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ELECTROMAGNETIC NUCLEON MASS DIFFERENCE, SUM RULES FOR DEEP INELASTIC SCATTERING AND EQUAL TIME COMMUTATORS

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Electromagnetic Nucleon Mass Difference,
Sum Rules for Deep Inelastic Scattering
and Equal Time Commutators

In one-photon approximation the connection between the finiteness of electromagnetic mass corrections and the light-cone behaviour of the product of electromagnetic currents has been studied. In the canonical case the complete removal of divergences leads to two sum rules for the scaling functions which are equivalent to vanishing of certain equal-time commutators. Using the technique of the Jost-Lehmann-Dyson representation problems with subtractions are avoided.

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1. Introduction

Both the usual approach to deep inelastic eN scattering[1] and the theory of electromagnetic mass corrections of nucleons leading to the so-called Cottingham formula[2]

$$\begin{aligned} \delta m &= \delta m^{p} - \delta m^{n} \\ \delta m^{p,n} &= \frac{\pi\alpha}{i} \int d^4 q \, \frac{-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}}{q^2 + i\varepsilon} \, T_{\mu\nu}^{p,n}(p,q) \end{aligned} \tag{1}$$

are based on the one-photon exchange approximation. When taking into account the experimental results for deep inelastic scattering there arises the possibility of a quadratical and a logarithmical divergence in the expression of the mass difference. Therefore two sum rules are found as the consistency conditions between the above-mentioned two approaches inserting the automodel or scaling behaviour of deep inelastic ep-resp. en- scattering into the virtual Compton amplitude of (1) and demanding finitness of the resulting mass difference.

During the last ten years many authors have studied the pn-mass difference, respectively its divergent parts only, obtaining quadratical or logarithmical sum rules. Applying a Wick rotation to the Cottingham formula they used dispersion relations for $T_{\mu\nu}(q,p)$ at arbitrary, fixed q^2 [3]. As a rule, however, these dispersion relations contain subtractions, so that further assumptions are required, in particular about the number and the q^2-independence of subtractions. In some cases these additional assumptions influence the final structure of the sum rules. Other authors investigating relations between the mass divergences and some equal-time commutators have used integral representations for the virtual Compton amplitude [4,5]. They have not succeeded, however, in establishing a unique relation between the logarithmical divergence and ETC.

In an extension of an earlier paper [6] we will study the connections between finiteness of the electromagnetic mass corrections, scaling behaviour and ETC applying a Dyson-Jost-Lehmann representation [7] for the scattering amplitude of virtual Compton scattering in forward direction. As basic physical assumption which is suggested by experiment we choose the canonical light-cone behaviour of the current commutator

$$\sum_\sigma \langle p\sigma|[j_\mu(x),j_\nu(0)]|p\sigma\rangle \approx 8\pi(g_{\mu\nu}\Box-\partial_\mu\partial_\nu)[G_0(x_0)\varepsilon(x_0)\delta(x^2)+G_1(x_0)\varepsilon(x_0)\theta(x^2)]$$
$$+8\pi[-p_\mu p_\nu\Box+(p_\mu\partial_\nu+p_\nu\partial_\mu)(p\partial)-g_{\mu\nu}(p\partial)^2]\,G_2(x_0)\varepsilon(x_0)\,\theta(x^2)\,, \tag{2}$$
$$x^2\approx 0\,,$$

with

$$\langle p'\sigma'|p\sigma\rangle = 2p_0(2\pi)^3\,\delta(\vec p-\vec p\,')\,\delta_{\sigma\sigma'}\,.$$

For the structure functions $V_i(q,p)$ of the current commutator defined by

$$W_{\mu\nu}(q,p)=\frac{1}{8\pi}\sum_\sigma\int dx\, e^{iqx}\langle p\sigma|[j_\mu(x),j_\nu(0)]|p\sigma\rangle \tag{3}$$
$$=(-g_{\mu\nu}q^2+q_\mu q_\nu)V_1+[p_\mu p_\nu q^2-(p_\mu q_\nu+p_\nu q_\mu)pq+g_{\mu\nu}(pq)^2]V_2$$

the ansatz (2) is equivalent to a scaling behaviour

$$V_1(\nu,\xi)\approx\frac{1}{\nu}h_0(\xi)+\frac{1}{\nu^2}h_1(\xi)\,,$$
$$V_2(\nu,\xi)\approx\frac{1}{\nu^2}h_2(\xi) \tag{4}$$

for $\nu=2pq\to+\infty$, $\xi=-\frac{q^2}{\nu}$ fixed.

In the language of the parton picture we take into account both spin 0 and spin 1/2 partons.

Under this assumption we established the following sum rules

$$\int_0^1 d\xi\, h_0^{(p-n)}(\xi)=0\,,$$
$$\int_0^1 d\xi\,[\xi^2 h_0(\xi)+\tfrac{1}{2}\xi h_2(\xi)-\xi h_1(\xi)]^{(p-n)}=0 \tag{5}$$

which guarantee the complete removal of divergences from the expression of the pn-mass difference (also in that case when both a quadratical and a logarithmical divergence exist). In terms of ETC the first sum rule is equivalent to vanishing of the operator Schwinger term

$$\sum_\sigma\langle p,\sigma|[j_0(x),j_i(0)]|p,\sigma\rangle^{(p-n)}_{x_0=0}=0 \tag{6}$$

(i spatial index).

In addition we have found a unique relation between the logarithmical sum rule and vanishing of a more complicated ETC, namely,

$$\sum_\sigma\langle p,\sigma|[\Box(\partial_k j_0(x)+\partial_0 j_k(x)),j_i(0)]|p,\sigma\rangle^{(p-n)}_{x_0=0}=0,\quad (k\neq i)\,. \tag{7}$$

We remark, however, that this latter equivalence holds only if $\int d\xi\,\xi^2 h_0(\xi)^{p-n}=0$, especially, if there does not appear a quadratic divergence from the beginning, $h_0(\xi)\equiv 0$, i.e. there are no spin 0 partons.

II. Scaling Behaviour in the Framework of DJL Representation

In the one-photon exchange approximation deep inelastic scattering is described by the invariant, causal structure functions $V_i(q,p)$ or $W_i(q,p)$ of the electromagnetic tensor

$$W_{\mu\nu}(q,p) = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) W_1 + \left(p_\mu - \frac{pq}{q^2} q_\mu\right)\left(p_\nu - \frac{pq}{q^2} q_\nu\right) W_2 \;, \tag{8}$$

For the measurable structure functions W_i the scaling behaviour reads

$$\begin{aligned} W_1(\nu,\xi) &\approx f_1(\xi) + \frac{1}{\nu} g_1(\xi) \;, \\ W_2(\nu,\xi) &\approx \frac{1}{\nu} f_2(\xi) + \frac{1}{\nu^2} g_2(\xi) \end{aligned} \qquad \text{for} \begin{cases} \nu = 2pq \to +\infty \\ \xi = -\frac{q^2}{\nu} \end{cases} \text{fixed.} \tag{9}$$

The scaling functions of W_i and V_i are connected by

$$\begin{aligned} h_0(\xi) &= \frac{1}{\xi}\left[\frac{1}{4\xi} f_2(\xi) - f_1(\xi)\right] \;, \\ h_1(\xi) &= \frac{1}{\xi}\left[\frac{1}{4\xi} g_2(\xi) - g_1(\xi)\right] \;, \\ h_2(\xi) &= -\frac{1}{\xi} f_2(\xi) \;. \end{aligned} \tag{10}$$

Bogolubov et al.[8] proved the causality of these structure functions so that with symmetry and spectrality conditions they could use a Dyson-Jost-Lehmann integral representation for each of the structure functions $V_i(q,p)$. (with $p=(1,0,0,0)$)

$$V_i(q,p) = \iint d\vec{u}\, d\lambda^2\, \varepsilon(q_0)\, \delta\!\left(q_0^2 - (\vec{q}-\vec{u})^2 - \lambda^2\right) \psi^i(\vec{u},\lambda^2) \tag{11}$$

with

$$(\vec{u},\lambda^2):\quad |\vec{u}| \le 1 \;,\quad \lambda^2 \ge \left(1-\sqrt{1-\vec{u}^2}\right)^2 .$$

On this basis they considered the structure and scaling functions as tempered distributions of ξ . In the sense of convergence in the space of tempered distributions[9] the scaling behaviour of the structure functions is defined to be the limit of the sequence of integrals $\int d\xi\, f(\xi)\, V(\nu,\xi)$ for $\nu \to +\infty$ with arbitrary $f \in S$.

To obtain a definite scaling behaviour the following sufficient conditions for the spectral functions of the DJL representation have been formulated

$$\text{I.} \quad \lim_{\lambda^2\to\infty} \frac{\psi(\vec{u},\lambda^2)}{\lambda^{2k}} = \frac{\psi_0(\vec{u})}{\Gamma(k+1)} \;,\quad k > -1 \tag{12}$$

or

$$\text{II.} \quad \lim_{\lambda^2\to\infty} \psi_{(s+1)}(\vec{u},\lambda^2) = \psi_0(\vec{u}) \;,\quad s = 0,1,2\ldots \tag{13}$$

$\psi_{(s+1)}(\vec{u},\lambda^2) = \frac{1}{\Gamma(s+1)} \int_0^{\lambda^2} dy\, \psi(\vec{u},y)(\lambda^2-y)^s$ is the $(s+1)$- primitive of $\psi(\vec{u},\lambda^2)$ with respect to λ^2 . Note, that conditions I and II imply a classical behaviour of $\psi(\vec{u},\lambda^2)$ for large λ^2 . These sufficient conditions lead to the following scaling behaviour for the structure functions

$$\begin{aligned} V(\nu,\xi) &\approx \nu^k\, V(\xi) \\ V(\xi) &= 2\pi \left[\xi\, \psi_0(|\xi|)\right]^{-k-2} \end{aligned} \tag{14}$$

or

$$\begin{aligned} V(\nu,\xi) &\approx \frac{1}{\nu^{s+1}}\, V(\xi) \\ V(\xi) &= (-1)^{s+1}\, 2\pi \left(\frac{d}{d\xi}\right)^{s-1} \left\{\xi\, \psi_0(|\xi|)\right\} \;,\quad s = 1,2,\ldots \\ V(\xi) &= 2\pi \int_\xi^1 d\rho\, \rho\, \psi_0(|\rho|) \;,\quad s = 0 \end{aligned} \tag{15}$$

respectively.

In eq. (14) the notation of generalized derivatives

$$f^{(\alpha)}(x) = \frac{1}{\Gamma(-\alpha)} \int_x^1 dy\, f(y)(y-x)_+^{-\alpha-1} \tag{16}$$

has been employed[9] . $\xi\chi_0(|\xi|)$ is defined as

$$\int_{-1}^{+1} d\rho\, \rho\chi_0(|\rho|)\varphi(\rho) = \int_0^1 d\rho\, \rho^2 \chi_0(\rho)\, \frac{\varphi(\rho)-\varphi(-\rho)}{\rho}, \tag{17}$$

$\varphi \in S$.

Let us consider the structure function $V_1(\nu,\xi)$ with the scaling behaviour

$$V_1(\nu,\xi) \approx \frac{1}{\nu} h_0(\xi) + \frac{1}{\nu^2} h_1(\xi) .$$

Here $h_0(\xi)$ is determined by case II (eq. 15) with $S = 0$. The determination of $h_1(\xi)$ requires further conditions for the spectral function in comparison with (12), (13) (see Appendix). In our special case (s = 0 in (13)) we demand

$$\chi_{(1)}(\vec{u},\varkappa^2) = \chi_0(\vec{u})\,\theta(\varkappa^2) + X(\vec{u},\varkappa^2) \tag{18}$$

with

$$\lim_{\varkappa^2\to\infty} \chi_{(1)}(\vec{u},\varkappa^2) = \chi_0(\vec{u}) \neq 0 . \tag{19}$$

From the analysis of Appendix we get

$$h_0(\xi) = 2\pi \int_\xi^1 d\rho\, \rho\, \chi_0(\rho) \tag{20}$$

and

$$h_1(\xi) = 2\pi\xi^3\chi_0(|\xi|) - 4\pi\int_\xi^1 d\rho\,\rho^2\chi_0(|\rho|) + 4\pi\int_\xi^1 d\rho\,\rho\,\chi_0(|\rho|)(\rho-\xi)_+ + 2\pi\xi\,\chi_0(|\xi|) . \tag{21}$$

III. Derivation of Sum Rules

Based on the idea about the electromagnetic origin of the nucleon mass difference the Cottingham formula (1) determines the electromagnetic mass corrections in terms of the amplitude for virtual Compton scattering in forward direction (for proton and neutron, respectively, averaged over nucleon spins)

$$T_{\mu\nu}^{p,n}(q,P) = \frac{i}{4\pi}\sum_\sigma \int dx\, e^{iqx} \langle P\sigma|T(j_\mu(x)j_\nu(0))|P\sigma\rangle . \tag{22}$$

Obviously the behaviour of $T_{\mu\nu}(q,P)$ as $q_\mu \to \infty$ is important for the convergence of (1). A more detailed analysis shows that just the behaviour in the Bjorken region and the properties of the scaling functions are crucial for convergence.

We remember[10] that the T-product of (22) contains an uncertainty at $x_\mu = 0$:

$$T(j_\mu(x)j_\nu(0)) = \theta(x_0)j_\mu(x)j_\nu(0) + \theta(-x_0)j_\nu(0)j_\mu(x) + Q_{\mu\nu}(x) \tag{23}$$

We choose the unknown quasilocal terms $Q_{\mu\nu}(x)$ so that the T-amplitude becomes gauge invariant. Then assuming canonical light-cone behaviour (eq. (2)) the following DJL-representation holds[11]

$$T_{\mu\nu}(q,P) = \frac{1}{4}(-g_{\mu\nu}q^2 + q_\mu q_\nu)T_1 + \frac{1}{4}[q^2P_\mu P_\nu - Pq(P_\mu q_\nu + P_\nu q_\mu) + (Pq)^2 g_{\mu\nu}]\,T_2 \tag{24}$$

with

$$T_i(q) = \iint d\vec{u}\, d\lambda^2 \, \frac{\psi^i(\vec{u},\lambda^2)}{q_0^2-(\vec{q}-\vec{u})^2-\lambda^2+i\varepsilon} .$$

The $\psi^i(\vec{u},\lambda^2)$ are the spectral functions corresponding to $V_i(q,p)$. Canonical light-cone behaviour (eq. (2)) can be reproduced by means of integrable spectral functions $\psi^i(\vec{u},\lambda^2)$ (see eq. (13)). To simplify the following considerations we will assume these sufficient conditions which, as one knows [12], are not necessary for the light-cone behaviour (eq. (2)). Then the representation (24) converges without any subtractions. As a matter of fact it is possible to obtain all our results without such an assumption, analysing the integral (1) in x-space (see final remark in section IV).

Inserting eq.(24) into eq.(1) and applying two partial integrations we get

$$\delta m = \frac{2\alpha}{i} \iint d\vec{u}\, d\lambda^2 \left\{ \psi^1_{(2)}(\vec{u},\lambda^2) \int d^4q \frac{3q^2}{q^2[q_0^2-(\vec{q}-\vec{u})^2-\lambda^2+i0]^3} + \psi^2_{(2)} \int d^4q \frac{-q^2-2q_0^2}{q^2[q_0^2-(\vec{q}-\vec{u})^2-\lambda^2]^3} \right\} \tag{25}$$

where the $\psi^i_{(2)}(\vec{u},\lambda^2)$ are the second primitives of the spectral functions $\psi^i(\vec{u},\lambda^2)$ with respect to λ^2.

Now the q-integrals in (25) converge and show the following behaviour for large values of λ^2

$$\int d^4q \, \frac{3q^2}{q^2[q_0^2-(\vec{q}-\vec{u})^2-\lambda^2+i0]^3} = -\frac{3\pi^2}{2\lambda^2}\, i \tag{26}$$

$$\int d^4q \, \frac{-q^2-2q_0^2}{(q^2+i0)[q_0^2-(\vec{q}-\vec{u})^2-\lambda^2+i0]^3} \approx \frac{3\pi^2}{4\lambda^2}\, i + O\!\left(\frac{1}{\lambda^4}\right). \tag{27}$$

The asymptotical behaviour of the spectral functions for large λ^2 was (see eq. (13), (18), (19))

$$\begin{aligned} \psi^1_{(2)}(\vec{u},\lambda^2) &\approx \psi^1_0(\vec{u})\lambda^2 + \chi_0(\vec{u}) , \\ \psi^2_{(2)}(\vec{u},\lambda^2) &\approx \psi^2_0(\vec{u}) . \end{aligned} \tag{28}$$

This means that the integral (25) contains both a logarithmic and a quadratic divergence for large λ^2

$$\delta m = 3\pi^2\alpha \int_{\Lambda^2}^{\infty} d\lambda^2 \int d\vec{u} \left\{ [\psi^1_0(\vec{u})\lambda^2 + \chi_0(\vec{u})]\left(\frac{-1}{\lambda^2}\right) + \psi^2_0(\vec{u})\frac{1}{2\lambda^2} \right\} + \int_0^{\Lambda^2} d\lambda^2 d\vec{u} \ldots \tag{29}$$

We demand finite mass difference, consequently,

$$\begin{aligned} &\int_{|u|\le 1} d\vec{u}\; \psi^1_0(\vec{u}) = 0 , \\ &\int_{|u|\le 1} d\vec{u}\; [\psi^2_0(\vec{u}) - 2\chi_0(\vec{u})] = 0 . \end{aligned} \tag{30}$$

Let us express these conditions in terms of the scaling functions $h_i(\xi)$. To this end, we apply the relations (see eqs. (15), (20), (21)) connecting the spectral functions with scaling functions

$$\begin{aligned} h_0(\xi) &= 2\pi \int_\xi^1 d\rho\, \rho\, \psi^1_0(|\rho|) , \\ h_1(\xi) &= 2\pi\xi^3\psi^1_0(|\xi|) - 4\pi\int_\xi^1 d\rho\,\rho^2\psi^1_0(|\rho|) + 4\pi\int_\xi^1 d\rho\,\rho\,\psi^1_0(|\rho|)(\rho-\xi) \\ &\quad + 2\pi\xi\,\chi_0(|\xi|) , \\ h_2(\xi) &= 2\pi\xi\,\psi^2_0(|\xi|) \end{aligned} \tag{31}$$

leading to

$$\begin{aligned} \psi^1_0(|\xi|) &= -\frac{1}{2\pi\xi}\frac{d}{d\xi}h_0(\xi) , \\ \chi_0(|\xi|) &= \frac{\xi}{2\pi}\frac{d}{d\xi}h_0(\xi) + \frac{1}{2\pi\xi}h_1(\xi) + \frac{1}{\pi}h_0(\xi) , \\ \psi^2_0(|\xi|) &= \frac{1}{2\pi\xi}h_2(\xi) . \end{aligned} \tag{32}$$

Now we rewrite the conditions (30) into the final form

$$\int_0^1 d\xi\; h_0^{(p-n)}(\xi) = 0 \,,$$

$$\int_0^1 d\xi \left[\xi^2 h_0(\xi) + \frac{1}{2}\,\xi h_2(\xi) - \xi h_1(\xi)\right]^{(p-n)} = 0 \,. \qquad (33)$$

The first sum rule guarantees the vanishing of the quadratic divergence in (29) and the second sum rule does so for the logarithmic one. So the divergences in the expression of pn-mass difference are completely removed.

In terms of the structure functions W_i (eq. (9)) relations (33) read

$$\int_0^1 d\xi \left[\frac{1}{\xi} f_1(\xi) - \frac{1}{4\xi}\, f_2(\xi)\right]^{(p-n)} = 0 \,, \qquad (34)$$

$$\int_0^1 d\xi \left[g_1(\xi) - \xi f_1(\xi) - \frac{1}{4} f_2(\xi) - \frac{1}{4\xi}\, g_2(\xi)\right]^{(p-n)} = 0 \,.$$

We see that the quadratic sum rule contains leading terms of (9) only. Therefore one can try to compare it with experimental data. The second sum rule appears as a relation between leading and nonleading terms in (9).

We derived the sum rules taking into account the general case $h_0(\xi) = \frac{1}{\xi}\left[\frac{1}{4\xi} f_2(\xi) - f_1(\xi)\right] \neq 0$. From experimental point of view it is not excluded that $h_0(\xi) \equiv 0$, that means

$$V_1(\nu,\xi) \approx \frac{1}{\nu^2}\, h_1(\xi) \quad \text{and} \quad V_2(\nu,\xi) \approx \frac{1}{\nu^2}\, h_2(\xi) \,.$$

In this case there is one sum rule only removing the logarithmic divergence in the expression of pn-mass difference [4],[6].

As to the mathematical structure of the sum rules (33), we note that they are well-defined without any further ad hoc regularization taking into account that the scaling functions involved are defined as distributions. It is well-known, for example, that the scaling function $h_2(\xi)$ at $\xi = 0$ behaves like $P.V.\frac{1}{\xi}$ [13] (constancy of the total cross section of real Compton scattering!). The singularities of the other scaling functions are unknown although the symmetry conditions $h_i(\xi) = \pm h_i(-\xi)$ already lead to some restrictions. In this connection it should be mentioned that division by ξ in equations (32) and (34) by no means gives rise to incertainties since the scaling limit is considered in the sense of distributions.

An actual problem is the possible existence of singular contributions of the type $\psi_0^i(\vec{u}) \sim \delta(\vec{u})$, $\chi_0(\vec{u}) \sim \delta(\vec{u})$ (see eq. (28)) and correspondingly $h_i(\xi) \sim \delta(\xi)$ or $\delta'(\xi)$, which are not measureable in deep inelastic scattering but nevertheless can contribute considerably to the sum rules. The dynamical origin of such terms would be Feynman diagrams without discontinuity in the s-channel. The missed experimental information could be obtained by measurements of Compton pair production $\gamma + N \rightarrow N + \mu^+ + \mu^-$ [14] .

IV. Sum Rules in Terms of Equal-Time Commutators

Connections between divergences of electromagnetic mass corrections and ETC have been studied by several authors[3],[4],[5]. At first Bjorken has established in 1966[3] that some ETC determine the contribution from virtual photons of very high momentum to the electromagnetic mass divergences. The quadratic divergence in the pn-mass difference is known to be connected with q-number Schwinger terms in the commutator of charge and current density.

As far as we know the logarithmic divergence was studied in that case only, when the quadratic divergence in the expression of mass difference does not appear. Cornwall and Norton[5], e.g., considered the ETC of current density and its time derivative, whereas Boulware and Deser[4] discussed the ETC $[\partial_0 j_k(x) - \partial_k j_0(x), j_k(0)]$ with k spatial index.

In order to rewrite our sum rules (eq. (33)) in terms of ETC it is necessary to formulate the relations between scaling behaviour and light-cone singularities.

The scaling behaviour found for the Fourier transform of the current commutator in the region $\nu = 2pq \to \infty$, $\xi = -\frac{q^2}{\nu}$ fixed, is uniquely correlated with the asymptotical behaviour of the commutator

$$\widetilde{W}_{\mu\nu}(x,p) = \frac{1}{8\pi} \sum_{\sigma} \langle p,\sigma | [j_\mu(x), j_\nu(0)] | p,\sigma \rangle$$
$$= (g_{\mu\nu}\Box - \partial_\mu \partial_\nu)\widetilde{V}_1(x,p) + [-p_\mu p_\nu \Box + (p_\mu \partial_\nu + p_\nu \partial_\mu)(p\partial) - g_{\mu\nu}(p\partial)^2]\widetilde{V}_2(x,p) \tag{35}$$

near the light cone $x^2 = 0$ [8,12].

To obtain the light-cone behaviour of a structure function $\widetilde{V}(x,p)$ we need the Fourier transform of the DJL-representation for the structure function $V(q,p)$:

$$\widetilde{V}(x,p) = -\frac{i}{2\pi} \int_0^\infty d\lambda^2 \, \Delta(x,\lambda^2) D(x,\lambda^2) \tag{36}$$

with

$$D(x,\lambda^2) = \frac{i}{(2\pi)^3} \int dq \, e^{-iqx} \varepsilon(q_0) \delta(q^2 - \lambda^2) , \tag{37}$$

$$\Delta(x,\lambda^2) = \int d\vec{u} \, e^{-i\vec{u}\vec{x}} \psi(\vec{u},\lambda^2)$$
$$= 4\pi \int_0^1 d\rho \, \rho \, \frac{\sin \rho \sqrt{x_0^2 - x^2}}{\sqrt{x_0^2 - x^2}} \psi(\rho,\lambda^2) . \tag{38}$$

For structure functions of type II(eq. (13)) one gets[8] the light-cone behaviour

$$\widetilde{V}(x,p) \approx \frac{-i}{4^s M \pi^2 \Gamma(s)} G(x_0) \varepsilon(x_0) (x^2)_+^{s-1} \tag{39}$$

where

$$G(x_0) = 4\pi \int_0^1 d\rho \, \rho \, \frac{\sin \rho r}{r} \psi_0(\rho) , \quad r^2 = x_0^2 - x^2 \approx x_0^2 . \tag{40}$$

Especially, for $\widetilde{V}_2(x,p)$ $(s=1)$:

$$\widetilde{V}_2(x,p) \approx \frac{-i}{16\pi^2} G_2(x_0) \varepsilon(x_0) \theta(x^2) \tag{41}$$

with

$$G_2(x_0) = 4\pi \int_0^1 d\rho \, \rho \, \frac{\sin \rho x_0}{x_0} \psi_0^2(\rho) . \tag{42}$$

In the case of $\widetilde{V}_1(x,p)$ we have to evaluate the next to leading singularity. From our ansatz (eq. (18), (19))

$$\psi^1(\rho,\lambda^2) = \psi_0^1(\rho) \delta(\lambda^2) + \frac{\partial}{\partial \lambda^2} \chi(\rho,\lambda^2) \tag{43}$$

and

$$D(x,\lambda^2) \approx \frac{1}{2\pi} \varepsilon(x_0) \delta(x^2) - \frac{\lambda^2}{2\pi} \varepsilon(x_0) \theta(x^2) + O(x^2) , \quad x^2 \approx 0 , \tag{44}$$

we get finally

$$\widetilde{V}_1(x,p) \approx -\frac{i}{4\pi^2} G_0(x_0) \varepsilon(x_0) \delta(x^2) - \frac{i}{16\pi^2} G_1(x_0) \varepsilon(x_0) \theta(x^2) , \tag{45}$$

where

$$G_0(x_0) = 4\pi \int_0^1 d\xi\, \xi\, \frac{\sin \xi x_0}{x_0}\, \gamma_0^1(\xi)\,,$$
$$G_1(x_0) = 4\pi \int_0^1 d\xi\, \xi\, \frac{\sin \xi x_0}{x_0}\, \chi_0(\xi) \tag{46}$$

In such a way we have reproduced the light-cone behaviour of the current commutator from eq. (2) relating the $G_i(x_0)$ to the asymptotical behaviour of the spectral functions. Relations (31), (32), (46) constitute a definite connection between the scaling functions and the coefficient functions of the light-cone representation.

Let us now discuss the quadratic sum rule given in terms of the spectral functions (eq. (30))

$$\int_0^1 d\xi\, \xi^2\, \gamma_0^1(\xi) = 0 \quad . \tag{47}$$

From eq. (46) we get

$$G_0(0) = 0 \tag{48}$$

We see that the quadratic sum rule is equivalent to $G_0(0)=0$.
In the following we use the formulae

$$\partial_0 \left[\varepsilon(x_0)\, \delta(x^2) \right]_{/x_0=0} = 2\pi\, \delta(\vec{x})\,,$$
$$\partial_0 \left[\varepsilon(x_0)\, \theta(x^2) \right]_{/x_0=0} = 0\,, \tag{49}$$
$$\Box \left[\varepsilon(x_0)\, \theta(x^2) \right] = 4\, \varepsilon(x_0)\, \delta(x^2)$$

and obtain at once

$$\sum_\sigma \langle p,\sigma | [j_0(x), j_i(0)] | p,\sigma \rangle_{/x_0=0} = 4i\, G_0(0)\, \partial_i\, \delta(\vec{x}) \quad . \tag{50}$$

The removal of the quadratic divergence in the pn-mass difference is therefore equivalent to vanishing of the operator Schwinger

term :

$$\sum_\sigma \langle p,\sigma | [j_0(x), j_i(0)] | p,\sigma \rangle_{/x_0=0} = 0 \quad . \tag{51}$$

To obtain the conditions resulting from the logarithmic sum rule (eq. (30))

$$\int_0^1 d\xi\, \xi^2 \left[\gamma_0^2(\xi) - 2\, \chi_0(\xi) \right] = 0 \tag{52}$$

which is equivalent to

$$G_2(0) - 2\, G_1(0) = 0 \tag{53}$$

we consider the commutators

$$\partial_k\, \Box\, [j_0(x), j_i(0)] \quad \text{and} \quad \partial_0 \Box\, [j_k(x), j_i(0)] \,.$$

From eq. (2) we get

$$\begin{aligned}
\sum_\sigma \langle p,\sigma | [\Box\, \partial_k\, j_0(x), j_i(0)] | p\sigma \rangle_{x_0=0} &= \frac{i}{2\pi}\, \partial_i \partial_k\, \Box\, \partial_0 \left[4 G_0(x_0)\, \varepsilon\, \delta(x^2) + G_1(x_0)\, \varepsilon\, \theta(x^2) \right]_{/x_0=0} \\
&\quad - \frac{i}{2\pi}\, \partial_i \partial_k\, \Box\, \partial_0 \left[G_2(x_0)\, \varepsilon(x_0)\, \theta(x^2) \right]_{/x_0=0} \\
&= \frac{i}{2\pi}\, \partial_i \partial_k \left[24\pi\, G_0''(0)\, \delta(\vec{x}) + 8\pi\, G_1(0)\, \delta(\vec{x}) - 8\pi\, G_2(0)\, \delta(\vec{x}) \right] \\
&= 12i\, G_0''(0)\, \partial_i \partial_k\, \delta(\vec{x}) - 4i\, \partial_i \partial_k\, \delta(\vec{x}) \left[G_2(0) - G_1(0) \right]
\end{aligned} \tag{54}$$

and for $i \neq k$

$$\begin{aligned}
\sum_\sigma \langle p,\sigma | [\partial_0 \Box\, j_k(x), j_i(0)] | p\sigma \rangle_{x_0=0} &= \frac{i}{2\pi}\, \partial_i \partial_k\, \partial_0\, \Box \left[4 G_0(x_0)\, \varepsilon(x_0)\, \delta(x^2) + G_1(x_0)\, \varepsilon(x_0)\, \theta(x^2) \right]_{/x_0=0} \\
&= 12i\, G_0''(0)\, \partial_i \partial_k\, \delta(\vec{x}) + 4i\, \partial_i \partial_k\, \delta(\vec{x})\, G_1(0) \quad .
\end{aligned} \tag{55}$$

Finally we obtain

$$\sum_{\sigma} \langle p\sigma | [\Box(\partial_k j_0(x) + \partial_0 j_k(x)), j_i(0)] | p\sigma \rangle_{/x_0=0} = \tag{56}$$
$$= 24i\, G_0''(0)\, \partial_i \partial_k\, \delta(\vec{x}) - 4i\, [G_2(0) - 2G_1(0)]\, \partial_i \partial_k\, \delta(\vec{x}).$$

If there is no quadratic divergence from the very beginning, that means that $G_2(x_0) \equiv 0$, $h_0(\xi) \equiv 0$ (or in other words, if there are spin 1/2 partons only) we have

$$\sum_{\sigma} \langle p\sigma | [\Box(\partial_k j_0(x) + \partial_0 j_k(x)), j_i(0)] | p\sigma \rangle_{x_0=0} = -4i [G_2(0) - 2G_1(0)]\, \partial_i \partial_k \delta(\vec{x}) \tag{57}$$

Under this condition the vanishing of the ETC (eq. (57)) is equivalent to the removal of the logarithmic divergence in the pn-mass difference.

Let us compare our result with those of [4], [5] , where the following ETC have been considered

$$\sum_{\sigma} \langle p\sigma | [\partial_0 j_k(x), j_k(0)] | p\sigma \rangle_{/x_0=0} = \tag{58}$$
$$= \frac{i}{2\pi} \partial_0 (\Box + \partial_k^2) [4G_0(x_0)\varepsilon\delta + G_1(x_0)\varepsilon\theta]_{/x_0=0} + \partial_0^3 [G_2(x_0)\varepsilon\theta]_{/x_0=0}$$
$$= 4i\, G_0(0)\, \partial_k^2 \delta(\vec{x}) + 12i\, G_0''(0)\, \delta(\vec{x}) + 4i [G_1(0) - G_2(0)]\, \delta(\vec{x})$$

and

$$\sum_{\sigma} \langle p\sigma | [\partial_k j_0(x), j_k(0)] | p\sigma \rangle_{/x_0=0} = \tag{59}$$
$$= \frac{i}{2\pi} \partial_k^2 \partial_0 [4G_0(x_0)\varepsilon\delta + G_1(x_0)\varepsilon\theta]_{/x_0=0} - \frac{i}{2\pi} \partial_k^2 \partial_0 [G_2(x_0)\varepsilon\theta]_{/x_0=0}$$
$$= 4i\, G_0(0)\, \partial_k^2\, \delta(\vec{x}) .$$

The ETC proposed in [4] is the combination

$$\sum_{\sigma} \langle p\sigma | [(\partial_0 j_k(x) - \partial_k j_0(x)), j_k(0)] | p\sigma \rangle_{/x_0=0} = \tag{60}$$
$$= 12i\, G_0''(0)\, \delta(\vec{x}) + 4i\, \delta(\vec{x})\, [G_1(0) - G_2(0)] .$$

Obviously there is no equivalence to expression (53), even if $G_0(x_0) \equiv 0$. The ETC $[\partial_0 j_k(x), j_i(0)]_{/x_0=0}$ mentioned in ref. [12] is essentially identical to Eq.(58) since from Eq.(2) for $k \neq i$ a contribution of $G_2(0)$ does not appear. Again there is no equivalence looked for.

But, in general, a quadratic divergence has to be supposed in the mass difference and consequently $G_0''(0) \neq 0$. In this general case it turns out to be impossible to express the logarithmic sum rule (52) by means of an appropriate ETC (because it is impossible to separate the term with $G_0''(0)$ by means of higher-order time derivatives). Such an equivalence with the ETC of Eq.(56) would be valid, however, if $G_0''(0) - 4\pi \int_0^{\infty} d\xi\, \xi^2 h_2(\xi) = 0$

As has already been remarked there is another approach to our results which avoids assumptions about classical asymptotic behaviour of the spectral functions. Starting with the singularities of the commutator, Eq.(2), we immediately get the singularities of the T-product near the light cone taking into account the correspondence

$$\varepsilon(x_0)\, \delta(x^2) \longrightarrow \frac{1}{x^2 + i\varepsilon}$$

and

$$\varepsilon(x_0)\, \Theta(x^2) \longrightarrow \log(x^2 + i\varepsilon) .$$

Consideration of the Fourier transformed Cottingham integral

$$\int d^4x\, \frac{g_{\mu\nu}}{x^2 + i0}\, \tilde{T}_{\mu\nu}(x, p)$$

shows that the divergences now arise at $x_\mu \to 0$. Analysing this integral in Euclidean metric we directly derive Eqs.(48),(53) as the necessary convergence conditions. Obviously these conditions follow without any assumption about the spectral functions.

In order to get the sum rules in the form (33) it is useful to refer to the technique of the quasi-limit [12] which establishes the asymptotic behaviour in the Bjorken region (i.e. the asymptotics of the functional $\int f(\xi) V(\nu,\xi)\, d\xi$ for $\nu \to \infty$) starting from the singularities on the light cone if defined in the sense of the q-limit. It is crucial that the connections between scaling functions $h_i(\xi)$ and coefficient functions $G_i(x_0)$ given by Eqs.(31) and (46) remain true in this approach and therefore allow to rewrite the conditions (48) and (53) in the form (33). Our derivation on the basis of the restricted class of spectral functions (this one considered in ref. [12] is a more general class) has been chosen for the reason of mathematical simplicity only.

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Appendix

To find the next term with $\frac{1}{\nu^{s+2}}$ in the asymptotic behaviour of structure functions

$$V(\nu,\xi) \approx \frac{1}{\nu^{s+1}} h_0(\xi) + \frac{1}{\nu^{s+2}} h_1(\xi) \tag{A.1}$$

we make the ansatz

$$\varphi_{(s+1)}(\rho,\lambda^2) = \varphi_0(\rho)\,\theta(\lambda^2) + \chi(\rho,\lambda^2) \quad \text{with} \quad \lim_{\lambda^2 \to 0} \chi(\rho,\lambda^2) = \chi_0(\rho). \tag{A.2}$$

This condition for $\chi(\rho,\lambda^2)$ is caused by the assumption we made that the next term in (A.1) decreases by an integer power in comparison to $\frac{1}{\nu^{s+1}} h_0(\xi)$.

On the basis of condition (A.2) we consider the scaling behaviour in the framework of [8] as a sequence of the distributions-structure functions in ν with $f(\xi)$ as a test function of ξ. We get the decomposition

$$\int d\xi\, V(\nu,\xi)\, f(\xi) = F_1(\nu) + F_2(\nu).$$

The integral

$$F_1(\nu) = \frac{2\pi}{\nu}\int_0^1 d\rho\,\rho^2\,\varphi_0(\rho)\int_0^\infty d\lambda^2\int_{-1}^{+1} d\mu\, f^{(s+1)}\left(-\frac{\lambda^2}{\nu} + \rho\mu + \frac{\Phi_1}{\nu}\right)\left(1+\frac{\Phi_2}{\nu}\right)\left[\frac{1}{\nu^{s+1}} + \frac{2\rho\mu(s+1)}{\nu^{s+2}}\right],$$

[Φ_1, Φ_2 : expressions which remain bounded for $\nu \to +\infty$], we evaluate in an analogous manner as it was done in the case I of spectral functions in [8] with $K=0$, taking into account additionally the (s+1)-derivative of $f(\xi)$ and all the terms which contribute to the next order in $\frac{1}{\nu}$. We get

$$F_1(\nu) = \frac{2\pi}{\nu^{s+1}}\int d\xi\, f(\xi)\Big\{\left[\xi\,\varphi_0(|\xi|)\right]^{(s-1)} + \frac{1}{\nu}\left[\xi^3\varphi_0(|\xi|)\right]^{(s)}$$
$$- \frac{2(s-1)}{\nu}\left[\xi\,\varphi_0(|\xi|)\right]^{(s-2)} + \frac{2(s-1)}{\nu}\left[\xi^2\varphi_0(|\xi|)\right]^{(s-1)}\Big\}.$$

Now

$$\mathcal{F}_2(\nu) = \frac{2\pi}{\nu^{s+2}} \int_0^1 d\rho\, \rho^2 \int_0^\infty d\lambda^2\, \chi(\rho,\lambda^2) \int_{-1}^{+1} d\mu\, f^{(s+1)}\left(-\frac{\lambda^2}{\nu} + \rho\mu + \frac{\Phi_1}{\nu}\right)\left(1 + \frac{\Phi_2}{\nu}\right)$$

and we determine this integral in completely the same way as that for the spectral functions of type II in [8] and obtain

$$\mathcal{F}_2(\nu) = \frac{2\pi}{\nu^{s+2}} \int d\xi\, f(\xi)\, \left[\xi \chi_0(|\xi|)\right]^{(s)}$$

with generalized derivatives

$$f^{(s)}(\xi) = \frac{1}{\Gamma(-s)_\xi} \int^1 d\rho\, f(\rho)(\rho - \xi)^{-s-1}$$

The final result is then

$$V(\nu,\xi) \approx \frac{1}{\nu^{s+1}} h_0(\xi) + \frac{1}{\nu^{s+2}} h_1(\xi)$$

with

$$h_0(\xi) = 2\pi \left[\xi\, \chi_0(|\xi|)\right]^{(s-1)}$$

$$h_1(\xi) = 2\pi \left[\xi^3 \chi_0(|\xi|)\right]^{(s)} + 4\pi(s-1)\left[\xi^2 \chi_0'(|\xi|)\right]^{(s-1)} - 4\pi(s-1)\left[\xi\, \chi_0(|\xi|)\right]^{(s-2)} + 2\pi\left[\xi\, \chi_0(|\xi|)\right]^{(s)}.$$

For $s = 0$ we get the formula used in eqs. (20), (21).

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