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A RELATIVISTIC MODEL OF COMPOSITE MESONS

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1. Introduction

The principal features of high energy hadronic reactions suggest a simple picture of hadrons as composite systems built up of point-like constituents C freely moving within a sphere of a finite, approximately universal diameter D. (For a recent discussion and references see, e.g. [1-3]). The data on elastic and inelastic hadronic reactions and on the pion and the proton electric-charge radii seem to be consistent with D of order 1 fm [1]. From the well-known results on deep-inelastic lepton-hadron scattering it follows that these constituents are fermions of spin ½ [1]. The absence of exotic mesons suggests that mesons are built up from fermion C and antifermion $\bar{C}$ having isospins 0 and ½, and that the admixture of exotic states (such as $(\bar{C}C)(\bar{C}C)$ e.a.) is small. The exact nature of these fermions is not known. They may be Gell-Mann-Zweig quarks, "colored" quarks, "bare" baryons or something else. All these possibilities were widely discussed (see, e.g. [4]), and here we consider only dynamical problems which are more or less independent of the exact properties of C particles (other than their spin, mass, and their mutual interaction). As composite models for baryons do essentially depend on symmetry properties of constituents, we restrict our discussion to mesons.

In building composite mesons the first dynamical question is: which formalism to use for describing tightly bound systems of C and $\bar{C}$. We choose the Bethe-Salpeter equation (BSE) for the relativistic wave function or, equivalently, the homogeneous Edwards equation for the bound state vertex function [5-7]. To give a physical interpretation of the wave function and to exclude the exotic states of the second kind[4] it is useful to consider the bound-state wave function with equal time coordinates of C and $\bar{C}$ (cf.[2]). Here these problems are not discussed.

The second question is: which interaction potential (kernel) to choose to glue C and $\bar{C}$. The most popular potential nowadays is the four-dimensional oscillator well which is capable of reproducing linear Regge trajectories and keeps constituents inside hadrons[2,8]. However, it is difficult to interpret this potential in any reasonable field theory[9] or in terms of particle exchanges.

An appealing idea is to treat the $\bar{C}C$ potential as a bootstrap potential, i.e.,to consider it as determined by exchange of resonances R , which are built up of the same constituents glued by the same potential. This idea was developed by several authors (see,e.g.[10]), and usually the kernel is approximated by low-lying resonances (π, ρ , ω , etc.). However, the corresponding potential rapidly varies with the relative distance r between C and C and is singular at $r=0$. This qualitatively disagrees with the picture of free constituents. Moreover, the singularity at the origin $r=0$ results in the well-known Goldstein difficulty[6], and to avoid it a cut-off is necessary. With such potentials, it is also impossible to explain why the Regge trajectories are growing, and there is no universal size D of hadrons. Our idea is to take into account exchanges of high-mass resonances. As far as the average number $N(m)dm$ of resonance states in the interval $(m, m+dm)$ is rapidly growing with their mass m , one may suspect that the contribution of high-mass resonances might be not negligible.

In fact, if the distribution $N(m)$ is asymptotically exponential[11], i.e., $N(m) \sim exp(ma)$, and if the average contribution of the individual resonance state is proportional to $r^{-1} e^{-mr}$, then the average total contribution of resonances of mass m is $\sim r^{-1} exp(ma-mr)$, and for $r \approx a$ the contribution of infinitely massive resonances is infinite (the integral of this expression over the interval $(m_0, +\infty)$) diverges for $r \leqslant a$.

The exponential growth of the resonance spectrum $N(m)$ in the interval $m \lesssim$ 1.5 GeV, with $a \propto 4\div6$ GeV^{-1} , is the empirical fact (see[11] and what follows), and thus we face two possibilities.1)The resonance spectrum is exponentially growing up to some finite mass M and for $m \geqslant M$ it is dying away (or its growth is less than exponential). In this case, the potential corresponding to the exchange of resonances with mass $\leqslant M$ forms a deep well (or a "core") of a radius of order $a \propto 4\div6$ GeV^{-1} . The average depth of the well is $\gtrsim M$, and therefore the mass of the constituents must be greater than M . This may give a justification of quark models with heavy quarks in terms of the bootstrap inter-quark potential (cf.[4]). 2) A much more interesting possibility is suggested by statistical and dual resonance models, in which the resonance spectrum is ideed asymptotically exponential, i.e., $M = \infty$. Here we construct the corresponding potential and investigate its simplest consequences for composite models of mesons.

2. The pion vertex

In what follows we treat in some details the composite pion, which is described by means of the Euclidean Bethe-Salpeter (or Edwards) equation with a local potential (kernel).

As explained above, we consider the potentials having exponentially infinite spectral functions, connected with the empirical mass spectrum of resonances. The $\bar{C}C$ potential in the coordinate space (the coordinate is $\tau = \tau_C - \tau_{\bar{C}}$ where τ_C and $\tau_{\bar{C}}$ are the Euclidean four-dimensional coordinates of C and $\bar{C}$) is the four-dimensional Fourier transform of $\bar{C}$-C scattering amplitude. For simplicity we consider only on-mass-shell amplitudes giving local potentials. The equation for the pion vertex $\Gamma(p,k)$ is presented in the diagram form on Fig.1. The corresponding equation for the BS wave function $\chi_K(P)$ follows from the relation[7]

$$\chi_K(p) = S'_F(p-k/2)\,\Gamma(p,k)\,S'_F(p+K/2), \qquad (1)$$

where S'_F is the exact propagator of the C-particle. As the first approximation we take here the bare fermion propagators, the dressed propagators will be discussed later. To simplify the discussion we solve here only the pion equation in which the dependence of the vertex Γ on k may be neglected. We simply put in the pion equation $k=0$ and $m_\pi = 0$, and in calculating the physical processes with the pions, $k \neq 0$, $m_\pi \neq 0$, we use an approximate vertex $\Gamma(p) \equiv \Gamma(p,0)$. This approximation is reliable if the C-particle mass M_C is much greater than m_π.

Now the most general form of the potential is[2]

$$V(-t,s) = (1\otimes 1)V_S(-t,s) + (\gamma_5\otimes\gamma_5)V_P(-t,s) + \tfrac{1}{4}(\gamma_\mu\otimes\gamma^\mu)V_V(-t,s) +$$
$$+\tfrac{1}{4}(\gamma_5\gamma_\mu\otimes\gamma_5\gamma^\mu)V_A(-t,s) + \tfrac{1}{\sqrt{6}}(\sigma_{\mu\nu}\otimes\sigma^{\mu\nu})V_T(-t,s), \qquad (1a)$$

where V_i are scalar functions, and for the Dirac γ-matrices the Bjorken-Drell[13] notation is used. (Here we consider only isoscalar potential, inclusion of the isovector one is quite obvious). The $k=0$ pion vertex has the form $\Gamma(p) = i\vec{\tau}\,\gamma_5 F(-p^2)$, and F obeys the equation

$$F(-p^2) = \int \frac{d^4q}{(2\pi)^4 i}\, V_\pi[-(p-q)^2](M_C^2 - q^2 - i0)^{-1} F(-q^2), \qquad (2)$$

where

$$V_\pi(-t) = \sum_i \varepsilon_i V_i(-t), \quad (i = S,P,V,A,T),$$
$$V_i(-t) \equiv V_i(-t,0), \quad \varepsilon_S = \varepsilon_V = -\varepsilon_T = 1, \quad \varepsilon_P = \varepsilon_A = -1. \qquad (3)$$

One can easily identify the particles (or the Regge trajectories) contributing to different potentials V_i by using the t-channel Regge formalism developed for treating the conspiracy problem[14]. Here we simply suppose that all V_i are of the same form, i.e., $V_i(-t) = f_i V(-t)$, and $V(-t)$ is determined by the mass spectrum of all resonances. Then $V_\pi(-t) = f_\pi V(-t)$, where $f_\pi = \sum_i \varepsilon_i f_i$, and by solving Eq. (2) the constant f_π will be determined in terms of M and the parameters characterizing $V(-t)$.

3. The potential in pion equation

Now let us discuss the potential $V(-t)$. If it is determined by exchange of a spinless particles with mass m_R, it has the form $g_R(m_R - t - i0)^{-1}$, and the corresponding Eq. (2) was carefully investigated by many authors[5-7]. The imaginary part of this potential on the cut $t>0$ (the spectral function) is $\pi g_R\,\delta(t-m_R^2)$. The potential in the Euclidean coordinate representation can be defined in terms of the spectral function as follows (this is the Källen-Lehmann representation of $U(\tau)$[13])

$$U(\tau) = \int_0^\infty dm^2\,\sigma(m^2)\,\Delta_F(m^2,\tau) = \int \frac{d^4k}{(2\pi)^4}\, e^{-ik\tau}\, V(k^2). \qquad (4)$$

Here $\Delta_F(m^2,\tau) = m K_1(m\tau)/4\pi^2\tau$ is the Euclidean Feynman propagator, and $\tau^2 = \vec{\tau}^2 + \tau_4^2$. For one-particle exchange $\sigma(m^2) = g_R\,\delta(m^2-m_R^2)$ and $U(\tau) = g_R\,\Delta_F(m_R^2,\tau)$. The spectral function corresponding to the exchange of many infinitely narrow resonances R may be approximated by the distribution

$$\sigma(m^2) = \sum_n g_n\,\delta(m^2-m_n^2)\,N_n\,, \tag{5}$$

where $N_n=(2J_n+1)$ is the number of different spin states of R_n. Smoothing off this distribution (replacing δ - functions by Gaussian exponentials, as in ref.[11]), we find

$$\sigma(m^2) \approx g(m^2)\,\rho(m^2), \tag{6}$$

where $\rho(m^2)$ is the average density of exchanged resonance states and $g(m^2)$ is the average $\bar{C}CR$ coupling constant.

To obtain some useful information from the empirical mass spectrum we make the simplest assumptions that g is independent of m and that $\rho(m^2)$ is proportional to the density of all observed resonance states $\rho_{tot}(m^2)$. As the pion is composed of constituents with isospin ½, only isoscalar and isovector resonances contribute to the pion equation, and the second assumption is true if the distributions $\rho_{B,I}(m^2)$ of baryons $(B=1;\, I=0,\frac{1}{2},1,\frac{3}{2})$ and mesons $(B=0;\, I=0,\frac{1}{2},1)$ are proportional to each other, i.e., $\rho_{B,I}(m^2) = A_{B,I}\,\rho_{tot}(m^2)$. For high-mass resonances this seems to be consistent with present experimental evidence[15].

In the above discussion we have included the spin of the resonance R_n only in the factor $2J_n+1$. To justify this assumption, imagine that we know the distribution $\tilde{\rho}(m^2,J)$ of resonance masses and spins. Then the spectral function of the

potential $U(\tau)$ may be written in the form (cf.[14]).

$$\sigma(m^2) = \int_0^\infty dm_R^2 \sum_{J=0}^{\infty} \tilde{\rho}(m_R^2,J)\,g_R\,\delta(m^2-m_R^2)\,(2J+1)\,P_J\left(1+\frac{2s}{m^2-4M_c^2}\right), \tag{7}$$

where $1+2s/(t-4M_c^2)=\cos\theta_t$, and θ_t is the t-channel scattering angle. At first sight, this spectral function depends on s, but we are interested in the dependence of σ on m for $m\to\infty$ and for $s<4M_c^2$ (bound states!), and it will be immediately shown that in this domain the s-dependence of σ can be neglected. Suppose that $\tilde{\rho}(m^2,J)$ is such that the average value $\bar{J}$ contributing to Eq. (7) for large m is bounded by m^α, where $\alpha<2$. This assumption is valid in statistical and dual resonance models[16], where the spin distributions are proportional to $\exp(-J^2/md)$, $d\sigma a$, and $\exp(-J/md)$, respectively. Then for $2s\,(m^2-4M_c^2)^{-1}\ll 1$

$$P_J\left(1+\frac{2s}{m^2-4M_c^2}\right) \approx 1 + \frac{2s}{m^2-4M_c^2}P_J'(1)+\ldots \approx 1+\frac{\bar{J}s}{m^2-4M_c^2}+\ldots \approx 1, \tag{8}$$

as $s\bar{J}/m^2 \to 0$ for $m\to\infty$, and we effectively obtain the equation (6) with

$$\rho(m^2) \approx \sum_{J=0}^{\infty}(2J+1)\,\tilde{\rho}(m^2,J).$$

All these considerations make sense if $\rho(m^2)$ is asymptotically exponential

$$\rho(m^2) \underset{m^2\to\infty}{\sim} c\,m^b \exp(ma). \tag{9}$$

In statistical models such behaviour of ρ is required by bootstrap conditions[11]. A very similar asymptotic behaviour of ρ was observed in the Veneziano model[12], where the parameter a was found to be equal to $a=2\pi(\frac{2}{3}\alpha')^{1/2}\approx 4.7\ GeV^{-1}$ (if we take for α' the slope of the ρ trajectory). The empirical density of states $\rho_{tot}(m^2)$ in the interval $0\leqslant m\leqslant 1$ GeV was obtained by R.Hagedorn[11], who fitted the smoothed experimental curve by the exponential

$$\rho_{tot}(m^2) \approx 0.83\,(2m)^{-1}(m_0^2+m^2)^{-5/4}\,\exp(ma),$$

where $m_0 \approx 0.5$ GeV and $a \approx 6.25\ \mathrm{GeV}^{-1}$.

On the basis of these observations $\rho(m^2)$ is supposed to be asymptotically exponential and the parameter a can be derived from the empirical distribution of resonance levels. To obtain a simple analytic form for the potential $U(r)$ for all r we use, instead of the exponential, the Bessel functions. Namely, we fit the empirical spectral density by the modified Bessel function of the first kind: $A_{tot}\cdot(2m)^{-1}\cdot I_1(ma)\ \mathrm{GeV}^{-2}$. Taking into account all well established resonances (see[15]) we find a good fit in the interval $0.2 \leq m \leq 1.4$ GeV with the parameters $a \approx 4\ \mathrm{GeV}^{-1}$ and $A_{tot} \approx 18\ \mathrm{GeV}^{-1}$. The potentials corresponding to this spectral function can easily be calculated for $r>a$ by substituting $gA_{tot}(2m)^{-1}I_1(ma)$ in Eq. (4) (see[17], Chap. 10, Sec.3, eq. (17)) and is equal to $\left(\frac{g}{4\pi^2}\right)A_{tot}\,a\,r^{-2}(r^2-a^2)^{-1}$. This potential, analytically continued to the interval $0 \leq r < a$ is singular at the origin and, as was discovered in ref.[6], the corresponding BSE has no discrete spectrum. To obtain a regular potential we therefore modify it by adding the term $\sim \delta(m^2)$ to the spectral function (alternatively one can use the function $J_1(ma) + I_1(ma)$ instead of $I_1(ma)$, obtaining the potential $\left(\frac{g}{4\pi^2}\right)2A_{tot}\,a\,(r^4-a^4)$ which is as good as the potential used here). Following these considerations, we finally use the spectral function

$$\sigma(m^2) = gA\cdot(2m)^{-1}\left[I_1(ma) + 4ma^{-1}\,\delta(m^2)\right], \tag{10}$$

giving the potential

$$U(r) = (g/4\pi^2)\,A a^{-1}(r^2-a^2)^{-1} \equiv f^2(r^2-a^2)^{-1}. \tag{11}$$

This potential is singular at the point $r=a$, and to calculate its Fourier transform a rule of integration over this point is required. The most natural one is the principal value prescription, i.e.,

$$U(r) = f^2\,\mathcal{P}\left\{\frac{1}{r^2-a^2}\right\}. \tag{12}$$

The origin of this recipe can be understood in the context of nonpolynomial field theories (see[18] and references quoted therein). The equation (4) is the Källen-Lehmann representation for the effective propagator describing resonance exchanges between C and $\bar{C}$, and $\sigma(p^2)\cdot$ is its imaginary part on the out $0 \leq p^2 \equiv p_0^2 - \vec{p}^2$. Therefore, to find the potential is to find the propagator with the exponentially rising imaginary part. This problem was solved in nonpolynomial field theories, and such propagators (or, strictly speaking, their Fourier transforms into momentum space) are usually called superpropagators x). The principal value prescription corresponds to constructing the minimally singular[18] superpropagator having Eq. (10) as its imaginary part.

4. Solution of the pion equation and pion decays

Using this recipe one can easily calculate the Fourier transform of Eq. (12) (this result can be obtained using[17], Chap. 8, Sec. 5, Eq. (12), and analytic continuation in a-plane):

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x) For example, in the theory with the interaction $\mathcal{L} = G{:}\bar{\Psi}\Psi\,\exp(g\varphi^+\varphi)$, where φ is the massless charged scalar (or pseudoscalar) field, the superpropagator $\langle T\,\mathcal{L}(r)\mathcal{L}(0)\rangle$ is proportional to $\mathcal{P}\{(r^4-a^4)^{-1}\}$, where $a^2 = g/4\pi$ (see[19]).

$$V[(p-q)^2] = \int d^4 z \, e^{i(p-q)z} \, U(z) = -4\pi^2 f^2 \tfrac{1}{2}\pi a \, |p-q|^{-1} \, Y_1(a|p-q|), \qquad (13)$$

where Y_1 is the Bessel function of the second kind. Now Eq. (2) for the pion vertex can be written in both coordinate and momentum representations. The latter is more convenient for out present purposes. Writing Eq. (2) in the Euclidean representation $(-p^2 = \vec{p}^2 - p_0^2 \Rightarrow p^2 = \vec{p}^2 + p_4^2, \, i dq_0 \to -dq_4)$ and putting in it the potential (13) we obtain after performing angular integrations

$$F(x) = \tfrac{1}{2}\pi f^2 \Big\{ \int_0^x dy \; x^{-1} y^2 (y^2+\mu^2)^{-1} Y_1(x) J_1(y) F(y) + \int_x^\infty dy \; x^{-1} y^2 (y^2+\mu^2)^{-1} J_1(x) Y_1(y) F(y) \Big\}, \qquad (14)$$

where

$$x = pa, \quad y = qa, \quad \mu = M_c a, \quad F(p^2) \equiv F(x), \quad F(q^2) \equiv F(y).$$

This integral equation is equivalent to the differential equation for $u(x) = x^{3/2} F(x)$

$$\frac{d^2 u}{dx^2} + \left[1 - f^2 (x^2+\mu^2)^{-1} - \tfrac{3}{4} x^{-2} \right] u(x) = 0, \qquad (15)$$

with boundary conditions

$$x^{1/2} u(x) \to 0, \; x \to 0 \; ; \quad u(x) \sim x^{1/2} Y_1(x), \; x \to \infty . \qquad (16)$$

The well-known normalization condition for $\chi_K(P)$ (see[5-7]) and Eq. (1) define the normalization condition for $u(x)$

$$\int_0^\infty dx \; u^2(x) (x^2+\mu^2)^{-2} = 8\pi^2. \qquad (17)$$

We have solved the equations (15) and (16) by using WKBJ method x). For $\mu=0$ the analytic solution evidently exists, which was used to test the WKBJ approximation. The eigenvalue problem (Eqs. (15) and (16)) has the discrete f^2 spectrum. For small and large values of μ the WKBJ approximations for f^2 are:

x) All numerical results were obtained in collaboration with D.Mavlo, I.Puzynin and N.Truskova.

$$f^2 = N(N+2)\{1 + \tfrac{1}{2}\mu^2 [N(N+2)+1]^{-1} + O(\mu^4)\}, \quad \mu^2 \ll 1,$$

$$f^2 = 2N\mu - \tfrac{1}{2}N^2 + O(\mu^{-1}), \quad \mu^2 \gg 1, \qquad (18)$$

where $N = 1,3,5, \ldots$, and the lowest eigenvalue corresponding to $N=1$ is to be chosen to describe the pion.

To obtain further information on the parameters f, a and M_c we calculated $\Gamma(\pi \to \mu\nu)$ and $\Gamma(\pi \to \gamma\gamma)$ by using one-loop diagrams Fig.2. For $C\bar{C} \to \mu\nu$ transition the nonrenormalized V-A vertex was used, and $C\bar{C} \to \gamma\gamma$ transition was approximated by the C-pole diagram with bare $C\bar{C}\gamma$ vertices. The results of the numerical calculations of $\Gamma(\pi \to \mu\nu)$ and $\Gamma(\pi \to \gamma\gamma)$ (with WKBJ approximation for $\bar{C}C\pi$ vertex) were compared with the experimental values taken from PDG[15]. Combining these with the eigenvalue conditions for f^2 we can estimate all the parameters. The results are the following: for C particles with integral charges

$$a \approx 4.2 \; GeV^{-1}, \quad f^2 \approx 8, \quad M_c \approx 0.9 \; GeV, \qquad (19a)$$

and for fractionally charged C (quarks)

$$a \approx 5.4 \; GeV^{-1}, \quad f^2 \approx 10, \quad M_c \approx 0.4 \; GeV. \qquad (19b)$$

These numbers should not be considered too seriously, due to the approximations used. Nevertheless, the value of a is consistent with our mass spectrum interpretation of the U- potential.

In the above discussion we have ignored the spin and isospin structure of U . Now let us briefly discuss this problem. In meson models with nonexotic quantum numbers the great degeneracy of levels, leading to the exponentially infinite density of states, is due to daughter trajectories[12]. Then, the highest trajectories (ρ and ω) with their daughters give the main contribution to the potential (assuming all $\bar{C}C$ coupling constant of the same order) determining its spin and isospin (or SU_3 spin) structure x). To treat particles other than the pion a careful analysis of this structure is necessary. In particular, we have to explain the absence of tightly bound CC states. This will be discussed elsewhere. (With vector forces this could probably be explained). Here we only stress one of the unusual features of our approach: the dominant part of the potential is connected with highest trajectories (as in the Regge theory) rather than with lightest exchanged resonances (contrary to the usual dispersion ideas).

Above we also ignored the finiteness of range of forces which glue constituents. In fact the forces corresponding to different structures in Eq. (1a) may have different radii. For example, the range of V_P must be greater than m_π^{-1} whereas that of V_V is $\gtrsim m_\rho^{-1}$ (correspondingly, $\sigma_P(m^2)=0$ for $m<m_\pi$ and $\sigma_V(m^2)=0$ for $m<m_\rho$, see Eq. (4)). As far as we consider only tightly bound states of C and $\bar{C}$ (like π or ρ) this does not significantly influence our results. However, for more massive states, or for possible bound states of two C-particles (which we have to forbid) this is quite important. We can understand this by considering the shape of the potential. From Fig.3 one can see that in the "attractive" case (Fig.3a)the effect of the short range on bound states of the potential is not crucial but for the "repulsive" case (Fig.3b) the situation changes drastically as tightly bound states may exist only for very light constituents.

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x)Therefore, one may suspect that the potential is dominated by the vector $\gamma_\mu \otimes \gamma^\mu$ structure.

The easiest way to demonstrate this is to consider the Schrödinger equation $u''=[\varkappa^2+U(r)]u$, where $\varkappa^2=M_c^2-\frac{1}{4}s$. A tightly bound state corresponds to $s \ll 4M_c^2$, i.e., $\varkappa \approx M_c^2$, but in case b) the eigenvalue $\varkappa^2$ must be small. This gives us a possibility of explaining the absence of CC states by assuming "attractive" vector interaction between C and C. The problem of the existence of CCC bound states requires much more delicate considerations and is not discussed here.

5. Fermion propagator

Here we briefly discuss the fermion propagator S_F' .Consider the approximate Johnson type equation for S_F' which in the diagram form, is represented by Fig.4. Suppose that the bare fermion mass is zero, and the physical mass is created by the virtual emission and absorption of the resonances, effectively described by the superpropagator $V[-(p-q)^2]$ (see Fig.4). The propagator S_F' has the form

$$S_F' = [\alpha(-p^2) - \hat{p}\,\beta(-p^2)]^{-1}, \tag{20}$$

where $\alpha=0$ and $\beta=1$ for the bare propagator S_F, and the scalar functions α and β satisfy the equations $(-p^2 \underset{Eucl.}{\to} p^2)$

$$\alpha(p^2) = -\frac{\pi a}{2} f_\alpha \int \frac{d^4q}{(2\pi)^4} \frac{\alpha(q^2)}{\alpha^2+q^2\beta^2} \frac{Y_1(a|p-q|)}{|p-q|}, \tag{21}$$

$$\beta(p^2) = 1 - \frac{\pi a}{2} \int \frac{d^4q}{(2\pi)^4} \frac{\beta(q^2)}{\alpha^2+q^2\beta^2} \frac{Y_1(a|p-q|)}{|p-q|} \frac{(pq)}{q^2}, \tag{22}$$

where

$$\begin{gathered} f_\alpha = \sum_i \varepsilon_\alpha^i f_i, \quad f_\beta = \sum_i \varepsilon_\beta^i f_i, \quad (i=S,P,V,A,T) \\ \varepsilon_\alpha^P = \varepsilon_\alpha^V = 1 = \varepsilon_\beta^P = \varepsilon_\beta^S; \quad \varepsilon_\alpha^S = \varepsilon_\alpha^A = \varepsilon_\alpha^T = -1, \\ \varepsilon_\beta^V = \varepsilon_\beta^A = 1/2; \quad \varepsilon_\beta^T = -2/3. \end{gathered} \tag{23}$$

These nonlinear equations have a solution satisfying the boundary conditions $\alpha \to 0$, $\beta \to 1$, $p^2 \to \infty$. Then, by iterating Eq. (22) one can prove that $\beta(p^2)$ may be represented in the form

$$\beta(p^2) = 1 - f_\beta [4\pi^2 a^2 p^2]^{-1} + (pa)^{-5/2}\, u_2(pa) . \tag{24}$$

Introducing the notation

$$\alpha(p^2) \equiv (pa)^{-3/2}\, u_1(pa) , \tag{25}$$

and neglecting the terms $\sim u_1^2$ and $\sim u_2^2$ in the denominators of Eqs. (21) and (22) we arrive at the approximate linear equations for u_1 and u_2, which in the differential form are ($x = pa$, $f_1 = -f_\alpha / 4\pi^2$, $f_2 = -f_\beta / 4\pi^2$, $\mu^2 = 2 f_2$):

$$u_1'' + [1 - f_1 (x^2 + \mu^2)^{-1} - \tfrac{3}{4} x^{-2}]\, u_1(x) = 0 ; \tag{26}$$

$$u_2'' + [1 - f_2 (x^2 + \mu^2)^{-1} - \tfrac{15}{4} x^{-2}]\, u_2(x) = 0 . \tag{27}$$

In deriving these equations we have neglected the nonhomogeneous term (in Eq. (27)) of order $\sim x^{-3/2}$. The boundary conditions for u_1 and u_2 are

$$u_1(x) \sim x^{1/2}\, Y_1(x) , \quad u_2(x) \sim x^{1/2}\, Y_2(x) , \quad x \to \infty ; \tag{28}$$

$$u_1(x) \to 0 , \qquad u_2(x) \to 0 , \qquad x \to 0 ,$$

immediately follow from the linearized form of Eqs. (21) and (22). The solutions of Eqs. (28) and (27) give a good approximation to those of Eqs. (21) and (22) for large x and not too much deviate from them for small x. Therefore we consider the solution of the linear equations (26) and (27) as a reasonable approximation to the solution of the nonlinear equations (21) and (22) for all x x). The solution of the linear problem was found numerically (in collaboration with I.V.Puzynin) and the result is

$$f_1 = 8.35 , \qquad f_2 = 9.25 , \qquad \mu = 4.30 . \tag{29}$$

Note that the possibility of determining the parameter μ is due to the relation between μ and f_2 resulting from the nonlinear nature of the original problem.

Let us discuss the physical interpretations of the solution. If the coupling is a purely vector (or axial-vector) one, Eq. (26) is identical to the equation for the massless pion vertex, if $\mu = M_c a$ and $f_1 = f$. This is a consequence of the general theorem on the spontaneous breakdown of γ_5-symmetry[21], if the pion is treated as the corresponding Goldstone particle. This interpretation is strongly supported by the striking coincidence of the parameters f_1 and μ given by Eq. (29) with the values given by Eq. (19a). Therefore it is not unreasonable to suppose that C-particles are something like bare nucleons, as in the Fermi--Yang model[22]. This conclusion is supported by the fact that $\bar{C}C\pi$ coupling constant (i.e., $F(-M_c^2)$) is found to be of the order 20÷25 (this result was obtained in collaboration with D.Mavlo and N.Truskova) and favours the vector forces giving the repulsive core of a radius $a \approx 0.8\ fm$ for CC-interaction, which in qualitative agreement with nuclear physics data[23].

6. Conclusion

Finally, we mention another remarkable feature of our model, which can be tested experimentally: the pion electromagnetic form-factor $G_\pi(-q_\gamma^2)$ has oscillating terms in the space-like asymptotic region $q_\gamma^2 = \vec{q}_\gamma^{\,2} + q_{\gamma 4}^2 \to +\infty$. We have shown this in the static non-relativistic approximation for G_π in which $(\tau_0 = 0, \vec{\tau}^{\,2} = \tau^2)$

$$G_\pi(q_\gamma^2) = \int_0^\infty d\tau\, \frac{2 \sin(q_\gamma \tau/2)}{q\tau}\, u^2(\tau) = G_0(q_\gamma^2) + c_1 (q_\gamma a)^{-\beta_1} \cos(q_\gamma a/2) + \ldots$$

where G_0 is a smoothly decreasing function of q_γ, determined by the behaviour of $u(\tau)$ near $\tau = 0$, and the second term depends on the infinite barrier at $\tau = a$ x). For our potential

x) For potential with a marginal singularity or for regular potentials there are no such oscillating terms (see, e.g. 1-3).

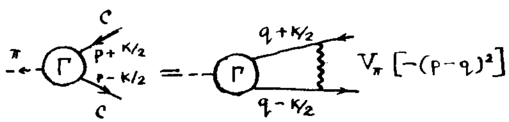

Fig. 1.

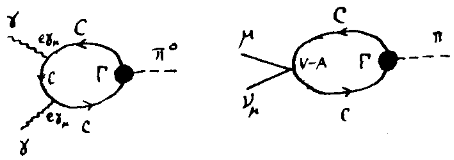

Fig. 2.

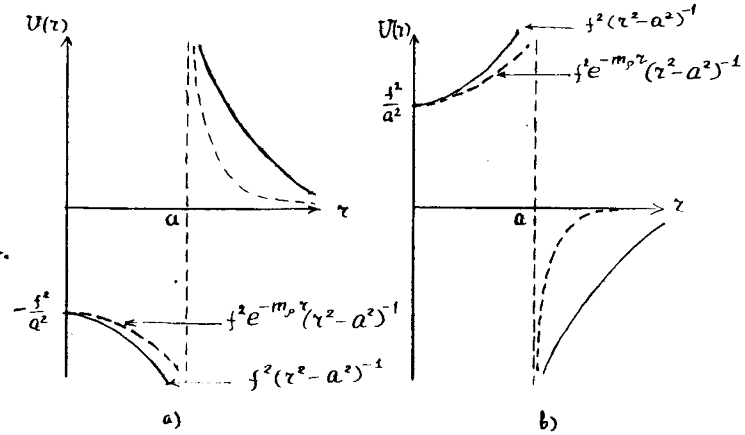

Fig. 3.

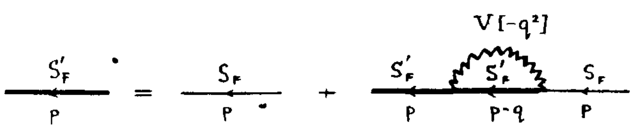

Fig. 4.

$\ell_1 \propto 3$ and c_1 is of order 4÷5 (for $M_c \approx 1$ GeV). This asymptotic expansion is valid for $\frac{1}{2}q_\gamma a \gg 1$, or $q_\gamma \gg 0.5$ GeV. The static nonrelativistic approximation is of course very crude but the principal qualitative fact, the existence of oscillations with the period of order 1 GeV, survives also in better approximations. The experimental discovery of such oscillations would be a very serious evidence in favour of the composite model discussed above.

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