ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

ДУБНА

E2 - 7916

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1974

ЛАБОРАТОРИЯ
ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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THE MULTIPLICITY MOMENTS IN A SEMI-CLASSICAL COMPOSITE MODEL FOR PROTON-PROTON COLLISIONS AT HIGH ENERGIES

Submitted *to Physics Letters*

An analysis of experimental data[1-3] on the high energy proton-proton scattering shows that the general shape of $d\sigma_{el}/dt$ is accounted for by the Van Hove[4] overlap function $d\sigma_{inel}/d^2b = G(b,s)$ being a Gaussian in impact parameter[5]

$$G(b,s) = \eta \exp(-b^2/R^2), \qquad (1)$$

where b is the length of the two-dimensional impact parameter vector and $d^2b = 2\pi b\, db$. In this framework elastic scattering is entirely due to the shadow effect with the neglect of the real part and spin flip effects. The value of the "opacity" η is close to unity and $R \approx 1$ fermi. It is of importance that in this Van Hove approach[5] σ_{el}/σ_{tot} remains close to a constant value, 0.185. This is practically true (for a summary of experimental data see, for example, ref.[3]) for an outstretched domain of energy: from Serpukhov through ISR energies, i.e., $40 \le p_L \le 1500$ GeV/c, in spite of σ_{el} and σ_{tot} are rising at ISR energies. Perhaps, it is an evidence for the limiting behaviour of the overlap function, which we call "the Van Hove regime", and it keeps up with increasing energy.

In the present note we are going to demonstrate that the early onset of the Van Hove regime could be a reason for the early onset/6/ of KNO-scaling/7/ in multiplicity. Nielsen and Olesen/8/ connected the KNO-scaling with the constancy of σ_{el}/σ_{tot} within a simple semi-classical absorption model in which a proton is built up from some kind of constituent partons and a probability of their "hits" was connected with the opacity of "the grey disk". There are two differences between their approach and ours. Firstly, instead of the uniform overlap function in the case of the grey disk, we propose a Gaussian (1) which is more appropriate within the Van Hove regime. Secondly, and what is more important, instead of postulating the Poisson distribution for the hit's number we rather consider the following quark-hadron picture of the hadron-hadron collisions. Assume a proton is built up out of a number ν "stable quarks". In our framework the total proton-proton collision is summed up of individual independent quark-proton collisions when a quark belonging to one proton strikes the other proton as a whole. In such a quark-proton collision a bunch of particles is produced and emitted. The independence of the quark-proton collisions can be understood on the basis of the assumption that a great number of virtual quark-antiquark pairs is involved in the proton-proton collision, and the original proton remains to be relatively undisturbed. Then the probability of N quark-proton collisions at a given impact parameter b , $p_N(b)$, is defined by the generating function for a binomial distribution

$$\pi(z,b) = \sum_{N=0}^{\infty} p_N(b)\, z^N = [p(b) + (1-p(b))\, z]^{\nu}, \tag{2}$$

where $p(b)$ is the probability of the absence of a quark-proton collision, $1-p(b)$ is the probability that this collision occurs, and z is an auxiliary variable within $0 \leq z \leq 1$. The total probability of the absence of the quark-proton collisions for quarks is equal to

$$p_0(b) = p^{\nu}(b) . \tag{3}$$

Our main assumption consists in the identification of this quantity with the "transparency" of protons at a given impact parameter

$$p_0(b) = 1 - G(b,s). \tag{4}$$

The total probability for the N number of quark-proton collisions is defined by

$$P_N = \frac{1}{\sigma_{inel}} \int_0^{\infty} d^2 b\, p_N(b), \quad N = 1,2,..., \tag{5}$$

where $\sigma_{inel} = \eta\, \pi\, R^2$, and the corresponding generating function is

$$H(z) = \sum_{N=1}^{\nu} P_N z^N = \frac{1}{\sigma_{inel}} \int_0^{\infty} d^2 b\, [\pi(z,b) - p_0(b)]. \tag{6}$$

Due to eqs. (1)-(4), with the help of a substitution $x = 1 - \eta \exp(-b^2/R^2)$, we obtain

$$H(z) = 1 + \frac{1}{\eta} \int_{\Lambda}^{1} \{[x^{1/\nu} + (1 - x^{1/\nu})\, z]^{\nu} - 1\} \frac{dx}{1 - x}, \tag{7}$$

where $\Lambda = 1 - \eta \ll 1$. In so far as we want to give an outline description of the multiple particle production we assume that $\eta = 1$. Then

$$P_N = \frac{1}{N!}\frac{d^N \Pi(z)}{dz^N}\Big|_{z=0} = \nu(\nu-1)\ldots(\nu-N+1)\Big[\sum_{r=0}^{N}\frac{(-1)^{r+1}}{r!(N-r)!}\psi\Big(2-\frac{N-r}{\nu}\Big)\Big] \quad (8)$$

Here there is a combination of the well-known Euler Ψ -functions/9/. The factorial moments of the distribution are

$$\phi_m \equiv \langle N(N-1)\ldots(N-m+1)\rangle = \frac{d^m \Pi(z)}{dz^m}\Big|_{z=1}$$

$$= \nu(\nu-1)\ldots(\nu-m+1)\Big[\sum_{r=0}^{m}(-1)^{r+1}\binom{m}{r}\psi\Big(1+\frac{r}{\nu}\Big)\Big], \quad (9)$$

where $\binom{m}{r}$ are the binomial coefficients. The amount of the quark-proton collisions cannot, obviously, exceed a quark number ν , and only first ν factorial moments are not vanishing. In the limit $\nu\to\infty$ ("parton model") the binomial distribution (2) is replaced by its limiting Poisson distribution. Then, instead of eqs. (7) - (9), we have

$$\Pi(z) = 1 - C - \psi(2-z), \qquad C \text{ - Euler constant}, \quad (10)$$

$$P_N = \zeta(N+1) - 1, \quad (11)$$

$$\phi_m = m!\,\zeta(m+1)\,. \quad (12)$$

Two latter eqs. (11) and (12) contain the Riemann ζ-function/9/.

Thus, if our assumption about the early onset of the Van Hove regime is true, then the distribution (8) or (11) for quark-proton collisions and its moments (9) or (12) are independent of energy. They are functions of the only parameter: the quark number ν .

To perform the transition from this distribution to the particle multiplicity one we propose, following Nielsen and Olesen/8/, that the width of the particle distribution in each individual bunch is much narrow compared with the overall multiplicity distribution. After that the relative moments, $c_m^{theor.} = \langle N^m \rangle / \langle N \rangle^m$, for the quark-proton collision distribution can directly be compared with the relative moments, $c_m^{exp.} = \langle n_{ch}^m \rangle / \langle n_{ch} \rangle^m$, for the charged particle multiplicity distribution. Some numerical grounds for this direct comparison will be given at the end of the paper. In table 1 the theoretical values $c_m^{Theor.}$ are compared with the experimental Slattery values c_m^{exp} /6/. We see that the best agreement is observed for the quark number ν equal to <u>three</u>. In this case a single, double and triple quark-proton collisions can occur with the probabilities $P_1 = 0.723$, $P_2 = 0.218$ and $P_3 = 0.059$, respectively. Further, if the quark-spectators, which do not participate in the collisions, take away their fraction of the incident momentum/10/, then we have an estimate for the inelasticity coefficient (with $\nu = 3$, $\langle N \rangle = 1.335$) $k \cong \langle N \rangle / \nu \cong 0.45$ in good agreement with experimental data (see,/11/). We would like to emphasize that the value of the inelasticity coefficient is expected to be a constant, owing to the stabilization of the Van Hove régime, which was just observed in an experiment/11/. This quantity ought to gather around the values 1/3, 2/3 and 1 with the above-mentioned probabilities. This assertion does not contradict the experimental data/11/. At last a correlation between the

Table 1

Calculated values $c_m^{theor.} = \langle N^m \rangle / \langle N \rangle^m$ for the quark number $\nu = 2, 3, 4, \infty$ and experimental Slattery/6/ values $c_m^{exp} = \langle n_{ch}^m \rangle / \langle n_{ch} \rangle^m$.

m	$c_m^{theor.}$ $\nu=2$	$\nu=3$	$\nu=4$	$\nu=\infty$	$c_m^{exp.}$ $\langle n_{ch} \rangle = 5.3 - 8.9$
2	I,II7	I,I9I	I,240	I,50	I,244±0,006
3	I,40	I,7I	I,92	3,45	I,8I ± 0,02
4	I,94	2,82	3,57	II,24	2,97 ± 0,06
5	2,89	5,I7	7,47	47,40	5,36 ± 0,I5
6	4,48	I0,I2	I6,98	244,0	I0,43 ± 0,4
7	7,I2	20,69	40,82	I479	2I,6 ± I,I
8	II,45	43,58	I02,4	I0300	47,0 ± 2,8

Table 2

Values $c_m^{theor.} = \langle n_{ch}^m \rangle / \langle n_{ch} \rangle^m$ calculated for the composite proton model with three quarks

m	$\langle n_{ch.} \rangle = 6.41$	$\langle n_{ch.} \rangle = 9.05$
2	I,250	I,240
3	I,88	I,85
4	3,25	3,I9
5	6,27	6,I5
6	I3,II	I2,86
7	29,I2	28,54
8	67,6	66,3

increase in the inelasticity coefficient and the increase in the secondary multiplicity is expected.

Finally, we try to compare directly the calculated charged particle multiplicity distribution with the experimental one. To this end, we need to make some additional assumptions about the particle production in each quark-proton collision. The distribution of the secondaries produced in this collision is supposed to be independent of the impact parameter, and is a Poisson distribution with the average particle number μ. Then the probability for production of n secondaries is

$$P_n = \sum_{N=1}^{\infty} P_N \frac{(N\mu)^n}{n!} \exp(-N\mu). \qquad (13)$$

To perform the transition from the overall multiplicity distribution to the charged particle multiplicity distribution we make use of the charge-independence hypothesis/12/. The calculation was done for the case $\nu = 3$ and for two values of the average charged particle multiplicity $\langle n_{ch} \rangle = 6.41$ and 9.05, which correspond to $p_L \cong 100$ and 300 GeV/c, respectively. The results of the calculations are presented in table 2. As we can see from a comparison of the data of table 1 and 2, the values of the relative moments for the charged particle multiplicity distribution are indeed close to the values of the relative moments for the quark-proton collision distribution. This confirms to some extent our assumption about the coincidence of these quantities. However, KNO-plot presented on Fig. 1 shows a disagreement between the theo-

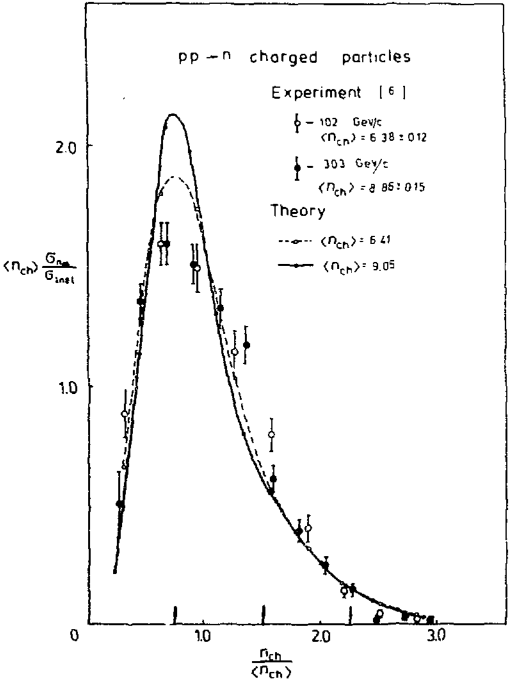

Fig. 1. KNO-plot: $\langle n_{ch}\rangle \sigma(n_{ch})/\sigma_{inel}$ vs $n_{ch}/\langle n_{ch}\rangle$. Though the distributions are discrete we joint by hand the calculated points by continuous curves. The arrows on the x-axis indicate the places where the maxima of individual distributions corresponding to a single, double and triple quark-proton collisions would be located.

retical and experimental values in the range of the maximum. This appears to be a consequence of our assumption that the secondary particle distribution in each quark-proton collision is a Poisson distribution. For example , it could be broken down by the production of resonances or clasters/13/. We hope that the details of the particle production mechanism affect only slightly the general behaviour of the multiplicity distribution. The latter depends mainly on the possibilities of quark-proton collisions. This may be a cause of the fact that the good theoretical values for the relative moments are obtained even from a "wrong" distribution.

Thus, on the basis of our suppositions about (i) the early onset of the Van Hove regime for the overlap function, (ii) the composite structure of protons and the independence of quark-proton collisions, and (iii) narrow single particle distribution for each of these collisions compared with the overall multiplicity distribution, we have obtained the scaling behaviour for the multiplicity moments. Their experimental values are found to be preferable for the construction of a proton out of three quarks.

The author would like to thank Prof. A.M.Baldin, Drs. S.B.Gerasimov and V.A.Suleymanov for valuable and helpful discussions.

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Received by Publishing Department
on May 6, 1974.