Euclidean case by the substitution $x_0 \to x_2 = ix_0$ and $\gamma_0 \to \gamma_2 = i\gamma_0$.

2. THE EQUATION OF MOTION FOR THE SUPERSYMMETRIC GENERALIZED NONLINEAR SIGMA MODEL

Consider the matrix superfield

$$G(x;\Theta) = g(x) + \bar{\Theta}\phi(x) + \frac{1}{2}\bar{\Theta}\Theta k(x)\,, \tag{2.1}$$

where $g(x)$ and $k(x)$ are $N\times N$ matrices with scalar matrix elements, $\phi(x)$ is an $N\times N$ matrix with the Majorana spinor matrix elements and Θ is a two-component Majorana spinor. The existence of $G^{-1}(x;\theta)$ will always be supposed throughout. In the most general case $G(x;\Theta)$ belongs to the Lie algebra of the general linear group $GL(N)$. In the case when G is an element of $U(N)$ or $O(N)$ Lie algebra we say that we have the principal chiral fields. The fields $G(x;\Theta)$ for which

$$G^{-1}(x;\Theta) = G(x;\Theta)$$

and consequently

$$G(x;\theta) = I - 2\Pi(x;\Theta) = G^{-1}(x;\Theta)\;;\ \text{i.e.}\ \ G^2 = I \tag{2.2}$$

are of special interest. Here $\Pi(x;\Theta)$ are the projection operators $(\Pi^2=\Pi)$ on certain $n\times N (n \leq N)$ dimensional subspaces. In the last two cases $G(x;\Theta)$ is transformed with respect to the adjoint representation of the global gauge $U(N)$ $(O(N))$ group. These cases include $O(N)$ supersymmetric chiral, CP^{N-1} models, and their generalization, i.e., the fields defined on the Grassmannian manifolds. In the last two cases it is convenient to introduce to the field Π (or G) because we consider only the local gauge invariant entities [5].

Substituting (2.1) in $G^{-1}G = GG^{-1} = I$ we have the following conditions for the component fields:

$$g^{-1}(x)\,g(x) = g(x)\,g^{-1}(x) = 1\,,$$

$$g^{-1}(x)\,\phi(x) + \phi^{-1}(x)\,g(x) = g(x)\,\phi^{-1}(x) + \phi(x)\,g^{-1}(x) = 0\,, \tag{2.3}$$

$$k^{-1}(x)\,g(x) + g^{-1}(x)\,k(x) - \bar{\phi}^{-1}(x)\,\phi(x) = g(x)\,k^{-1}(x) + k(x)\,g^{-1}(x) - \bar{\phi}\phi^{-1} = 0.$$

In the case of generalized chiral fields, i.e., (2.2) the constraint $G^2 = 1$ can be written in terms of components fields in the form (2.3) with the substitution $g^{-1}=g$, $\phi^{-1}=\phi$ and $k^{-1}=k$.

Consider the action

$$\begin{aligned} S &= \frac{1}{2}\int d^2x\,d^2\Theta\,\mathrm{tr}\{D^{\alpha}G^{-1}(x;\Theta)D_{\alpha}G(x;\Theta)\}, \\ &= -\frac{1}{2}\int d^2x\,d^2\Theta\,\mathrm{tr}\{G^{-1}(x;\Theta)D^{\alpha}G(x;\Theta)G^{-1}(x;\Theta)D_{\alpha}G(x,\Theta)\}, \end{aligned} \tag{2.4}$$

where

$$D_{\alpha} = i\frac{\partial}{\partial\Theta^{\alpha}} + (\bar{\Theta}\partial\!\!\!/)_{\alpha}\,, \quad \partial\!\!\!/ = \gamma^{\mu}\partial_{\mu} \quad (\alpha = 1,2) \tag{2.5}$$

are supercovariant derivetives, $\bar{\Theta} = \Theta^T C$. For the Dirac γ-matrices we use the following representations

$$\gamma_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma_1 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \gamma_5 = \gamma_1\gamma_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \; C = \gamma_1 \tag{2.6}$$

in Minkowski space, and $\gamma_2^E = i\gamma_0$, $\gamma_1^E = \gamma_1$, $C^E = C$, $\gamma_5^E = \gamma_2^E\gamma_1^E$ in Euclidean space.

From the action (2.4) we derive the following equations of motion

$$D^{\alpha}A_{\alpha}(x;\Theta) = 0, \tag{2.7}$$

where

$$A_{\alpha}(x;\Theta) = G^{-1}(x;\Theta)D_{\alpha}G(x;\Theta)\,. \tag{2.8}$$

It can be checked without difficulties that (2.7) can be written in the following equivalent form

$$D^{\alpha}D_{\alpha}G(x;\Theta) + G(x;\Theta)D^{\alpha}G^{-1}(x;\Theta)D_{\alpha}G(x;\Theta) = 0, \tag{2.9}$$

which is the equation of motion for the supersymmetric generalized nonlinear sigma model.

Substituting (2.1) in (2.9) we have the equations of motion for the component fields

$$\begin{aligned} &\Box g - g\partial^{\mu}g^{-1}\partial_{\mu}g + \frac{1}{2}g(\partial^{\mu}\bar{\phi}^{-1}\gamma_{\mu}\phi - \bar{\phi}^{-1}\gamma^{\mu}\partial_{\mu}\phi) + \\ &+ \frac{}{2}\bar{\phi}\gamma^{\mu}\phi^{-1}\partial_{\mu}g + \frac{}{2}\bar{\phi}\gamma^{\mu}(\partial_{\mu}g^{-1}.)\phi + \frac{1}{4}g\bar{\phi}^{-1}(\bar{\phi}\phi^{-1})\phi = 0, \end{aligned} \tag{2.10a}$$

$$i\partial^\mu\gamma_\mu\phi + \frac{i}{2}g\partial^\mu g^{-1}(\gamma_\mu\phi) + \frac{i}{2}g(\gamma_\mu\phi^{-1})\partial^\mu g -$$

$$- \frac{1}{4}[\phi(\bar\phi^{-1}\phi) + (\bar\phi\phi^{-1})\phi] = 0, \tag{2.10b}$$

where the $k(x)$ component of G is excluded because

$$k(x) = \frac{1}{2}g(x)\,\bar\phi^{-1}(x)\,\phi(x), \tag{2.11}$$

which follows from (2.9).

The equation (2.7) means that $A_\alpha(x;\Theta)$ appears as a conserved supercurrent. Substituting (2.1) in (2.8) and taking into account the equation of motion (2.10) and constraints (2.3), we write $A_\alpha(x;\Theta)$ in terms of field component:

$$\begin{aligned} a_\alpha(x) &= ig^{-1}(x)\,\phi_\alpha(x), \\ r(x) &= \frac{i}{2}\bar\phi^{-1}(x)\,\gamma_5\,\phi(x), \\ v_\mu(r) &= g^{-1}(x)\,\partial_\mu g(x) - \frac{i}{2}\bar\phi^{-1}(x)\,\gamma_5\,\phi(x), \\ b(x) &= -g^{-1}(\not\partial\phi) - (\gamma^\mu\phi^{-1})\partial_\mu g + \frac{i}{2}[(\bar\phi\phi^{-1})\phi + \phi(\bar\phi^{-1}\phi)], \end{aligned} \tag{2.12}$$

where $a(x)$ and $b(x)$ are spinor, $r(x)$ pseudoscalar and $v_\mu(x)$ vector components of $A_\alpha(x;\Theta)$. Because of equation (2.7) the scalar component of $A_\alpha(x;\Theta)$ vanishes, and the components a, b and v_μ satisfy the following equations

$$\begin{aligned} i\not\partial a(x) &= b(x), \\ \partial^\mu v_\mu(x) &= 0, \end{aligned} \tag{2.13}$$

and the pseudoscalar component $r(x)$ is arbitrary. Equations (2.13) are equivalent to equations (2.10) when a, b, v_μ and r are determined by (2.12). However, they are satisfied for any conserved supercurrent also.

3. INFINITE NUMBER OF NONLOCAL CONSERVED SUPERCURRENTS

It will be proved here that there exists an infinite number of conserved nonlocal supercurrents. For this purpose it is convenient to introduce as in papers [3,10] the matrix spinor covariant derivative

$$\nabla_\alpha = D_\alpha + A_\alpha(x;\Theta) \qquad (\alpha = 1,2), \tag{3.1}$$

where D_α is the supercovariant derivative (2.5) and $A_\alpha(x;\Theta)$ is given by formula (2.8). When the field G satisfies (2.9), A_α satisfies (2.7). Because of definition (2.8) for $A_\alpha(x;\Theta)$ and equations of motion (2.7), the following identities hold:

$$\begin{aligned} \mathcal{F}_{\alpha\beta} &= \{\nabla_\alpha, \nabla_\beta\} = D_\alpha A_\beta + D_\beta A_\alpha + \{A_\alpha, A_\beta\} = \\ &= G^{-1}\{D_\alpha, D_\beta\}G + \{D_\alpha, D_\beta\} = 2i(C\gamma^\mu)_{\alpha\beta}(\partial_\mu + G^{-1}\partial_\mu g), \end{aligned} \tag{3.2}$$

$$D^\alpha\nabla_\alpha = -\nabla_\alpha D^\alpha = \nabla^\alpha D_\alpha, \tag{3.3}$$

where in the r.h.s. of (3.2) the anticommutation relations

$$\{D_\alpha, D_\beta\} = D_\alpha D_\beta + D_\beta D_\alpha = 2i(C\gamma^\mu)_{\alpha\beta}\partial_\mu \tag{3.4}$$

are used. For the representations of γ-matrices given by (2.6) we have $(C\gamma_\mu)_{12} = (C\gamma_\mu)_{21} = 0$ and consequently

$$\mathcal{F}_{12} = \mathcal{F}_{21} = \{\nabla_1, \nabla_2\} = D_1 A_2 + D_2 A_1 + \{A_1, A_2\} = 0, \tag{3.5}$$

For the supersymmetric theory, in contradiction with the ordinary case [8], generally the curvature tensor $\mathcal{F}_{\alpha\beta} \neq 0$ is a consequence of the curved character of the superspace. However, it has been shown in [10] for the existence of an infinite number of nonlocal currents it is sufficient that $\mathcal{F}_{12} = \mathcal{F}_{21} = 0$ only. Then we make the following:

<u>Ansatz (A)</u>: There exist two functions $X(x;\Theta)$ and $Y(x;\Theta)$ being $N \times N$-matrices with matrix elements transforming as scalar superfields and coupled by the equations

$$D_\alpha X(x;\Theta) = (\gamma_5\nabla)_\alpha Y(x;\Theta). \tag{A}$$

By means of these functions we define the following spinor quantity

$$J^{(X,Y)}_{\alpha}(x;\Theta) = A_{\alpha}(x;\Theta)X(x;\Theta) + (\gamma_5 A)_{\alpha} Y(x;\Theta). \qquad (3.6)$$

Then it is easy to check that there holds the following:

Theorem. The Ansatz (A) is the necessary and sufficient condition for the conservation of the supercurrent (3.6) in a weak sense, i.e.,

$$D^{\alpha} J^{(X,Y)}_{\alpha}(x;\Theta) = 0 \qquad (3.7)$$

if the equation of motion (2.7) is satisfied.

Indeed, substituting (3.6) in (3.7) we have

$$D^{\alpha} J^{(X,Y)}_{\alpha}(x;\Theta) = (D^{\alpha}A_{\alpha})X + A^{\alpha}(D_{\alpha}X) + D^{\alpha}(\gamma_5 A)_{\alpha} Y + (\gamma_5 A)^{\alpha} D_{\alpha} Y =$$

$$= A^{\alpha}(\gamma_5 \nabla)_{\alpha} Y + D^{\alpha}(\gamma_5 A)_{\alpha} Y + (\gamma_5 A)^{\alpha} D_{\alpha} Y = \qquad (3.8)$$

$$= A^{\alpha}(\gamma_5 \nabla)_{\alpha} Y - A^{\alpha}(\gamma_5 A)_{\alpha} Y - A^{\alpha}(\gamma_5 D)_{\alpha} Y = 0.$$

Here we take into account the equation of motion (2.7), Ansatz (A) and the following identities

$$D^{\alpha}(\gamma_5 A)_{\alpha} = D_1 A_2 + D_2 A_1 = -\{A_1, A_2\} = -A^{\alpha}(\gamma_5 A)_{\alpha},$$

$$A^{\alpha}(\gamma_5 D)_{\alpha} = -(\gamma_5 A)^{\alpha} D_{\alpha}.$$

The necessity of the Ansatz (A) can be proved analogously.

Substituting in (A)

$$X = X^{(n+1)}, \quad Y = X^{(n)} \qquad (3.9)$$

and denoting

$$j^{(n)}_{\alpha} = (\gamma_5 D)_{\alpha} X^{(n)}(x;\Theta), \quad (\alpha = 1,2), \qquad (3.10)$$

which is conserved if X is an arbitrary smooth function /10/ we have

$$j^{(n+1)}_{\alpha}(x;\Theta) = \nabla_{\alpha} X^{(n)}(x;\Theta), \quad (\alpha = 1,2). \qquad (A')$$

(A') was a starting point for derivation of one infinite set of conserved currents in /10/ . As will be seen, the Ansatz in the form (A) is more profound than (A'). Moreover, it allows us to derive the infinite number of nonlocal currents from the variation of action. Indeed following /12/ we make some transformation

$$G' = G + \delta G \approx G + X$$

with respect to these transformations the action is not invariant, but because of (A) and (3.4) the change of action can be represented as the superdivergence of some quantity. In such cases it is known /13/ that there exist conserved currents, given by (3.6).

One current of this type is obtained if we substitute $X = I$ and $Y = 0$ in (3.6). These functions are a trivial solution of (1) and corresponding current coincides with (2.8).

Let us find the other functions satisfying (A). Taking into account (3.4), from (A) the condition on the function

$$\nabla^{\alpha} D_{\alpha} X(x;\Theta) = D^{\alpha} \nabla_{\alpha} X(x;\Theta) = 0 \qquad (3.11)$$

follows for any Y. However, the function Y also cannot be completely arbitrary. From the "integrability" condition of the system of equations (A) we have

$$D^{\alpha}(\gamma_5 D)_{\alpha} X = (D_1 D_2 + D_2 D_1)X = D^{\alpha}\nabla_{\alpha} Y = \nabla^{\alpha} D_{\alpha} Y = 0 \qquad (3.11')$$

for any smooth function X. Because of (3.5), (3.6) the functions $\tilde{X}$ and $\tilde{Y}$ satisfy also equation (3.11).

In order to solve the equation (3.11) it is convenient to write them in terms of component fields. For this aim the functions $X(x;\Theta)$ and $Y(x;\Theta)$ are represented as

$$X(x;\Theta) = \chi(x) + \bar{\Theta}\kappa(x) + \frac{1}{2}\bar{\Theta}\Theta\zeta(x), \qquad (3.12)$$

where $\chi(x)$ and $\zeta(x)$ are $N\times N$ matrices with scalar elements and $\kappa(x)$ is $N\times N$ matrix with Majorana spinor elements. Substituting (3.12) in (3.11) we have

$$\Box\chi + v^{\mu}\partial_{\mu}\chi - \frac{1}{2}\bar{a}\,\partial\!\!\!/\kappa - \frac{i}{2}\bar{b}\kappa = 0, \qquad (3.13a)$$

$$2i\partial\!\!\!/\kappa + ia\zeta + (a\gamma_\mu)\partial^\mu\chi + iv^\mu(\gamma_\mu\kappa) - ir(\gamma_5\kappa) = 0, \tag{3.13b}$$

$$2\zeta + i\bar{a}\kappa = 0, \tag{3.13c}$$

where $a(x)$, $b(x)$, $v_\mu(x)$ and $r(x)$ are components of $A_\alpha(x;\Theta)$ given by (2.12).

The system of partial differential equations (3.13) is of third order. As is well known from the theory of differential equations, this system has an infinite number of solutions. Consequently from these solutions one can construct an infinite number of conserved supercurrents (3.6).

This problem can be solved in two ways:

a) To find all the independent solutions of (3.11) ((3.13)) and between them the pairs satisfying the Ansatz (A).

b) Using the substitution (3.9), the Ansatz (A) can be written in the form

$$D_\alpha X^{(k+1)}(x;\Theta) = (\gamma_5\nabla)_\alpha X^{(k)}(k;\Theta). \tag{3.14}$$

We denote that (3.14) is a more restrictive form of Ansatz (A) because we limit ourselves to the numerable set of functions $X^{(k)}$. Then starting with some independent solutions $X^{(0)}_m (m=1,\dots,M)$ of equation (3.11), substituting them in the r.h.s. of (3.14), and solving the last equations we arrive at the functions $X^{(1)}_m(x;\Theta)$ which are solutions also to equations (3.11). Consequently $X^{(1)}_m(x;\Theta)$ can be substituted in the r.h.s. of (3.14) and so on. In such a way we are able to construct the following infinite sequences

$$X^{(0)}_m, X^{(1)}_m, \dots, X^{(k)}_m, \dots \quad (m=1,\dots,M), \tag{3.15}$$

every element of which satisfies the equation (3.11) and any two neighbours are coupled by (3.14). Because of the fact that Ansatz (A) (or (3.14)) is the first order nonhomogeneous differential equation (more strictly, the system of differential equations) the construction of the sequences (3.15) must take into account also the solution of the corresponding to (3.14) homogeneous equation, i.e.,

$$D_\alpha X^{(k+1)}_m(x;\Theta) = 0, \tag{3.16}$$

which is constant. Substituting this constant in (3.6) we obtain the term $A_\alpha C$ which is self-conserved because of the equation of motion (2.7). It should be noted that the Ansatz (A) ((3.14)) is a mutual constraint, i.e., it can be solved also with respect to Y if X is known. In the last case the solutions of the homogeneous equation

$$(D_\alpha + A_\alpha(x;\Theta))X^{(k)}_m(x;\Theta) = 0, \quad (\alpha = 1,2), \tag{3.17}$$

which are not constants can be taken into account.

In such a way the sequence (3.15) can be continued to negative values of k, i.e., we have the sequences

$$\dots, X^{(-k)}_m, \dots, X^{(-1)}_m, X^{(0)}_m, X^{(1)}_m, \dots, X^{(k)}_m, \dots \quad (m=1,\dots,M). \tag{3.18}$$

When eq. (3.17) is satisfied for some $X^{(\ell)}_m$, the sequence (3.18) can be truncated with this term.

For our purposes only the linear independent sequences (3.18) are of interest. We say that two sequences $\{X^{(k)}_\ell\}$ and $\{X^{(k)}_m\}$ are linear independent if any of them contains not less than one element $X^{(p)}_{\ell(m)}$ linear independent of all elements of other series $\{X^{(k)}_{m(\ell)}\}$.

Using (3.6) we see that to any sequence (3.18) there corresponds one infinite series of conserved supercurrents. In the case of linear independent sequences (3.18) the corresponding currents are also linear independent.

4. EXPLICIT FORM OF NONLOCAL CONSERVED SUPERCURRENTS

According to the recipe given in the previous section we are able to find the explicit form of any term of three N^2-parameter (in all probability) linear-independent series of conserved supercurrents. For this purpose consider equation (3.11). Suppose that some numbers of conserved currents $J^{(m)}_\alpha$ $(m=1,\dots,M)$ are found. Then eq. (3.11) is equivalent to the system of first order linear equations

$$(D_\alpha + A_\alpha)X^{(0)}_m = J^{(m)}_\alpha(x;\Theta)C, \tag{4.1}$$

where C is a constant $N\times N$ -matrix. If $C=0$, (4.1) coincide with the equations (3.17). In the case when $C\neq 0$ because of (3.10), (4.1) is equivalent to the Ansatz (A). For this reason we restrict our consideration to the case $C=0$. In the last case the equation (4.1) (or (3.17)) has three linear independent solutions. The first one is trivial:

$$X_1^{(0)} = 0 . \tag{4.2}$$

The second is

$$X_2^{(0)} = G^{-1}(x;\Theta)\mathbf{C}_2 . \tag{4.3}$$

To find the third solution of (4.1) we first write down Ansatz (A) in terms of components of $X(x;\Theta)$:

$$\partial_\mu \chi^{(k+1)}(x) = \epsilon_{\mu\nu}\,[(\partial^\nu + v^\nu)\chi^{(k)}(x) + \frac{1}{2}\bar{a}\gamma^\nu \kappa^{(k)}] , \tag{4.4a}$$

$$\kappa^{(k+1)}(x) = \gamma_5(\kappa^{(k)} - ia\chi^{(k)}) , \tag{4.4b}$$

$$\zeta^{(k+1)}(x) = ir\chi^{(k)} - \frac{i}{2}\bar{a}\gamma_5\kappa^{(k)} . \tag{4.4c}$$

Here the "integrability" condition (3.13) is taken into account and $a(x)$, $v_\mu(x)$ and $r(x)$ are components of A_a given by (2.12). Taking into account (4.4) and (2.12) eq. (4.1) (if $C = 0$) becomes:

$$(\partial_\mu + g^{-1}\partial_\mu g)\chi^{(0)}(x) = 0 , \tag{4.5a}$$

$$\kappa^{(0)}(x) = -g^{-1}(x)\chi^{(0)}(x) , \tag{4.5b}$$

$$\zeta^{(0)}(x) = -\frac{1}{2}g^{-1}(x)\bar{\phi}\phi^{-1}g(x)\chi^{(0)}(x) . \tag{4.5c}$$

It can be checked immediately that (4.2) and (4.3) satisfy (4.5). Moreover from (4.5) we obtain the following solution

$$\chi_3^{(0)} = UV\mathbf{C}_3 , \tag{4.6}$$

where $\mathbf{C}_3$ is an $N\times N$ constant matrix,

$$U = P\exp\{-\int_{-\infty}^{x_0} dy_0\, g^{-1}(y_0,x_1)\,\partial_0 g(y_0,x_1)\} \tag{4.7}$$

and

$$V = P\exp\{-\int_{-\infty}^{x_1} dy^1 (Ug^{-1}\partial_1 gU^{-1})(x_0,y_1)\} .$$

Here P is the Wilson ordering operator.

Substituting (4.2), i.e., $\chi^{(0)} = \kappa^{(0)} = \zeta^{(0)} = 0$ in the r.h.s. of (4.4) and solving them we have

$$\chi^{(1)} = \mathbf{C}_1 ,\quad \kappa^{(1)} = 0 ,\quad \zeta^{(1)} = 0 , \tag{4.8}$$

where $\mathbf{C}_1$ is a constant $N\times N$ matrix. Using (4.8) we have from (4.9)

$$\partial_\mu \chi_1^{(2)}(x) = \epsilon_{\mu\nu} v^\nu(x)\,\mathbf{C}_1 , \tag{4.9a}$$

$$\kappa_1^{(2)}(x) = -i\gamma_5 a(x)\,\mathbf{C}_1 , \tag{4.9b}$$

$$\zeta_1^{(2)}(x) = ir(x)\,\mathbf{C}_1 . \tag{4.9c}$$

The solution of (4.9a) is given by

$$\chi_1^{(2)}(x) = -\int_{-\infty}^{x_1} dy^1 V_0(x_0,y_1)\mathbf{C}_1 = -\int_{-\infty}^{x_1} dy^1 (g^{-1}\partial_0 g - \frac{i}{2}\bar{\phi}^{-1}\gamma_0\phi) . \tag{4.10}$$

In such a way the sequence $\{X_1^{(k)} | k = 0,1,\ldots\}$ can be constructed. The conserved currents (3.6) corresponding to this sequence, in the case when $\mathbf{C}_1 = I$ coincide, up to linear combinations, with those found in /9, 10/.

The construction of the sequences $\{X_2^{(k)}\}$ and $\{X_3^{(k)}\}$ uses the symmetry of the Ansatz (A) discussed in Appendix A. For this aim it is sufficient to require the nonsingularity of the matrices $\mathbf{C}_2$ and $\mathbf{C}_3$. Then with the substitution

$$X_a^{(-k)} = X_a^{(0)}\tilde{X}_a^{(k)} \quad (a = 2,3) \tag{4.11}$$

for the $\tilde{X}^{(k)}$ Ansatz A reads (see (A.3))

$$D_\alpha \tilde{X}_a^{(k)} = (\gamma_5)_\alpha^{\ \beta}(D_\beta - \tilde{A}_\beta^{(a)})\,\tilde{X}_a^{(k+1)} \quad (a = 2,3) , \tag{4.12}$$

where

$$\tilde{A}_\alpha^{(a)} = (X_a^{(0)})^{-1} A_\alpha X_a^{(0)} . \tag{4.13}$$

Consequently, any term of sequences $\{\tilde{X}_a^{(k)}\}$ can be obtained from the corresponding term of $\{X_1^{(k)}\}$ with the substitution $A_\alpha \to -\tilde{A}_\alpha^{(a)}$.

The first two terms are given by (4.8-10). In such a way we are able to construct three sequences $\{X_1^{(k)}\}$, $\{X_2^{(-k)}=(X_2^{\circ})\tilde{X}_2^{(k)}\}$ and $\{X_3^{(-k)}=(X_3^{\circ})\tilde{X}_3^{(k)}\}$ which in all probability are linear independent because of (4.3), (4.6) and (4.10). To these sequences there correspond three infinite series of conserved nonlocal supercurrents. If we substitute one of the currents thus found into (4.1), it is easy to see that no new sequence arises. But it is not proved that there do not exist other linear independent sequences, and, consequently, other linear independent series of conserved currents.

Because of the fact that the field $G(x;\Theta)$ is transformed according to the adjoint representation of the non-Abelian group $GL(N)$, $U(N)$ or $O(N)$ when $N>1$ the corresponding charges are not in involution [1]. In all probability, conserved supercurrents thus obtained are generated from some infinite-parameter non-Abelian group [14] one manifestation of which is the dual symmetry discussed in paper [15]. However, the last question requires a more thorough study.

APPENDIX A

The Ansatz (A) possesses the following symmetry. Suppose that there exists a nonsingular matrix satisfying the equation

$$\nabla_\alpha U(x;\Theta) = (D_\alpha + A_\alpha)\, U(x;\Theta) = 0. \tag{A.1}$$

Then the Ansatz (A) with the substitution

$$X = U\tilde{X} \qquad \text{and} \qquad Y = U\tilde{Y} \tag{A.2}$$

is written in the following form

$$D_\alpha \tilde{Y} = (\gamma_5 \nabla)_\alpha \tilde{X} = (\gamma_5)_\alpha^\beta\, (D_\beta - \tilde{A}_\beta)\tilde{X}(x;\Theta), \tag{A.3}$$

where

$$\tilde{A}_\alpha(x;\Theta) = U^{-1} A_\alpha U\ . \tag{A.4}$$

From (A.1) and (A.4) it follows that

$$D^\alpha \tilde{A}_\alpha = U^{-1} D^\alpha A_\alpha\ U = 0, \tag{A.5}$$

if eq. (2.7) is satisfied and

$$\{\tilde{\nabla}_\alpha, \tilde{\nabla}_\beta\} = U^{-1}\{\nabla_\alpha, \nabla_\beta\} U\ . \tag{A.6}$$

Consequently, $\tilde{X}$ and $\tilde{Y}$ must also satisfy the second-order equation (3.11).

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