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THE SATURATION OF MOST STRINGENT ISOSPIN BOUNDS IN πN-SCATTERING

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In a recent paper [1] it was investigated the isospin bounds on polarization (P) and spin rotation parameters (A,R) in the pion-nucleon scattering using a set of bilinear forms which can be constructed from the scattering amplitudes of two charge or (s, t, u) -isospin channels. Thus, it was obtained the most stringent isospin bounds on H (see definitions (2b), (5c) and the bounds (6a,b,c)) which include also the recent result of Doncel et al. [2]. These isospin bounds are exactly saturated on the zeros-trajectories of the real and imaginary parts of the different bilinear forms. In this paper we discuss the exact saturation of the most stringent isospin bounds on H using the CERN-phase shift solutions [3].

Therefore, let f_i^{++} and f_i^{+-} be the helicity non-flip and helicity flip scattering amplitudes corresponding to different charge ($i=+,-,CE$) or (s, t, u) -isospin channels ($i = 2I_s, 2I_t, 2I_u$). Let us consider the following bilinear forms [1]:

$$Z'^{(0)}_{ij} = [f_i^{++}]^* f_j^{++} + [f_i^{+-}]^* f_j^{+-} , \tag{1a}$$

$$Z'^{(1)}_{ij} = i\{[f_i^{++}]^* f_j^{+-} - [f_i^{+-}]^* f_j^{++}\} , \tag{1b}$$

$$Z'^{(2)}_{ij} = [f_i^{++}]^* f_j^{+-} + [f_i^{+-}]^* f_j^{++} , \tag{1c}$$

$$Z'^{(3)}_{ij} = [f_i^{++}]^* f_j^{++} - [f_i^{+-}]^* f_j^{+-} , \tag{1d}$$

$$Y_{ij} = f_i^{++} f_j^{+-} - f_i^{+-} f_j^{++} , \tag{1e}$$

and let us calculate $|Z'^{(n)}_{ij}|^2$ and $|Y_{ij}|^2$, $n=0,1,2,3$, in terms of measurable quantities.

Then we obtain

$$|Z'^{(0)}_{ij}|^2 = \frac{1}{2}(1+\vec{P}_i\cdot\vec{P}_j)\,\sigma_i\sigma_j = -H_{ij}+\sigma_i\sigma_j \ge 0, \tag{2a}$$

$$|Y_{ij}|^2 = \frac{1}{2}(1-\vec{P}_i\cdot\vec{P}_j)\,\sigma_i\sigma_j \equiv H_{ij} \ge 0, \tag{2b}$$

$$|Z'^{(n)}_{ij}|^2 = H_{ij} + Z'^{(n)}_{ii}Z'^{(n)}_{jj} \ge 0, \quad n=1,2,3, \tag{2c}$$

where

$$Z'^{(1)}_{kk} = P_k\sigma_k;\quad Z'^{(2)}_{kk} = -A_k\sigma_k;\quad Z'^{(3)}_{kk} = R_k\sigma_k, \tag{3}$$

and σ_k, $\vec{P}_k \equiv (P_k, A_k, R_k)$, $k=i,j$ are the unpolarized differential cross-sections and final polarization (when incident polarization is 100%) corresponding to the i,j charge or isospin channels.

Now, from (2a) and (2c) and the positivity of $[\mathrm{Im}\,Z'^{(n)}_{ij}]^2$ and $[\mathrm{Re}\,Z'^{(n)}_{ij}]^2$ we obtain the following inequalities:

$$\max\{L'_{ij}\} \le H_{ij} \le \min\{U'_{ij}\}, \tag{4a}$$

where L'_{ij} and U'_{ij} are defined by

$$L'_{ij} \equiv \{0;\ [\mathrm{Re}\,Z'^{(n)}_{ij}]^2 - Z'^{(n)}_{ii}Z'^{(n)}_{jj};\ [\mathrm{Im}\,Z'^{(n)}_{ij}]^2 - Z'^{(n)}_{ii}Z'^{(n)}_{jj};\ n=1,2,3\}, \tag{4b}$$

$$U'_{ij} \equiv \{\sigma_i\sigma_j - [\mathrm{Re}\,Z'^{(0)}_{ij}]^2;\ \sigma_i\sigma_j - [\mathrm{Im}\,Z'^{(0)}_{ij}]^2\}. \tag{4c}$$

From the isospin invariance alone we obtain (see the relations (24), (25), (28) and (29) from ref. /1/):

$$[(\mathrm{Re}\,Z'^{(n)}_{+-})^2 - Z'^{(n)}_{++}Z'^{(n)}_{--}] = 2[(\mathrm{Re}\,Z'^{(n)}_{+CE})^2 - Z'^{(n)}_{++}Z'^{(n)}_{CE\,CE}] = 2[(\mathrm{Re}\,Z'^{(n)}_{-CE})^2 - Z'^{(n)}_{--}Z'^{(n)}_{CE\,CE}] = \ldots = \frac{1}{4}\lambda[Z'^{(n)}_{++}, Z'^{(n)}_{--}, 2Z'^{(n)}_{CE\,CE}] \tag{5a}$$

$n=0,1,2,3$, $Z'^{(0)}_{kk} = \sigma_k$;

$$[\mathrm{Im}\,Z'^{(n)}_{+-}]^2 = 2[\mathrm{Im}\,Z'^{(n)}_{+CE}]^2 = 2[\mathrm{Im}\,Z'^{(n)}_{-CE}]^2 = \ldots = \begin{cases} -H - \frac{1}{4}\lambda(\sigma_+, \sigma_-, 2\sigma_{CE}) \ge 0 & \text{for } n=0 \\ H - \frac{1}{4}\lambda(Z'^{(n)}_{++}, Z'^{(n)}_{--}, 2Z'^{(n)}_{CE\,CE}) \ge 0 & \text{for } n=1,2,3. \end{cases} \tag{5b}$$

and

$$H \equiv H_{+-} = 2H_{+CE} = 2H_{-CE} = \frac{4}{9}H_{13s} = 4H_{02t} = \frac{4}{9}H_{13u}, \tag{5c}$$

where

$$\lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz. \tag{5d}$$

Therefore, using (4a,b,c) and (5a,b,c) we obtain

$$\max\{L'\} \le H \le \min\{U'\} \tag{6a}$$

where

$$L' \equiv \{0;\ \frac{1}{4}\lambda[Z'^{(n)}_{++}, Z'^{(n)}_{--}, 2Z'^{(n)}_{CE\,CE}];\ \frac{1}{4}\Lambda'^{(n)};\ n=1,2,3\} \tag{6b}$$

$$U' \equiv \{-\frac{1}{4}\lambda(\sigma_+, \sigma_-, 2\sigma_{CE}),\ \frac{1}{4}\Lambda'^{(0)}\} \tag{6c}$$

and

$$\Lambda'^{(0)} \equiv 4H + \lambda(\sigma_+, \sigma_-, 2\sigma_{CE}) + 4\min\{\sigma_+\sigma_-, 2\sigma_+\sigma_{CE}, 2\sigma_-\sigma_{CE}, \ldots\} \tag{6d}$$

$$\Lambda'^{(n)} \equiv 4H - \lambda(Z'^{(n)}_{++}, Z'^{(n)}_{--}, 2Z'^{(n)}_{CE\,CE}) + 4\max\{-Z'^{(n)}_{++}Z'^{(n)}_{--}, -2Z'^{(n)}_{++}Z'^{(n)}_{CE\,CE}, -2Z'^{(n)}_{--}Z'^{(n)}_{CE\,CE}, \ldots\} \tag{6e}$$

$n=1,2,3$.

The bilinear forms $Z'^{(n)}_{ij}$ have the following relevant properties

(i) $Z'^{(0)}_{ij}$ are invariant under rotations of the spin reference frame while $Z'^{(1)}_{ij}$, $Z'^{(2)}_{ij}$, $Z'^{(3)}_{ij}$ one transform as the components of the polarization vector;

(ii) the zeros-trajectories of $\mathrm{Im}Z'^{(n)}_{ij}$ in the plane (s,t) are independent of the charge or isospin indices i,j, this property being independent of the isospin reference frame for any $n=0,1,2,3$.

Note that there are no restrictions on H when $\mathrm{Re}Z'^{(n)}_{ij} = 0$, $n=0,1,2,3$ (see the relations (5a)). Therefore, for the study of the exact saturation of the isospin bounds (6a,b,c) we have used CERN-phase shift solutions /3/ in order to calculate the zeros-trajectories of $\mathrm{Im}Z'^{(n)}_{ij}$, $n=0,1,2,3$ in the $(p_{LAB}, \cos\theta)$ -plane, where θ is the angle in the centre mass reference frame. These trajectories are shown in Figs. 1a,b. In order to estimate the errors and nearness to the bounds we have defined (see also ref. /4/)

$$F' \equiv [1-8H/(\sum_{i=+,-,CE}\sigma_i)^2]^{1/2}, \qquad (7a)$$

$$F'^{(0)} \equiv [1+2\lambda(\sigma_+, \sigma_-, 2\sigma_{CE})/(\sum_{i=+,-,CE}\sigma_i)^2]^{1/2}, \qquad (7b)$$

$$F'^{(n)} \equiv [1-2\lambda(Z'^{(n)}_{++}, Z'^{(n)}_{--}, 2Z'^{(n)}_{CECE})/(\sum_{i=+,-,CE}\sigma_i)^2]^{1/2}, \qquad (7c)$$

in terms of which the bounds (6a,b,c) can be written as

$$F'^{(0)} \le F' \le \min_{1\le n\le 3}\{1, F'^{(n)}\}. \qquad (8)$$

Then ,we have chosen a standard distance $|F'^{(n)} - F'| = 0.1$ for all the $F'^{(n)}$, $n=0,1,2,3$, and we have determined the hatched regions, shown in Figs. 1a,b, from the conditions $|F'^{(n)} - F'| \ge 0.1, n=0,1,2,3$. In the same way we have obtained the zeros-trajectories of $\mathrm{Im}Z^{(n)}_{ij}$ and the hatched regions presented in Figs. 2a,b, where $Z^{(n)}_{ij}$ are obtained

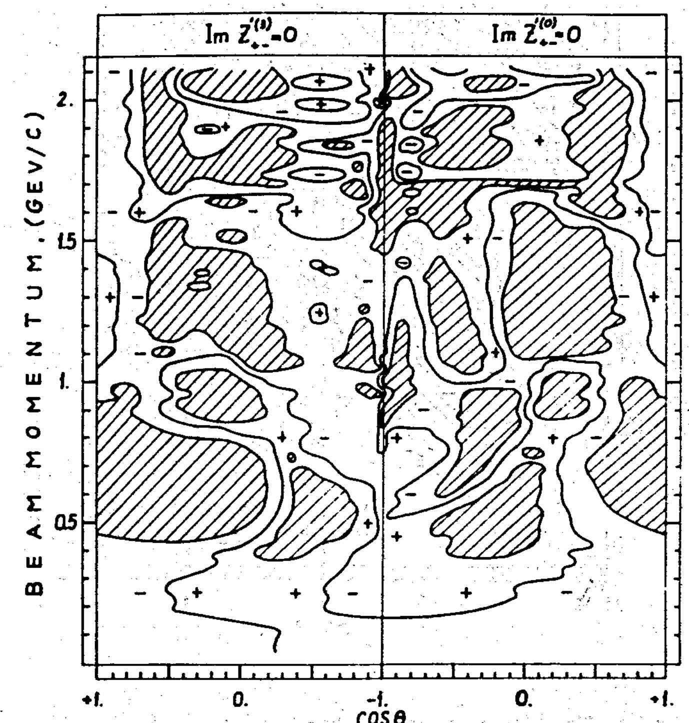

Fig. 1a. The zeros-trajectories of $\mathrm{Im}Z'^{(0)}_{ij}$ and $\mathrm{Im}Z'^{(3)}_{ij}$ in the $(p_{LAB}, \cos\theta)$ -plane. The hatched regions correspond to $|F'^{(n)} - F'| \ge 0.1$ and the signs refer to the sign of $\mathrm{Im}Z'^{(n)}_{+-}$, $n=0,3$.

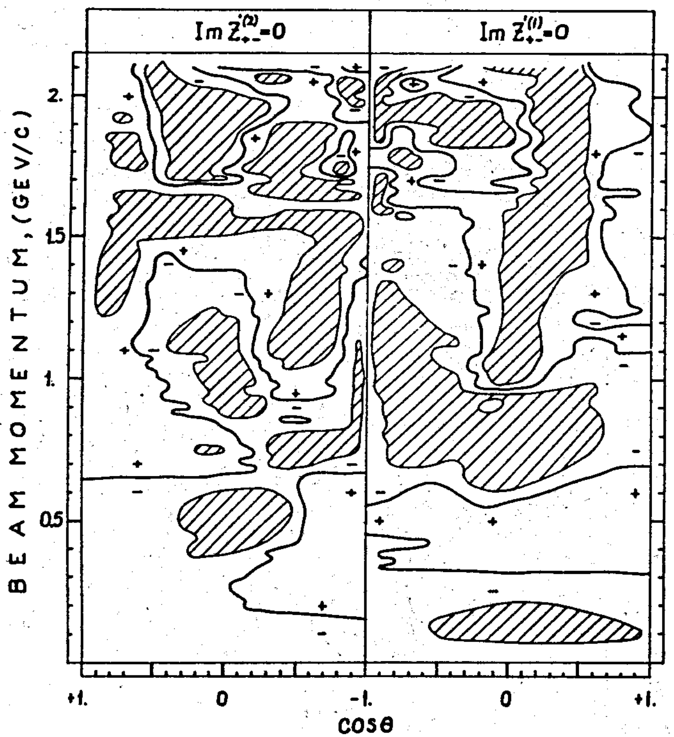

Fig. 1b. The zeros-trajectories of $\mathrm{Im} Z_{ij}^{\prime(1)}$ and $\mathrm{Im} Z_{ij}^{\prime(2)}$ in the $(p_{LAB}, \cos\theta)$ -plane. The hatched regions correspond to $|F^{\prime(n)} - F^{\prime}| \geq 0.1$ and the signs refer to the sign of $\mathrm{Im}\, Z_{+-}^{\prime(n)}$, n = 1,2.

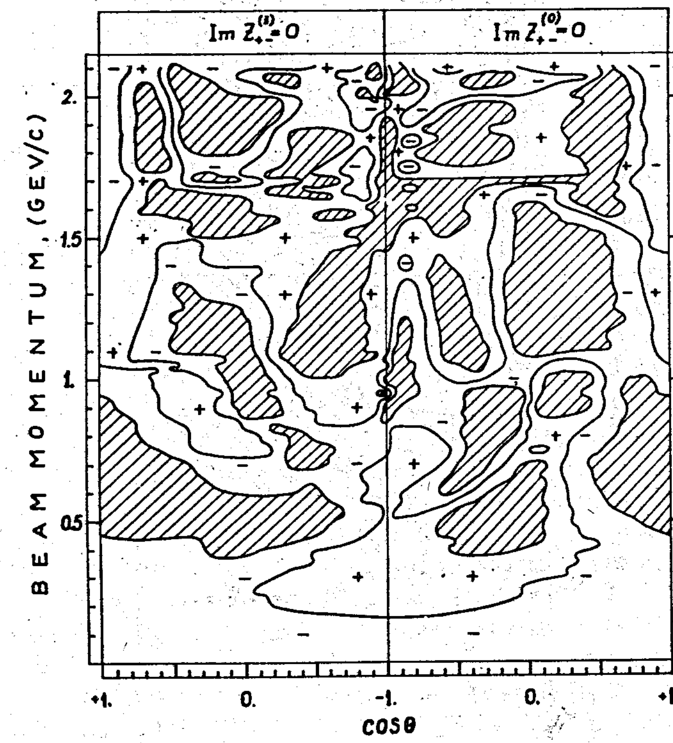

Fig. 2a. The zeros-trajectories of $\mathrm{Im} Z_{ij}^{(0)}$ and $\mathrm{Im} Z_{ij}^{(3)}$ in the $(p_{LAB}, \cos\theta)$ -plane. The hatched regions correspond to $|F^{(n)} - F| \geq 0.1$ and the signs refer to the sign of $\mathrm{Im}\, Z_{+-}^{(n)}$, n = 0,3.

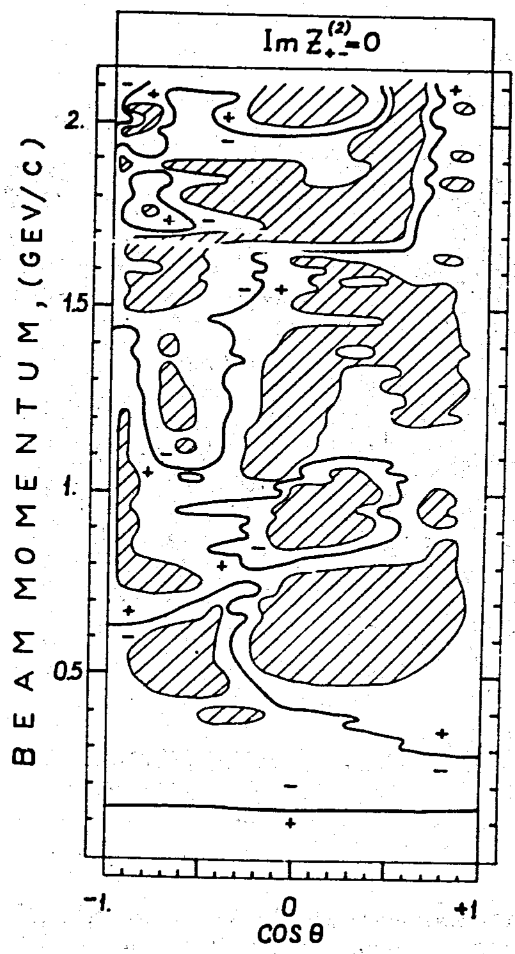

Fig. 2b. The zeros-trajectories of $\mathrm{Im}Z_{ij}^{(2)}$ in the $(p_{LAB}, \cos\theta)$ -plane. The hatched regions correspond to $F^{(2)} - F \geq 0.1$ and the signs refer to the sign of $\mathrm{Im}Z_{+-}^{(2)}$.

from (1a,b,c) by the substitution $f^{++} \to f$, $f^{+-} \to g$, where f and g are the usual spin non-flip and spin-flip pion-nucleon scattering amplitudes. We remark that,

(iii) at the boundaries $\theta=0°$ and $180°$, the zeros-trajectories of $\mathrm{Im}Z_{ij}^{\prime(3)}$ and $\mathrm{Im}Z_{ij}^{(3)}$ are smooth continuation of the zeros-trajectories of $\mathrm{Im}Z_{ij}^{\prime(0)}$ (so that in this points the isospin bounds (8) are degenerated). We note that $Z_{ij}^{\prime(0)} = Z_{ij}^{(0)}$ and $Z_{ij}^{\prime(1)} = Z_{ij}^{(1)}$. The signs shown in Figs. 1a,b, 2a,b, refer to the signs of $\mathrm{Im}Z_{+-}^{\prime(n)}$ and $\mathrm{Im}Z_{+-}^{(n)}$, respectively, from which the signs of any $\mathrm{Im}Z_{ij}^{\prime(n)}$ ($\mathrm{Im}Z_{ij}^{(n)}$) are determined according to the relations (21) from ref. /1/.

As we can see from Figs. 1a,b (an 2a,b) the isospin bounds are exactly saturated at certain values of $\cos\theta$ and p_{LAB} on the zeros-trajectories of $\mathrm{Im}Z_{ij}^{\prime(n)}$ ($\mathrm{Im}Z_{ij}^{(n)}$) and are nearly saturated in the unhatched regions. We expect that the true structure of the zeros-trajectories is much simpler and smoother so that the small details in Figs. 1a,b, 2a,b cannot be taken seriously because of the uncertainties of phase shifts. Thus the exact positions of the zeros-trajectories of all $\mathrm{Im}Z_{ij}^{\prime(n)}$, $n=0,1,2,3$ depend mainly on the theoretical assumptions. Therefore, any suggestion, based on intuition (see ref. /4/) or a theoretical approach, for a simple zeros-trajectories pattern for all $\mathrm{Im}Z_{ij}^{\prime(n)}$ should be useful in order to solve the problem of the ambiguities present in the phase shift analysis. On the other hand, if the zeros-trajectories of all $\mathrm{Im}Z_{ij}^{\prime(n)}$, $n=0,1,2,3$ will be well known from the experimental data these can be used for the construction (or as a fundamental test) of the different theoretical models.

Next, our results on the zeros-trajectories of $\mathrm{Im}Z_{ij}^{\prime(n)}$ can be compared with the results obtained in refs. /5,6,7/ and /8/ . We find that the isospin bounds (6a,b,c) are nearly saturated in the entire $\cos\theta$ -region below one pion threshold where all the differences, $|F^{\prime(n)} - F^{\prime}| < 0.1$ (respectively $|F^{(n)} - F| < 0.1$) for all $n=0,1,2,3$ are zero within the experimental error limits. This result is in agreement with the saturation of the isospin bounds on unpolarized integrated cross sections observed by Roy /8/

in the same energy region. Also, it will be interesting to obtain the zeros-trajectories of all $\mathrm{Im}Z'^{(n)}_{ij}$ ($\mathrm{Im}Z^{(n)}_{ij}$) from Saclay [9] phase shifts and to compare these results with our results presented in Figs. 1a,b, 2a,b since it was pointed out by Höhler et al [10] that these phase shifts lead to quite different predictions for spin rotation parameters in certain kinematic regions.

Finally we remark that the continuations of Figs. 1a,b, 2a,b to higher energies are of great interest for an amplitude analysis and a phenomenological investigation of the pion-nucleon scattering.

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