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SOME CONSEQUENCES OF THE FOURIER ANALYSIS ON THE LORENTZ GROUP FOR RELATIVISTIC QUANTUM MECHANICS

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Contents

Introduction

States of a quantum system are described by normalized rays (or wave functions) in the rigged Hilbert space $\mathcal{H}$ with positive definite metric /1/. In the case of nonrelativistic quantum mechanics a representation of the Galilei group is realized in the space $\mathcal{H}$. The requirement of relativistic invariance means that in the space $\mathcal{H}$ there is realized a Poincare group representation /3/.

The nonrelativistic quantum mechanics is now a well developed scheme, effective in describing the atomic systems. This is due to the fact that the Schrödinger equation exists and the way for introducing the interaction potential is known.

Analysing the two-particle stationary system on the basis of the Schrödinger equation, after separating the variables into centre-of-mass coordinate $\vec{X} = \frac{\vec{x}_1 + \vec{x}_2}{2}$ and relative coordinate $\vec{r} = \vec{x}_1 - \vec{x}_2$* it can be found that the dynamical infromation is contained in the wave function of the relative motion. Thus the two-particle problem is reduced to the problem for one ''effective'' particle in a potential field.

In the space of ''relative'' wave functions unit representation of the Galilei group is realized if one is not interested in rotations. This also holds in momentum representation, if the momentum and coordinate are canonical, i.e., if the wave functions in momentum and coordinate representations are connected by the usual Fourier transform

$$\psi(\vec{p}) = \frac{1}{(2\pi)^{3/2}} \int d^3\vec{r}\, e^{-i\vec{p}\vec{r}}\, \psi(\vec{r}). \qquad (1)$$

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*The case of equal masses in chosen here.

Formula (1) is an expansion of the "Galilei boosts" group representation in momentum space over the irreducible representations in coordinate space /4/.

In the variant of quasipotential approach proposed by Kadyshevsky /5/, on the basis of Hamiltonian formulation of quantum field theory, relativistic analogs of Schödinger and Lippmann-Schwinger equations were obtained. The only difference between the nonrelativistic and relativistic cases is the change of the three-dimensional Euclidean momentum space by the three-dimensional Lobachevsky space

$$p^2 = p_0^2 - \vec{p}^2 = m^2 ,$$

i.e., by the mass shell of one "effective" particle in the Minkovsky space. The Lorentz transformations in the Lobachevsky space play the role of the Galilei boosts in the Euclidean space. An analogous to (1) expansion in the relativistic case /6/ permitted to introduce the relativistic relative coordinate /7/, that led to the formulation of a quasipotential theory in the relativistic coordinate space*.

Formulae of type (1) in group theory are called Fourier analysis /4,8,9/.

The relativistic Fourier analysis formulated in a suitable for quasipotential approach parametrization (see the first of the papers /14/), was successfully exploited for parametrization of the nucleon form factor /10/.

In the present paper the analogy between nonrelativistic and relativistic Fourier analysis is studied in order to obtain some physical consequences.

In Section 1 the formulae of nonrelativistic Fourier analysis are discussed from the group-theoretical point of view /4,8,9,11/ and also a formal derivation of the uncertainty relation between momentum and coordinate is presented /12,13/.

The formulae of relativistic Fourier analysis (Fourier analysis on the Lorentz group /14/) are given in Section 2.

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* In ref. /7/ a complete list of references on the quasipotential approach is given.

In Section 3 the radial dependence of the relativistic wave functions of particle with arbitrary spin is studied as a methodological example of application of such a technique. Further, a formal derivation of the uncertainly relation between the rapidity and the relativistic coordinate is given. It should be pointed out that an example of similar uncertainty relations is actually following from the solution of the potential well problem /7/. The idea of the paper /7/, that the rapidity and coordinate are related by the Fourier transform, was also used in connection with the parton model in papers /15/.

In Section 3C an attempt is made to interprete the transition of exponential to power dependence of the differential elastic scattering cross section with increasing transfered momentum as an effects of transition of the Euclidean geometry to Lobachevsky one. It appears that in the Born approximation this always takes place for some class of quasipotentials satisfying the Jordan lemma /16/ in coordinate space.

1. Nonrelativistic Fourier Analysis.

A. Plancherel's theorem and its group-theoretical sence

The Plancherel's theorem /11/

Let a function $\psi(\vec{p})$ be square integrable *

$$\int d^3\vec{p}\ |\psi(\vec{p})|^2 < \infty .$$

Then there exists a function $\psi(\vec{r})$, such that

$$\int d^3\vec{r}\ |\psi(\vec{r})|^2 < \infty$$

and

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* The same symbol is chosen to denote functions in p - and r -representations.

$$\psi(\vec{r}) = \frac{1}{(2\pi)^{3/2}} \int d^3\vec{p}\, e^{i\vec{p}\vec{r}}\, \psi(\vec{p}). \tag{2}$$

Moreover

$$\int d^3\vec{r}\, |\psi(\vec{r})|^2 = \int d^3\vec{p}\, |\psi(\vec{p})|^2 \tag{3}$$

and we have

$$\psi(\vec{p}) = \frac{1}{(2\pi)^{3/2}} \int d^3\vec{r}\, e^{-i\vec{p}\vec{r}}\, \psi(\vec{r}). \tag{1}$$

We would like to emphasize some group-theoretical aspects of this theorem.

Let in p-representation a representation of the Galilei boosts be given

$$T_q\, \psi(\vec{p}) = \psi(\vec{p}-\vec{q}). \tag{4}$$

The Casimir operator of this representation is the usual three-dimensional Laplace operator. With the help of formula (1) and the "addition theorem" for the plane wave

$$e^{-i\vec{p}.\vec{r}}\, e^{i\vec{q}.\vec{r}} = e^{-i(\vec{p}-\vec{q}).\vec{r}} \tag{5}$$

one can obtain that the representation T_q is decomposed in terms of irreducible representations $T_q^{[r]}$ which act on functions in coordinate space in the following way

$$T_q^{[r]}\, \psi(\vec{r}) = e^{i\vec{q}.\vec{r}}\, \psi(\vec{r}). \tag{6}$$

The orthogonality and completeness conditions

$$\frac{1}{(2\pi)^{3/2}} \int d^3\vec{r}\, e^{i(\vec{p}-\vec{q}).\vec{r}} = \delta(\vec{p}-\vec{q}),$$

$$\frac{1}{(2\pi)^{3/2}} \int d^3\vec{p}\, e^{i\vec{p}.(\vec{r}-\vec{r}')} = \delta(\vec{r}-\vec{r}') \tag{7}$$

follow from eqs. (1) and (2). As is well known in quantum mechanics, the formulae (7) can be proved using the expansion

$$e^{i\vec{p}\cdot\vec{r}} = \sum_{\ell=0}^{\infty} (2\ell+1)\, i^{\ell}\, j_{\ell}(pr)\, P_{\ell}(\vec{p}\cdot\vec{r}) \qquad (8)$$

and orthogonality and completeness conditions for Bessel and spherical functions. It is interesting to point out that the usual exponent is the generating function (see (8)) of the boosts matrix element (the Bessel functions) if the basis functions are chosen to be the spherical functions.

B. Parseval's theorem and its group-theoretical sense

The Parseval's theorem states that if the functions $\psi(\vec{p})$, $\phi(\vec{p})$ and their convolution $\int d^3\vec{p}\,\overline{\psi(\vec{p})}\,\phi(\vec{p}-\vec{q})$ are square integrable, then

$$\int d^3\vec{r}\,\overline{\psi(\vec{r})}\,\phi(\vec{r})\,e^{i\vec{q}\cdot\vec{r}} = \int d^3\vec{p}\,\overline{\psi(\vec{p})}\,\phi(\vec{p}-\vec{q}).$$

That is, the Parseval's theorem is "isometric" statement for p- and r-spaces with resepct to the boosts

$$\int d^3\vec{r}\,\overline{\psi(\vec{r})}\,T_q^{|r|}\,\phi(\vec{r}) = \int d^3\vec{p}\,\overline{\psi(\vec{p})}\,T_q\,\phi(\vec{p}). \qquad (9)$$

C. Uncertainty relation

Formally, the uncertainty relation is a consequence of the fact that the commutator of the momentum and coordinate operators is a c-number

$$[\hat{p},\vec{r}] = -i\,, \qquad (10)$$

and that the integral /13/

$$\int d^3\vec{r}\,|(\alpha\vec{r} + i\hat{p})\,\psi(\vec{r})|^2,$$

where α is an arbitrary positive number, is positive definite.

On the other hand, commutation relations (10) can be derived making use of the diagonality of momentum and coordinate operators in p- and r-representations, respectively, and that the wave functions in p- and r-representation are connected by Fourier transform (1). Thus, formally, commutations (10) are a consequence of the type of the plane wave. So, the uncertainty relation, from such formal point of view, also results from the type of the nonrelativistic plane wave

$$\xi^{(0)}_{NR}(\vec{p},\vec{r}) = e^{i\vec{p}\cdot\vec{r}} ,$$

2. Relativistic Fourier Analysis

A. Fourier transforms connecting p -space with the relativistic r -space

Let the function $\psi^{(s)}_{\mu}(\vec{p})$ be relativistic wave function of a particle with spin s, spin projection μ, mass m and momentum $\vec{p}$ belonging to the Lobachevsky space.

The representation of the Lorentz group

$$T_q \psi^{(s)}(\vec{p}) = D^{(s)}[V(q,p)]\psi^{(s)}(\overrightarrow{p(-)q}), \tag{11}$$

where the Wigner rotation $V(q,p)$ is

$$V(q,p) = B^{-1}_p B_q B_{p(-)q} = V^{-1}(p,q) \tag{12}$$

and

$$B_p = \frac{1+\underline{p}}{\sqrt{2(1+p_0)}}, \quad \underline{p} \equiv p\sigma \equiv p_0 - \vec{p}\cdot\vec{\sigma} ,$$
$$p(-)q \equiv \Lambda^{-1}_q p = \begin{Bmatrix} pq \\ \vec{p} - \vec{q}\,\frac{p_0 + pq}{1+q_0} \end{Bmatrix} \tag{13}$$

is unitary, if the scalar product is

$$\int \frac{d^3\vec{p}}{2p_0}\,\overline{\psi^{(s)}_{\mu}(\vec{p})}\,\phi^{(s)}_{\mu}(\vec{p}) . \tag{14}$$

In spherical coordinates one has

$$p = \begin{Bmatrix} \operatorname{ch}\chi \\ \operatorname{sh}\chi\ \vec{n}_p \end{Bmatrix} ,$$

or

$$\chi_p = \ln(p_0 + p) = \frac{1}{2}\ln\frac{p_0+p}{p_0-p}\,. \qquad (15)$$

From eq. (13) and (15) it is clear that the transformation of the rapidity

$$\chi_p \to \chi_{p(-)q} = \chi_{q(-)p}$$

corresponds to the Lorentz boost

$$p \to p(-)q\,.$$

The reduction of the representation (12) into irreducible $\{\nu, r\}$ representation is given by the formulae

$$\psi_\mu^{(s)}(\vec{p}) = \frac{1}{(2\pi)^{3/2}}\int(\mu^2 + r^2)\,dr\, d^2\vec{r}\;\xi^{+(s)}_{\mu\nu}(\vec{p},\vec{r})\,\psi_\nu(\vec{r})\,, \qquad (16a)$$

$$\psi_\mu^{(s)}(\vec{r}) = \frac{1}{(2\pi)^{3/2}}\int\frac{d^3\vec{p}}{2p_0}\,\xi^{(s)}_{\mu\nu}(\vec{p},\vec{r})\,\psi_\nu(\vec{p})\,, \qquad (16b)$$

where

$$d^2\vec{r} = \sin\theta\, d\theta\, d\phi\,,\quad \vec{r} = r\vec{n}\,,\quad \vec{n} = \begin{pmatrix}\sin\theta & \cos\phi\\ \sin\theta & \sin\phi\\ \cos\theta & \end{pmatrix}$$

and

$$\xi^{(s)}_{\mu\nu}(\vec{p},\vec{r}) = (p_0 - \vec{p}\cdot\vec{n})^{-1-ir}\, D^{(s)}_{\mu\nu}(\overline{\vec{r}(-)\vec{p}}) = \xi^{(0)}(\vec{p},\vec{r})\, D^{(s)}_{\mu\nu}(\overline{\vec{r}(-)\vec{p}})\,,$$

where (17)

$$\overline{\vec{r}(-)\vec{p}} = r\,\overline{\vec{n}(-)\vec{p}}\,,\quad \overline{\vec{n}(-)\vec{p}} = \frac{1}{pn}\left(\vec{n} - \vec{p}\,\frac{1+pn}{1+p_0}\right),\quad pn = p_0 - \vec{p}\cdot\vec{n}\,.$$

The function $\xi^{(s)}_{\mu\nu}(\vec{p},\vec{r})$ plays the role of "relativistic plane wave". The orthogonality and completeness conditions for $\xi^{(s)}(\vec{p},\vec{r})$ have the form

$$\frac{1}{(2\pi)^3}\int(\mu^2 + r^2)\,dr\, d^2\vec{r}\;\xi^{+(s)}_{\mu\nu}(\vec{p},\vec{r})\,\xi^{(s)}_{\nu\sigma}(\vec{q},\vec{r}) = 2\delta_{\mu\sigma}\,\delta(\overline{p(-)q})\,,$$

(18)

$$\frac{1}{(2\pi)^3}\int\frac{d^3\vec{p}}{2p_0}\,\xi^{(s)}_{\mu\nu}(\vec{p},\vec{r})\,\xi^{(s)}_{\nu\sigma}(\vec{p},\vec{r}\,') = \delta_{\mu\sigma}\frac{r^2}{\mu^2 + r^2}\,\delta(\vec{r} - \vec{r}\,')\,.$$

As in the nonrelativistic case, these relations can be proved on the basis of the expansion

$$\xi^{(s)}_{\mu\nu}(\vec{p},\vec{r}) = \sum_{J=s}^{\infty} \sum_{M=-J}^{J} d^{[\nu,r]}_{Js\mu}(\chi_p) \overset{+}{D}{}^{(J)}_{\mu M}(\vec{p}) D^{(J)}_{M\nu}(\vec{r}) \tag{19}$$

using the orthogonality and completeness relations for the matrix elements of rotation group $D^{(J)}$ and Lorentz boosts $d^{[\nu,r]}_{Js\mu}$.

Unlike the addition theorem for usual exponents, the addition theorem for the relativistic plane wave takes place under the sign of integral

$$\int d^2\vec{r}\, \xi^{+(s)}(\vec{p},\vec{r})\, \xi^{(s)}(\vec{q},\vec{r}) = D^{(s)}[V(q,p)] \int d^2 \vec{q}\, \xi^{+(s)}(\overline{\vec{p}(-)\vec{q}},\vec{r}) D^{(s)}(\vec{r}). \tag{20}$$

The nonrelativistic limit of formulae (16) is just (up to the rotations in $\vec{r}$ -space) the usual Fourier transforms (1) and (2) for every spin projection component. The limits of formulae (18) and (19) are exactly formulae (7) and (8).

The space conjugated by means of formulae (16) to the Lobachevsky space is called [7] the relativistic coordinate space. The scalar product in this space has the form

$$\int (\nu^2 + r^2)\, dr\, d^2\vec{r}\, \overline{\psi^{(s)}_{\nu}(\vec{r})}\, \phi^{(s)}_{\nu}(\vec{r}). \tag{21}$$

B. Plancherel's theorem

In relativistic Fourier analysis [1,8] the Plancherel's theorem is the equality

$$\int (\nu^2 + r^2)\, dr\, d^2\vec{r}\, |\psi^{(s)}_{\nu}(\vec{r})|^2 = \int \frac{d^3 p}{2 p_0} |\psi^{(s)}_{\nu}(\vec{p})|^2.$$

This can be obtained easily from eq. (16) and (18).

C. Fourier transform of the Lorentz boosts

In nonrelativistic case the irreducible representation of the boost group was derived by calculating the Fourier transform of the function

$$T_q \psi(\vec{p}) = \psi(\vec{p}-\vec{q}). \tag{4}$$

Analogously, in the relativistic case the type of the irreducible representation in r -space can be derived from the conditions

$$T_q \psi_\mu^{(s)}(\vec{p}) = \frac{1}{(2\pi)^{3/2}} \int (\nu^2 + r^2) dr d^2\vec{r}\, \xi_{\mu\nu}^{+(s)}(\vec{p},\vec{r})\, T_q^{[\nu,r]} \psi_\nu^{(s)}(\vec{r}),$$

$$T_q^{[\nu,r]} \psi_\mu^{(s)}(\vec{r}) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3\vec{p}}{2p_0}\, \xi_{\mu\nu}^{(s)}(\vec{p},\vec{r})\, T_q \psi_\nu^{(s)}(\vec{p}).$$

The invariance of the measures

$$\frac{d^3\vec{p}}{2p_0} = \frac{d^3\overrightarrow{p(-)q}}{2(p(-)q)_0}, \quad d^2\vec{r} = (qn)^2 d^2\overrightarrow{r(-)q} \tag{22}$$

and the properties of the Wigner rotation (12) permits one to obtain

$$T_q^{[\nu,r]} \psi_\nu^{(s)}(r) = \xi^{(0)}(\vec{p},\vec{r})\, \psi_\nu^{(s)}(\overrightarrow{r(-)q}). \tag{23}$$

Thus, the Lorentz boosts do not change the spin projection and modulus of relative coordinate, but change its direction.

D. Parseval's theorem

From formulae (16), (18) and (22) it follows that:

$$\int (\nu^2 + r^2) dr d^2 r\, \overline{\psi_\nu^{(s)}(\vec{r})}\, T_q^{[\nu,r]} \phi_\nu^{(s)}(\vec{r}) =$$

$$= \int (\nu^2 + r^2) dr d^2\vec{r}\, \overline{\psi_\nu^{(s)}(\vec{r})}\, \phi_\nu^{(s)}(\overrightarrow{r(-)q})\, \xi^{(0)}(\vec{q},\vec{r}) =$$

$$= \int \frac{d^3\vec{p}}{2p_0}\, \overline{\psi_\nu^{(s)}(\vec{p})}\, D_{\nu\sigma}^{(s)}[V(q,p)]\, \phi_\sigma^{(s)}(\overrightarrow{p(-)q}) \equiv$$

$$= \int \frac{d^3\vec{p}}{2p_0}\, \overline{\psi_\nu^{(s)}(\vec{p})}\, T_q\, \phi_\nu^{(s)}(\vec{p}), \tag{24}$$

in agreement with the nonrelativistic case (7).

However, one can note that in the relativistic case the Fourier transform of two wave functions in r -space is not the convolution of wave functions in p - space. This is connected with the fact that the Lorentz boosts change the direction of the relative coordinate, and the wave function is multiplied by the spin zero plane wave $\xi^{(0)}(\vec{p},\vec{r})$ and not by the spin s plane wave $\xi^{(s)}(\vec{p},\vec{r})$ (see(23)). And, finally, the integral over r -space of more than two plane waves is not δ -function in contrary to the nonrelativistic case.

3. Some Physical Consequences

A. Radial dependence of the relativistic wave function of a particle with arbitrary spin

Let us examine formulae (16) under the assumption that in the right-hand side the wave function in the integrals has no angle dependence. Then, taking into account formulae (12), (22) and expansion (19) and integrating over angles, one can obtain that the left-hand sides of eqs. (16) are equal to zero when the spin projection is non-zero. This contradiction is an illustration of the intuitively obvious physical fact that the wave function always has an angle dependence when the spin is halfinteger or integer, but the projection is non zero.

B. Uncertainty relation for the rapidity and the relativistic coordinate

Let $s = 0$ and $\psi(\vec{r}) = \psi(r)$. After integrating eq. (16a) over angles one obtains

$$\psi(p) = \frac{2}{\sqrt{2\pi}}\, \frac{1}{p} \int_0^\infty r\, dr\, \sin \chi_p r\, \psi(r),$$

where (15)

$$\chi_p = \frac{1}{2} \ln \frac{p_0 + p}{p_0 - p} .$$

Further on, the rapidity operator $\hat{\chi}$ is diagonal in p-representation and the coordinate operator $\hat{r}$ is diagonal in r-representation. Proceeding in the same way as in Sect. 3, we obtain the form of rapidity operator in r-representation /12/

$$\chi = -i (\partial_r + \frac{1}{r}).$$

Therefore the commutation relation between the rapidity and relativistic coordinate operators are of the same form as the commutation relation between nonrelativistic momentum and coordinate operators

$$[\hat{\chi}, \hat{r}] = -i .$$

From here, in complete analogy with quantum mechanics (see Sect. 1B) the uncertainty relation formally follows for the rapidity and relativistic coordinate

$$\Delta \chi \Delta r \gtrsim \frac{1}{2} \frac{h}{mc} . \qquad (25)$$

C. Interpretation of the behaviour of the elastic scattering amplitude as a geometrical effect

Let us consider for simplicity the case $s = 0$. The plane wave, with all dimensional constants has the form

$$\xi^{(0)} (\vec{p}, \vec{r}) = \frac{\exp\{-i \frac{mc}{h} r \ln [\frac{\sqrt{m^2 c^2 + \vec{p}^{\,2}} - \vec{p} \cdot \vec{n}}{mc}]\}}{\frac{\sqrt{m^2 c^2 + \vec{p}^{\,2}} - \vec{p} \cdot \vec{n}}{mc}} .$$

The nonrelativistic limit $|p| \ll mc$ of the plane wave is the usual exponent

$$\lim_{|p| \ll mc} \xi^{(0)}(\vec{p},\vec{r}) = e^{-\frac{i}{\hbar}\vec{p}\cdot\vec{r}} \tag{26}$$

In Born approximation, if the quasipotential is given, the elastic scattering amplitude has the form

$$T(\vec{p},\vec{q}) = \int d^3\vec{r}\, \overset{+}{\xi}{}^{(0)}(\vec{p},\vec{r})\, V(\vec{r},E)\, \xi^{(0)}(\vec{q},\vec{r}).$$

The addition theorem (20) permits one to obtain

$$T(\vec{p},\vec{q}) = \int d^3\vec{r}\, \overset{+}{\xi}{}^{(0)}(\overrightarrow{p(-)q},\vec{r})\, V(\vec{r}). \tag{27}$$

Let us introduce the Mandelstam variable $t = (p-q)^2$. Then we have

$$(p(-)q)_0 = 1 - t/2, \qquad |p(-)q| = \sqrt{-t(1-t/4)},$$

i.e., the condition $|p(-)q| \ll 1$ is equivalent to the condition $|t| \ll 1$. Therefore when $|t| \ll 1$ one can take limit (26) and amplitude (27) will be a nonrelativistic Fourier transform of the quasipotential. This is in agreement with the statement that the local geometry of the Lobachevsky space is Euclidean.

If we choose the quasipotential in the form

$$V(\vec{r},E) = \frac{\lambda}{r^2 + a^2}, \tag{28}$$

after the integration over angles in (27) we obtain [17]

$$T(\vec{p},\vec{q}) = \frac{4\pi\lambda}{|p(-)q|} e^{-a\chi_{p(-)q}}.$$

Finally, in terms of the variable t we have

$$T(s,t) = \frac{\pi^2\lambda}{\sqrt{-t(1-t/4)}} \begin{cases} e^{-a\sqrt{-t(1-t/4)}} & \text{at } |t| \ll 1, \\ [1 - t/2 + \sqrt{-t(1-t/4)}]^{-a} & \text{at } |t| \gtrsim 1. \end{cases} \tag{29}$$

Formula (29) explains qualitatively the proton-proton elastic scattering experimental data [18].

The power behaviour when the momentum transfer is high was obtained by many authors. We would like to mention the paper [19], where the dimensionality arguments in the picture of composite hadrons lead to the power behaviour of the differential cross section and the paper [20], where this behaviour was obtained in the framework of quantum field theory with constant curvature momentum space.

In connection with the possibility of quantitative description of the experimental data, the following remark is interesting. Let the quasipotential do not depend on the angles. Then the calculation of the amplitude in the Born approximation is reduced to calculation of the integral

$$\int_{-\infty}^{\infty} r\, dr\, e^{i\chi r} V(r). \qquad (30)$$

If the function $rV(r)$ satisfies the Jordan lemma and has some finite set of poles (for definiteness in the upper half plane) the residues theorem gives oscillation power behaviour. From formula (30) it is clear, that the form of the quasipotential does not influence the change of the exponential behaviour to the power one when $|t|$ is increasing.

Conclusion

In this paper an attempt has been made to discuss the analogy between nonrelativistic quantum mechanics and relativistic theory on the basis of the Fourier analysis. We found that the fundamental role plays the fact that the relative momentum space in the nonrelativistic case is Euclidean and in the relativistic case is a Lobachevsky one.

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