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In recent experiments[1] the muonless events:

$$\nu_\mu + N \to \nu_\mu + \ldots \tag{1}$$

$$\bar{\nu}_\mu + N \to \bar{\nu}_\mu + \ldots \tag{2}$$

have been observed. A single event has been observed too[2] which is most probably interpretable as the process:

$$\bar{\nu}_\mu + e \to \bar{\nu}_\mu + e . \tag{3}$$

Possibly, weak neutral currents are experimentally discovered. Their possible existence was discussed[3-5] for a long time and has recently become of a special interest due to the development of the unified theories of weak and electromagnetic interactions.

In this note we consider the processes:

$$\ell + N \to \ell + \ldots \tag{4}$$

and

$$\bar{\ell} + N \to \bar{\ell} + \ldots \tag{5}$$

$(\ell = e, \mu)$.

If weak neutral currents contain terms with charged leptons, effects of parity violation in the processes (4) and (5) will appear [6-9]. We would like to emphasize here that observation of such effects would make it possible to determine the sign of the weak coupling constant [x].

For the part of the Hamiltonian of weak interactions, which contains neutral currents we take the following expression:

$$\mathcal{H}_0 = 2\frac{G}{\sqrt{2}}\{[\frac{1}{2}(\bar{\nu}_e\gamma_\alpha(1+\gamma_5)\nu_e)+(\bar{e}\gamma_\alpha(g_V+g_A\gamma_5)e)]j^\circ_\alpha + \frac{1}{2}(\bar{\nu}_e\gamma_\alpha(1+\gamma_5)\nu_e)(\bar{e}\gamma_\alpha(g_V+g_A\gamma_5)e)+ +(e\to\mu)+\dots\}, \quad (6)$$

where j°_α is the hadronic neutral current and g_V and g_A are constants. The effective neutral current Hamiltonian in the theory of Weinberg and Salam [10,11] has the structure of eq. (6). The constant G is just that one that appears in the charge-exchange interaction Hamiltonian and equals

$$G = \frac{g^2}{8m_W^2} > 0, \quad (7)$$

where m_W is the mass of the intermediate charged boson and g is the coupling constant of the isotriplet current with the triplet

[x] The sign of the constant G has been discussed by J.B.Zeldovich at the IX Conference on High Energy Physics, Kiev, 1959 (Proceedings of the Conference, v.II, p. 309).

of vector bosons. The parameters g_V and g_A in the Weinberg theory are:

$$g_V = -\frac{1}{2} + 2\sin^2\Theta_W$$
$$g_A = -\frac{1}{2}, \quad (8)$$

and the hadronic neutral current has the following structure [12]

$$j^\circ_\alpha = j^3_\alpha - 2\sin^2\Theta_W\, j^{em}_\alpha + j^s_\alpha . \quad (9)$$

In this expression j^3_α is the third component of the strangness conserving "V-A" isospin current and j^{em}_α is the electromagnetic hadronic current.

The last term in eq. (9) is an isoscalar. It arises in a theory [13] with a new quantum number ("charm"). The matrix element for the process (4) is

$$\langle f|S|i\rangle = i\frac{1}{(2\pi)^3}\left(\frac{m^2}{k_0k_0'}\right)^{1/2}\frac{e^2}{q^2}[\bar{u}(k')\gamma_\alpha u(k)\langle p'|j^{em}_\alpha|p\rangle - -\rho\,\bar{u}(k')\gamma_\alpha(g_V+\gamma_5 g_A)u(k)\langle p'|j^\circ_\alpha|p\rangle]\times \times(2\pi)^4\delta(p'-p-q), \quad (10)$$

where k and k′ are the momenta of the initial and final leptons respectively, p is the momentum of the incoming nucleon and p′ is the total momentum of the final hadrons, $q = k - k'$, m is the lepton mass and

$$\rho = \frac{G}{\sqrt{2}} \frac{q^2}{2\pi a}. \qquad (11)$$

The matrix element for the process (5) can be obtained from (10), replacing the spinors $u(k)$ and $\bar{u}(k')$ by the spinors $u_c(k)$ and $\bar{u}_c(k')$, describing the initial and final antileptons and $g_A \to -g_A$ and multiplying eq. (10) by (-1).

We shall suppose that the initial lepton is polarized. If $k_0 \gg m$ and $k_0' \gg m$ only the longitudinal polarization of the initial leptons survives (helicity is conserved in interaction (6)).

The corresponding density matrix is

$$\rho(k) = \Lambda(k) \frac{1}{2}(1+\lambda\gamma_5), \qquad (12)$$

where λ is the longitudinal polarization and Λ(k) is the projecting operator. The differential cross-sections for processes (4) and (5) equal respectively (we retain only the linear in ρ terms):

$$\frac{d^2\sigma}{dq^2 d\nu} = \frac{2\pi a^2}{q^4} \frac{1}{k_0^2} [L_{\alpha\beta} \mathfrak{M}^{em}_{\alpha\beta} - \rho (g_V L_{\alpha\beta} \pm$$

$$\pm g_A e_{\alpha\beta\mu\nu} k_\mu k_\nu') \mathfrak{M}^{I}_{\alpha\beta} +$$

$$+\lambda\rho (\pm g_A L_{\alpha\beta} + g_V e_{\alpha\beta\mu\nu} k_\mu k_\nu') \mathfrak{M}^{I}_{\alpha\beta} . \qquad (13)$$

Here k_0 is the energy of the initial lepton in the laboratory frame, $\nu = \frac{-pq}{M}$ (M is the mass of the nucleon), $L_{\alpha\beta} = k_\alpha k_\beta' + \frac{1}{2}\delta_{\alpha\beta} q^2 + k_\alpha' k_\beta$ and

$$\Sigma \int \langle p' | j^{em}_\alpha | p \rangle \langle p | j^{em}_\beta | p' \rangle \delta(p' - p - q) d\Gamma =$$

$$= - \frac{1}{(2\pi)^6} (\frac{M}{p_0}) \mathfrak{M}^{em}_{\alpha\beta} \qquad (14)$$

$$\Sigma \int [\langle p' | j^{em}_\alpha | p \rangle \langle p | j^{\circ}_\beta | p' \rangle + \langle p' | j^{\circ}_\alpha | p \rangle \langle p | j^{em}_\beta | p' \rangle] \times$$

$$\times \delta(p' - p - q) d\Gamma = - \frac{1}{(2\pi)^6} (\frac{M}{p_0}) \mathfrak{M}^{I}_{\alpha\beta}. \qquad (15)$$

The tensor $\mathfrak{M}^{em}_{\alpha\beta}$ is expressed in terms of the structure functions W_1^{em} and W_2^{em} :

$$\mathfrak{M}^{em}_{\alpha\beta} = W_1^{em}\left(\delta_{\alpha\beta} - \frac{q_\alpha q_\beta}{q^2}\right) + \frac{1}{M^2} W_2^{em}\left(p_\alpha - \frac{pq}{q^2} q_\alpha\right)\left(p_\beta - \frac{pq}{q^2} q_\beta\right). \tag{16}$$

Further on we shall average the cross-section over p and n :

$$\frac{d^2\sigma}{dq^2 d\nu} = \frac{1}{2}\left[\left(\frac{d^2\sigma}{dq^2 d\nu}\right)_p + \left(\frac{d^2\sigma}{dq^2 d\nu}\right)_n\right]. \tag{17}$$

As is well known, the interference of the isovector and isoscalar parts of the hadronic current does not contribute to the averaged cross-section (17). Let us write the cross-sections of the considered processes in the form:

$$\frac{d^2\sigma}{dq^2 d\nu} = \left(\frac{d^2\sigma}{dq^2 d\nu}\right)_0 (1 + \lambda A), \tag{18}$$

where $\left(\frac{d^2\sigma}{dq^2 d\nu}\right)_0$ is the cross-section for the unpolarized particles. From eq. (13) it follows that retaining only the linear in the small parameter ρ terms, the asymmetries A_ℓ and $A_{\bar{\ell}}$ for the scattering of leptons and antileptons take the forms:

$$A_{\ell(\ell')} = \rho \frac{(\pm g_A L_{\alpha\beta} + g_V e_{\alpha\beta\mu\nu} k_\mu k'_\nu)\mathfrak{M}^{I}_{\alpha\beta}}{L_{\alpha\beta}\mathfrak{M}^{em}_{\alpha\beta}}. \tag{19}$$

Evidently the asymmetry A depends on the structure of the neutral currents. It was shown in ref. /14/ that information about the structure of the neutral current can be obtained through the joint study of the processes (1), (2) and

$$\nu_\mu + N \to \mu^- + \dots \tag{20}$$

$$\bar{\nu}_\mu + N \to \mu^+ + \dots \tag{21}$$

In order to make our further consideration more concrete we shall assume that the neutral current is of the form:

$$j^\circ_\alpha = y\, j^{em}_\alpha + A^3_\alpha + \tilde{j}^s_\alpha , \tag{22}$$

where y is a real parameter (in the Weinberg model $y = 1 - 2\sin^2\Theta_W$), A^3_α is an axial current, the third component of the isovector A^1_α (in the charged currents the $A^{1\pm i2}_\alpha$ components enter) $\tilde{j}^s_\alpha$ is an isoscalar current.

Note that the data on neutrino experiments are consistent with this assumption. Equations (15) and (22) yield the following expression for the averaged over p and n quantity

$$\mathfrak{M}^{I}_{\alpha\beta} = 2y\,\mathfrak{M}^{em}_{\alpha\beta} + \mathfrak{M}^{V;A}_{\alpha\beta} + \mathfrak{M}^{S}_{\alpha\beta}. \tag{23}$$

The pseudotensor $\mathfrak{M}_{\alpha\beta}^{V;A}$ is determined as follows

$$\Sigma \int [<p'| V_\alpha^3 |p><p| A_\beta^3 |p'> + <p'| A_\alpha^3 |p><p| V_\beta^3 |p'>] \times$$

$$\times \delta(p' - p - q) d\Gamma = \tag{24}$$

$$= -\frac{1}{(2\pi)^6} \left(\frac{M}{p_0}\right) \mathfrak{M}_{\alpha\beta}^{V;A}$$

(V_α^3 is the isovector part of the electromagnetic current).

Further on, let us assume that the contribution of the isoscalar current to the cross section can be neglected at high energies we are interested in. This assumption is in agreement with the existing data/1,15/. Substituting (24) into (19) one obtains

$$A_{\ell(\bar{\ell})} = \rho [\pm 2 g_A y + g_V \frac{e_{\alpha\beta\mu\nu} \; k_\mu k_\nu' \mathfrak{M}_{\alpha\beta}^{V;A}}{L_{\alpha\beta} \mathfrak{M}_{\alpha\beta}^{em}}]. \tag{25}$$

Using (11) we find

$$A_\ell - A_{\bar{\ell}} = 2 g_A y \frac{G}{\sqrt{2}} \frac{q^2}{\pi\alpha}, \tag{26}$$

$$A_\ell + A_{\bar{\ell}} = g_V \frac{G}{\sqrt{2}} \frac{q^2}{\pi\alpha} \frac{e_{\alpha\beta\mu\nu} \; k_\mu k_\nu' \mathfrak{M}_{\alpha\beta}^{V;A}}{L_{\alpha\beta} \mathfrak{M}_{\alpha\beta}^{em}}. \tag{27}$$

The parameter y can be related to the total cross sections of the processes (1), (2), (20) and (21). If (22) is valid and the contribution of the isoscalar part of the neutral current can be neglected, it is easy to show/16/ that y is equal to

$$y = 2 \frac{\sigma_\nu - \sigma_{\bar{\nu}}}{\sigma_\mu - \sigma_{\bar{\mu}}}, \tag{28}$$

where σ_ν, $\sigma_{\bar{\nu}}$, σ_μ, $\sigma_{\bar{\mu}}$ are the averaged over p and n total cross sections of the processes (1), (2), (20) and (21), respectively.

The factor

$$\frac{e_{\alpha\beta\mu\nu} \; k_\mu k_\nu' \mathfrak{M}_{\alpha\beta}^{V;A}}{L_{\alpha\beta} \mathfrak{M}_{\alpha\beta}^{em}}$$

on the right-hand side of (27) can be related to the differential cross sections of the processes (20) and (21), as well as to the cross section of deep inelastic lepton-nucleon scattering.

One has

$$A_\ell + A_{\bar{\ell}} = g_V \frac{G}{\sqrt{2}} \frac{q^2}{\pi\alpha} \left(\frac{2\pi^2\alpha^2}{G^2 q^4}\right) \frac{\left(\frac{d^2\sigma}{dq^2 d\nu}\right)_\mu - \left(\frac{d^2\sigma}{dq^2 d\nu}\right)_{\bar{\mu}}}{\left(\frac{d^2\sigma}{dq^2 d\nu}\right)_{em}}. \tag{29}$$

The asymmetry A depends also on the values of g_V and g_A. If the interaction Hamiltonian has the form (6), information about these constants may be extracted from the $\nu - e$ scattering experiments. The total cross sections of the processes

$$\nu_\mu + e \to \nu_\mu + e , \quad (30)$$

$$\bar{\nu}_\mu + e \to \bar{\nu}_\mu + e \quad (31)$$

are equal to

$$\sigma(\nu_\mu e) = \sigma_0 (g_+^2 + \frac{1}{3} g_-^2), \quad (32)$$

$$\sigma(\bar{\nu}_\mu e) = \sigma_0 (g_-^2 + \frac{1}{3} g_+^2), \quad (33)$$

where

$$\sigma_0 = \frac{G^2}{\pi} s,$$

$$g_+ = \frac{1}{2}(g_V + g_A), \quad (34)$$

$$g_- = \frac{1}{2}(g_V - g_A).$$

From (32) and (33) we obtain

$$\frac{8}{3}\sigma_0 g_+^2 = 3\sigma(\nu_\mu e) - \sigma(\bar{\nu}_\mu e),$$

$$(35)$$

$$\frac{8}{3}\sigma_0 g_-^2 = 3\sigma(\bar{\nu}_\mu e) - \sigma(\nu_\mu e).$$

If, as in the Weinberg-Salam theory in addition to the Hamiltonian (6) there is diagonal interaction of the form

$$\mathcal{H}_d = \frac{G}{\sqrt{2}} (\bar{\nu}_e \gamma_\alpha (1+\gamma_5) e)(\bar{e}\gamma_\alpha (1+\gamma_5)\nu_e) \quad (36)$$

then the total Hamiltonian of the $\nu_e - e$ interaction can be written as

$$\mathcal{H}_{\nu_e e} = \frac{G}{\sqrt{2}} (\bar{\nu}_e \gamma_\alpha (1+\gamma_5)\nu_e)(\bar{e}\gamma_\alpha (g_V^e + g_A^e \gamma_5) e), \quad (37)$$

where

$$g_V^e = g_V + 1,$$

$$(38)$$

$$g_A^e = g_A + 1.$$

The total cross sections of the processes

$$\nu_e + e \to \nu_e + e , \quad (39)$$

$$\bar{\nu}_e + e \to \bar{\nu}_e + e \quad (40)$$

can be obtained from (32) and (33) if we put

$$g_+ \to g_+^e = \frac{1}{2}(g_V^e + g_A^e),$$

$$(41)$$

$$g_- \to g_-^e = \frac{1}{2}(g_V^e - g_A^e).$$

From (38) we have

$$g_+^e = g_+ + 1, \quad (42)$$

$$g_-^e = g_- . \quad (43)$$

Using (35) and (42) one finds

$$3\sigma(\nu_e e) - \sigma(\bar{\nu}_e e) - 3\sigma(\nu_\mu e) + \sigma(\bar{\nu}_\mu e) - \frac{8}{3}\sigma_0 = \frac{16}{3}\sigma_0 g_+ . \quad (44)$$

Note that (43) leads to[17]

$$3\sigma(\bar{\nu}_\mu e)-\sigma(\nu_\mu e)=3\sigma(\bar{\nu}_e e)-\sigma(\nu_e e). \quad (45)$$

The test of this relation is one of the possibilities to check (38).

From (35) and (44) we can determine the parameters g_+ and $|g_-|$. The sign of g_- can be found from polarization experiments.

Recall that in the Weinberg theory

$$g_+ = -\frac{1}{2}+\sin^2\Theta_W$$
$$g_- = \sin^2\Theta_W . \quad (46)$$

Thus, in eqs. (26) and (29) the constant G and experimentally observable quantities enter. The sign of the constant G can be determined from (26) and (29) if the assumptions we have made are correct.

As yet we did not make any assumptions about structure functions of $\mathfrak{M}^{em}_{\alpha\beta}$ and $\mathfrak{M}^{V;A}_{\alpha\beta}$ Suppose the following equalities take place

$$2xMW_1^{em} = \nu W_2^{em}$$
$$-xW_3 = 2W_2^{em}, \quad (47)$$

where $x=\frac{q^2}{2M\nu}$ and the function W_3 is determined as follows

$$\mathfrak{M}^{V;A}_{\alpha\beta} = \frac{1}{2M^2} e_{\alpha\beta\mu\nu} q_\mu p_\nu W_3 . \quad (48)$$

One can derive (47) by using parton model for spin 1/2 partons[18,19].

Comparison with the data on total cross sections of the processes (20) and (21), as well as with the data on deep inelastic e-p scattering, shows that relations (47) are consistent with experiment.

From (25), using (47) one has

$$A_{\ell(\bar{\ell})} = \rho[\pm 2g_A y + g_V \frac{2\nu(2k_0-\nu)}{\nu^2+2k_0(k_0-\nu)}]. \quad (49)$$

In (49) we have neglected the terms of an order $\frac{M}{k_0}$, ($k_0 \gg M$).

The differential cross section $\frac{d\sigma}{dq^2}$ for the polarized initial leptons is of the form:

$$\left(\frac{d\sigma}{dq^2}\right)_\lambda = \left(\frac{d\sigma}{dq^2}\right)_0 (1+\lambda\,\mathfrak{A}). \quad (50)$$

After integrating over ν we obtain the following expression for the asymmetry $\mathfrak{A}$:

$$\mathfrak{A}_{\ell(\bar{\ell})} = \rho[\pm 2g_A y + g_V]. \quad (51)$$

In the Weinberg theory eq. (51) takes the form

$$\mathfrak{A}_\ell = 1{,}6\cdot 10^{-4}\frac{q^2}{M^2}\left(-\frac{3}{2}+4\sin^2\Theta_W\right),$$
$$\mathfrak{A}_{\bar{\ell}} = 0{,}8\cdot 10^{-4}\frac{q^2}{M^2}. \quad (52)$$

It may be observed that in virtue of relations (47) the asymmetry $\mathfrak{A}_\ell$ is independent of the Weinberg angle. The asymmetry $\mathfrak{A}_{\bar{\ell}}$ is estimated to be

$$\mathfrak{A}_\ell = 1,6 \cdot 10^{-5} \frac{q^2}{M^2}$$

$$(\sin^2\Theta_W = 0,4^{/1/}), \qquad (53)$$

i.e., five times less than $\mathfrak{A}_{\bar{\ell}}$. Equation (52) implies also that observation of the asymmetry effects, we have discussed here, requires polarized high-energy lepton beams (> 100 GeV). In conclusion we would like to thank V.I.Ogievetsky, B.Pontecorvo and Ja.A.Smorodinsky for helpful discussions.

References

1. F.J.Hesert et al. Phys.Lett., 46B, 138 (1973).
2. F.J.Hesert et al. Phys.Lett., 46B, 121 (1973).
3. S.Bludman. Nuovo Cim., 9, 433 (1958).
4. J.B.Zeldovich. JETP, 36, 1964 (1959).
5. I.M.Vasilevsky, V.I.Veksler, V.V.Vishnyakov, B.Pontecorvo and A.A.Tyapkin. Phys.Lett., 1, 345 (1962).
6. N.N.Nikolaev, M.A.Shifman, M.Zh.Shmatikov. JETP Lett., 18, 70 (1973).
7. M.A.Shifman, M.Zh.Shmatikov. Lett.Nuovo Cim., 8, 201 (1973).
8. A.Love, G.G.Rose, D.V.Nanopoulos. Nucl. Phys., B49, 513 (1972).

9. S.M.Berman, J.R.Primack. Preprint SLAC-PUB-1360 (1973).
10. S.Weinberg. Phys.Rev.Lett., 19, 1264 (1967) and 27, 1688 (1971).
11. A.Salam. Elementary Particle Physics, Stockholm, 1968.
12. S.Weinberg. Phys.Rev., D5, 1412 (1972).
13. S.Glashow, J.Iliopoulos, L.Maiani. Phys.Rev., D2, 1285 (1970).
14. A.Pais, S.Treiman. Preprint, COO-3072-21, 1973.
15. D.M.Perkins. Proceedings of the XVI Internat. Conference on High Energy Physics, v. 4, 189 (1972).
16. E.Paschos, L.Wolfenstein. Phys.Rev., D7, 91 (1973).
17. L.M.Sehgal. Phys.Lett., 48B, 60 (1974).
18. C.G.Callan, D.G.Gross. Phys.Rev.Lett., 22, 156 (1969).
19. R.P.Feynman. Photon-Hadron Interactions. W.A.Benjamin INC, 1972.

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