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HIGH-ENERGY PROCESSES IN SCALE INVARIANT QUARK MODEL

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Summary

Within the framework of field theory of quarks, interacting with a scalar (or pseudoscalar) field, a series of hadron processes are considered, employing the consequences of the short distance scale invariance (SDSI) hypothesis for the on-mass-shell processes. A weak bare quark interaction is required for theoretical results to be consistent with experiment on large $-q^2$ nucleon electromagnetic form factor, high-energy large-angle elastic pp-scattering, large-$q_\perp$ pion inclusive production and deep inelastic ep-scattering. A number of predictions are formulated concerning analogous processes with other particles.
A possibility for experimental check of the SDSI hypothesis is discussed, as well.

I. Introduction. SDSI and physical processes

The idea of quarks has turned out to be rather fruitful for understanding a lot of experimental data both on the properties of elementary particles and resonances and on the relations between different reactions [1]. The question here arises whether this idea can help one to explain, e.g., the following properties of high energy processes: rapid decreasing of electromagnetic form factors and large-angle elastic scattering with increasing energy, the Bjorken scaling in deep inelastic scattering processes, the multiperipheral Regge character of multiple production and quasielastic scattering in diffraction region.

We have attempted to investigate the problem formulated above within the framework of field theory of quarks interacting with a scalar field ("dilaton"):

$$\mathcal{L}_{int} = g\bar{q}(x)q(x)\varphi(x) \tag{1}$$

assuming the short distance scale invariance (SDSI) (which is equivalent to the finite charge renormalization) and using the consequences of the SDSI hypothesis for physical processes. These consequences are derived by the method we have developed earlier[2]. And the question stated above appears to be answered "yes".

The SDSI hypothesis itself means that far from the mass shell when all the external momenta are large, i.e., when all scalar products of the external momenta are as follows:

$$p_i \cdot p_j \sim p_i^2 >> m_q^2 \,, \tag{2}$$

all the amplitudes (or the Green functions) are homogeneous functions of momenta:

$$T_n(\Lambda p_1, \cdots \Lambda p_n) = \Lambda^{2\delta_n} T_n(p_1, \cdots p_n) \qquad (3a)$$

the index of homogeneity being defined through the scale dimensions of n external fields:

$$2\delta_n = 4 + \sum_i^n d_i \; ; \qquad (3b)$$

where $d_q = -3/2 - \mathcal{E}_q$, $d_\varphi = -1 - \mathcal{E}_\varphi$ are the scale dimensions of quark and dilaton, respectively.

In other words, the asymptotic behaviour of invariant amplitudes in region (2) should be

$$T(p_i p_j) \sim (p^2)^{\delta_n} C\left(\frac{p_i p_j}{p^2}\right). \qquad (4)$$

This hypothesis, however, concerns the behaviour of amplitudes far from the mass shell. Our method of studying its role for physical processes is based on investigation of Feynman diagram asymptotics of these processes [3]. The result of this rather complicated investigation (which incorporates the summation of logarithms of divergent parts through the use of renormalization group and SDSI hypothesis, resulting in the appearance of bare coupling constant, g_o, and anomalous dimensions, $\mathcal{E}_q$, $\mathcal{E}_\varphi$) can be formulated as three rather simple rules [2].

To begin with, let us consider diagrams of a process in a region where some of variables are larger than the others: $s_1, \ldots, s_k \gg t_1, \ldots, t_r, m_q^2$, and define as S-BLOCK, $V_{s_{i_1}, \ldots s_{i_m}}$, such a block which, being contracted into a point, "kills" the dependence on the large variables, $s_{i_1}, \ldots, s_{i_m}$.

One of the most important observations made in perturbation theory is that the asymptotic behaviour is connected with such blocks.

It appears to be more convenient, for our considerations, to work not with the amplitude itself, but just with its Mellin transform with respect to each of the large variables:

$$T(s_1, \ldots s_k, t_r) = \left(\frac{i}{2}\right)^k \int_{\delta - i\infty}^{\delta + i\infty} \prod \frac{dj_i \, (-s_i)^{j_i}}{\sin \pi j_i \, \Gamma(j_i + 1)} \, \Phi(j_1 \cdots j_k, t_r). \qquad (5)$$

The SDSI hypothesis then results in the following rules:

Rule I: The scale regime of every of the s-blocks, $V_{s_{i_1} \ldots s_{i_m}}$, generates in the Mellin transform the single pole, $(j_{i_1} + \ldots + j_{i_k} - \delta_V)^{-1}$, where δ_V is the scale dimension of s-block, V, given by exp.(3b), defined by the number and sort of external lines of V.

The "scale regime" means integration over the region of small distances (or, more correctly, over small α-parameters in the so-called Schwinger representation) inside that block.

In other words, the Mellin transform of the amplitude is now written in the following form:

$$\Phi(j, t) = \frac{C(j, t)}{(j_{i_1} + \cdots + j_{i_m} - \delta_V)} + R_V(j, t), \qquad (6)$$

where the first term corresponds to the scale contribution from the block V, the second one is due to the nonscale contribution from V and is defined by a subtraction procedure. Note, that both C(j,t) and R (j,t) can possess a singularity due to the scale regime of other s-blocks.

From exp.(3a) it is clear that for the leading singularity (the most right one) just the s-blocks with minimal number of external lines are responsible. And the more such "essential" s-blocks (with maximal dimensions) are simultaneously in the scale regime, the higher the order of the pole.

Rule II: The coefficient for the leading pole of (6) is a product of functions, one of which is determined by V in the scale regime, depends upon g_0 and is independent of t , and the other functions are determined by weakly connected parts, G_i, resulting from the contraction of V, i.e.,

$$C_q(j,t) = \chi_V(j, g_0) \prod \phi_{G_i}(j_i, t_r) . \qquad (7)$$

Rule III: Only those s-blocks are allowed to be simultaneously in scale regime which either have no common lines or are wholly one inside the other.

Note, that in symmetric asymptotical region (2) the only essential s-block is the whole diagram. In scale regime (according to Rule I) it generates the single pole in the sum of Mellin parameters of all variables. Therefore the asymptotics in this region is of the form of (4).

In more complicated kinematical regions a lot of the s-blocks can be simultaneously in scale regime. This rises the order of the pole, and it becomes necessary to sum up over all the pole orders (i.e., over all possibilities of scale and nonscale regimes) and over the number of blocks [x)]. Rules II and III make it possible to perform this summation. The summation method is slightly different for various kinematical situations (see refs.[2)]). Here (in Appendix) we will demonstrate the method only by the case of pion electromagnetic form factor, in model (1). As to the other cases, we confine ourselves only to the finite results presented

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x) This summation is equivalent to summation of all logarithmic terms for the senior power of large variables over all diagrams.

in Section II. And in Section III these results are discussed from the experimental point of view.

II. Results

Within the framework of quark model (1), we have not yet learned how to construct real baryons and mesons as bound states of quarks, and we are considering them merely as a collinear beam of quarks, and antiquarks, projected on the spin and isospin states of the particle in question. This substitution, of course, does not influence the asymptotic behaviour. Applying Rules I-III gives the following results.

Pion form factor, $q^2 \gg m_q^2$, $p'^2 = p^2 = m^2$.

The block structure defining the asymptotics is given in Fig.1, where the shaded blocks are two-particle irreducible ones. Each of the blocks, V_q, drawn in Fig.1 gives the pole at $j=-2-2\mathcal{E}_q$.

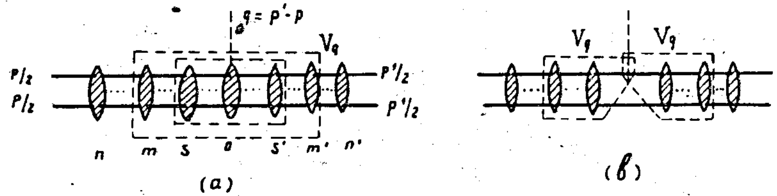

Fig.1

And summing up over the poles (for calculation procedure see App.) results in the following expression for the pion electromagnetic form factor:

$$F_\pi(q^2) \sim (-q^2)^{-(2+2\mathcal{E}_q)} \left[(-q^2)^{1-k_\pi^2(g_0^2)} - a(g_0^2)(-q^2)^{1-k_\pi(g_0^2)} \ln(-q^2) \right], \qquad (8)$$

where k_π and $\mathcal{E}_q$ are some functions of g_0^2. If we assume that $g_0^2 \ll 1$ then $\mathcal{E}_1 \approx g_0^2$, $a(g_0^2) \approx g_0^2$, $k^2-1 \approx g_0^2$ and the pion form factor acquires the form:

$$F_\pi(q^2)\sim(-q^2)^{-2+O(g_o^2)}. \qquad (9)$$

<u>Nucleon form factor (Fig.2)</u>

In the same manner, for the nucleon form factor we obtain:

$$F_N(q^2)\sim(-q^2)^{-(2+3\varepsilon_q)}\left[(-q^2)^{1-k_N^2}+a_N(g_o^2)(-q^2)^{1-k_N}\ln(-q^2)\right]\approx(-q^2)^{-2+O(g_o^2)} \qquad (10)$$

when $g_o^2 \ll 1$. The block structure, including also the essential s-blocks, is drawn in Fig.2.

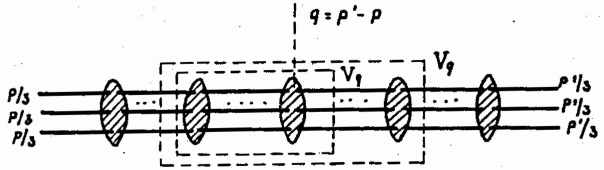

Fig.2

<u>Large-angle elastic scattering</u>, $s,t \gg m^2,\ m_q^2$ (Fig.3).

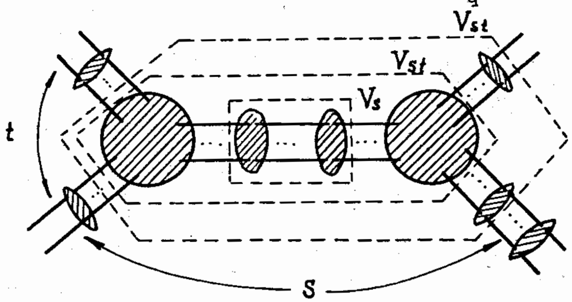

Fig.3

The essential s-blocks, V_{st}, define the asymptotics in t for $s/t \lesssim 1$ [x]

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x) Note here should be made that in the limit $g_o \to 0$ this result would turn into the analogous result of the papers [4] if the authors would take notice of the pseudoscalar nature of pions. The projecting factors, $\bar{v}(p)\gamma_5 u(p)\sim m_\pi$ "eat up" extra powers of momenta in the processes with pion.

$$T(s,t)\sim \varphi\left(\tfrac{s}{t}\right)t^{-\delta}, \qquad (11)$$

where
$$\delta=\begin{cases}4+4\varepsilon_q-(k^4_\pi \quad -1) & \text{for } \pi\pi\to\pi\pi\\ 4+5\varepsilon_q-(k^2_\pi\ k^2_N \quad -1) & \text{for } N\pi\to N\pi\\ 4+6\varepsilon_q-(k^4_N \quad -1) & \text{for } NN\to NN.\end{cases}\approx\ 4+O(g_o^2)$$

For the differential cross section, this behaviour corresponds to the following shape:

$$\frac{d\sigma}{dt}\sim t^{-10+O(g_o^2)}\chi\left(\tfrac{s}{t}\right), \qquad (g_o^2 \ll 1),$$

that is in good agreement with experiment [5], and so does the $(q^2)^{-2}$-behaviour of the nucleon form factor [6].

It is of especial interest that SDSI also permits one to predict the behaviour of $\varphi(s/t)$ in the region $s \gg t \gg m^2,\ m_q^2$, which is defined by the s-blocks, V_s (Fig.3):

$$\varphi\left(\tfrac{s}{t}\right)\sim\frac{I_1\left(\alpha_o \ln\frac{s}{t}\right)}{\ln s/t}, \qquad (12)$$

where I_1 is the Bessel function and $\alpha_o \approx g_o \ll 1$. The experimental verification of this behaviour is one of the direct tests for the SDSI hypothesis [7].

<u>Scattering in the diffraction region,</u> $s \gg t,\ m_q^2,\ m^2$.

Here the essential s-blocks are those of Fig.4 which lead immediately to a sort of the multiperipheral structure for the amplitude.

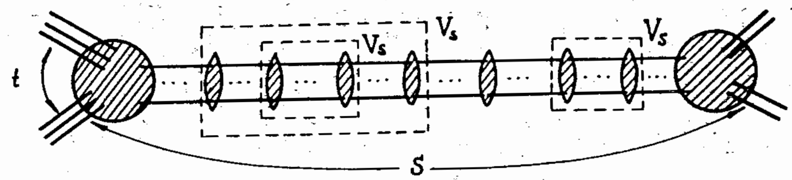

Fig. 4

For the Mellin transform of the amplitude the quasi-Regge-type formula then arises

$$\Phi(j,t) = C(t)\left[U(j) - B(t)\right]^{-1}\tilde{C}(t), \tag{13}$$

where C, B and U are matrices in the spin space of scattered particles, C and B being known only as a series in the renormalized coupling constant, g, and U(j) having square root branch points, the positions of which depend on g_o [8]. Here we point out but some of the properties of this representation:

a/ In addition to the moving Regge poles, due to det(U(j) -B(t))=0, it has fixed square root branch points of the type $\sqrt{j-\alpha_o}$.

b/ The factorization theorem holds both for the moving poles and for the fixed branch points.

c/ The channel with the exchange of vacuum quantum numbers appears to stand out against the others due to the possible 2φ- exchange.

d/ The channels with the exchange of exotic quantum numbers are much suppressed.

To all appearance, in the limit $g_o^2 \ll 1$ just the Regge poles are the leading singularities.

<u>Deep inelastic scattering</u>, $|q^2|$, $s \gg m^2, m_q^2$.

For $\omega = s/{-q^2} \approx 1$, the essential s-blocks are V_{qs}-blocks of Fig.5:

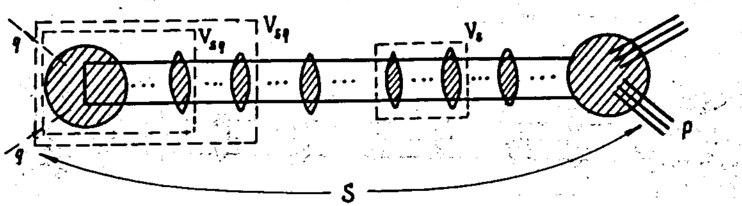

Fig. 5

One can write only the foolowing expression:

$$W_{L,T}(q^2,\omega) = \frac{i}{2}\int_{\xi-i\infty}^{\xi+i\infty} dj\,(-q^2)^{k(j)-1-\mathcal{E}_q}\,\omega^j\,K_{L,T}(j), \tag{14}$$

where $W_{L,T}$ are the form factors of deep inelastic scattering for the longitudinal and transversal virtual photons. Expression (14) is equivalent to the sum rule [9]:

$$\int_1^\infty W_{L,T}(q^2,\omega)\frac{d\omega}{\omega^{j+1}} = (-q^2)^{k(j)-1-\mathcal{E}_q}\,K(j) \tag{15}$$

and breaks the scaling law (automodelity [10]) due to the factor $(-q^2)^{(k(j)-1)-\mathcal{E}_q}$. In the limit $g_o^2 \ll 1$ the power of that factor is small ($\approx g_o^2$), and we have the approximate scaling law.

In the region $\omega \gg 1$ those leading singularities of K(j) are essential which arise due to scale regime of the s-blocks, V_s, and prove to be the same α (o) (or α_o) as in exp.(13), i.e., $K(j) \simeq (U(j)-B(o))^{-1}$, the form factors behaving here as follows:

$$W_{L,T} \sim \omega^{\alpha}(-q^2)^{k(\alpha)-1-\mathcal{E}_q}, \qquad (\omega \gg 1). \tag{16}$$

This region also is very suitable for experimental check of SDSI.

<u>Inclusive processes</u>. For these the modified Müller picture naturally arises [11,2], with $(U(j)-B(t))^{-1}$ instead of $(j-\alpha(t))^{-1}$. The Feynman scaling of normalized cross section $\frac{E_c}{\sigma_{tot}}\frac{d\sigma}{d^3\vec{q}_c} = f(s,\vec{q}_c)$ holds automatically for the leading Regge pole and is broken logarithmically in the pionization region for the fixed cut. However, SDSI tells nothing about scaling of the absolute inclusive cross sections. This scaling is entirely connected with the intercept of leading Regge pole, α (o)=1. For the leading term of $f(s,q_c)$ in the region of <u>large transverse momenta</u>, $q_\perp^2 \gg m_q^2, m^2$, $1-x_\perp \ll 1$ (the central region, Fig.6) we can obtain

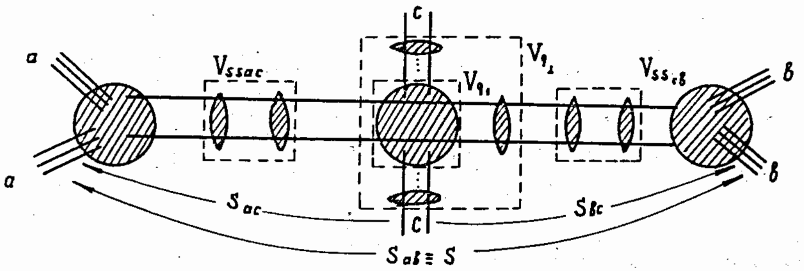

Fig.6

$$f(s,\vec{q}_c) \sim (m_\perp^2)^{\delta_c}\, \varphi_c\left(1-\frac{2m_\perp}{\sqrt{s}}\,\mathrm{ch}\,y^*\right) \tag{17a}$$

$$\delta_c = \begin{Bmatrix} 4 \\ 3 \\ 2 \end{Bmatrix} + 2\varepsilon_q\left(\frac{n_c}{2}+1\right) + \left(1-k^2_{(c)}k^2_c\right) \quad \begin{matrix} \text{for } N, \pi, \\ \text{for vector mesons,} \\ \text{for } \gamma, \end{matrix} \tag{17b}$$

where y^* is the c.m.s. rapidity of particle c, $m_\perp = \sqrt{m_c^2 + q_\perp^2}$, n_c the number of quarks constituting the particle c.

When $g_0^2 \ll 1$ the second and third terms in (17b) are of an order of g_0^2. Therefore, for the reaction $pp \to \pi$ or $\rho + X$, e.g., we obtain the expression

$$f(s,q_c) \sim m_\perp^{-8+O(g_0^2)}\, \varphi\left(1-\frac{2m_\perp}{\sqrt{s}}\,\mathrm{ch}\,y^*\right)$$

which is in good agreement with experiment [12,18].

III. Discussion.

The general conclusion on the previous section is that the quark model, with the SDSI hypothesis and weak bare coupling constant, $g_0^2 \ll 1$, pretends not only to the qualitative description of main features of high-energy phenomena but also to the quantitative description of small-distance effects. These are the

electromagnetic form factors, large-angle elastic scattering and inclusive productions with large transverse momenta. The natural question here arises: What comes from the quark model itself and what from SDSI? What, for instance, would the asymptotical freedom in the quark model give? The answer is almost evident: The results which are conserved in the limit $g_0^2 \to 0$ are proper to the quark model itself (may be, up to some logarithmic factors). Thus, the real experimental test for SDSI would be discovering the consistent deviation of the powers from those integer numbers which arise in the limit $g_0^2 \to 0$.

The experimental data presently available on the nucleon form factor, NN large-angle scattering, deep inelastic ep-scattering and inclusive pion production seem to testify to a small value of such extra powers ($\sim 0.1 \div 0.3$). However, the experimental errors, being of the same order of magnitude, do not allow us to distinguish between SDSI and asymptotical freedom. Therefore, increasing accuracy, at least, of two of these experiments may turn out to be crucial (one of these experiments will provide the value of g_0, the second will allow the check of theoretical extra powers). Theoretical calculations for these extra powers are now in progress.

It is well known that the quark model itself raises a series of problems: Why is the ratio $R = \sigma(e^+e^- \to \text{hadr})/\sigma(e^+e^- \to \mu^+\mu^-)$ still growing up to $s \approx 25$? Why can nobody observe quarks ? Why does the backward $\pi^+p(\pi^-n)$-scattering cross section, with the main contribution from the nucleon quantum number exchange, slow its decrease [13] from s^{-3} ($\alpha_N(0) = -1/2$), below 2o GeV, to $\sim s^{-3/2}$

($\alpha_N(o)\approx 1/4$), in the region 20 ÷ 40 GeV ?

The first question should be hardly considered seriously at the present time. First, our experience on the nucleon form factor, viz., its large distinction in the space-like and time-like regions, shows that in the time-like region the asymptotics, probably, sets in much later. Second, scale invariance tells only that $R \to$ const., but does not link this constant limit with the charges of constituents, like the parton model. Thus, it seems too early to draw any serious conclusion from the growth of that ratio R.

The latter question, however, may turn out to be crucial for any quark (and, in general, for any composite) model of nucleon. Really, no high-lying trajectory with nucleon quantum numbers which could explain such a slow decrease of the cross section is observed now in the resonance region. And a sudden curvature of N_α (or N_γ)-trajectory in the region $u < 1$ GeV is also not grounded well enough. Let us conjecture for a moment that the region 20 ÷ 40 GeV is a transitional one and that some NAL-experiment at higher energy will give us the s^{-1}-law of decrease(and absence of the cone shrinkage). <u>The only explanation for this would be an elementary (nonreggeized) particle with nucleon quantum numbers</u>. Such a hypothesis even now agrees very well with the presently available data [14]. This explanation is justified also by the absence of MacDowel symmetry of resonances $1/2^{\pm}$: there are three resonances $1/2^+$ and only two resonances $1/2^-$ [15] (the same is true for Λ-resonances, but Σ-resonances obey that symmetry). Then, a tempting question inevitably arises: Is one of the $1/2^+$-resonances,1400 or 1700,the very quark which is so persistently searched for? (A nucleon itself cannot be quark because of very rapid decrease of formfactor and large-angle scattering cross section and because of the automodelity in deep inelastic scattering.) Why not ? The integer electric and baryon charge ? But quarks can be "coloured". Too small mass ? But why must it be large ? And what is more, the attempt to obtain rising Regge trajectories within the quark model resulted in the quark mass $m_q \approx 1$ GeV [16]. Surely, to explain the backward pion-nucleon scattering baryons should be constituted of two quarks and one antiquark as, e.g., in the Van Hove model [17].

We are, of course, conscious of that there is a lot of fantasy above. The matter is that we would like to draw attention of experimentalists to the backward $\pi^+ p(\pi^- n)$-scattering and its great interest for the whole elementary particle physics. The same can be said about inclusive processes $p \to \pi^+$ in the region $1-x < 1/40$.

Those, who do not like this philosophy, are free to disregard the recent experimental points on backward scattering and can try to put the disobedient quarks into an "asymptotical prison". Who knows, however, how to reach the ideal ?

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Appendix

Here the application of Rules I-III is illustrated by calculating the pion electromagnetic form factor. The s-blocks for $q^2 \gg m_1^2$ are those whose contracting converts the initial diagram drawn in Fig.7a into the diagram on Fig.7b

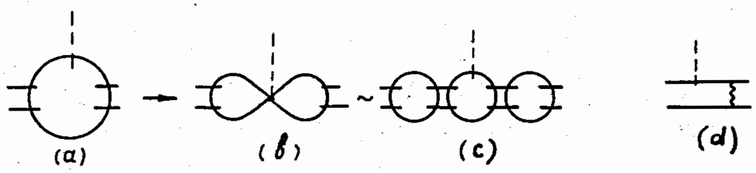

Fig.7

The number of external lines of each block has to be not less than five (including the photon line), i.e., the leading singularity is due to scale regime of the blocks V_q drawn in Fig.7c, the simplest of them is shown in Fig.7d. The dimension of each of the blocks, according to (3b), is $\delta_V=-3/2-2\varepsilon_q$ ($\varepsilon_g=0$). Taking into account that

$$F_\pi(q^2)(p+p')_\mu \approx Sp\left(\gamma^5 \overset{1}{F}{}^{\mu}_{q\bar{q},\gamma,q\bar{q}} \gamma_5\right)$$

we find that scale regime of V_q generates the pole $(j+2+2\varepsilon_q)^{-1}$ in the Mellin trasnform of $F_\pi(q^2)$. It is easy to understand that the maximal possible number of blocks V_q is determined by the number of irreducible kernels of Fig.1. Any connected union of kernels, including the kernel 0, is the s-block, V_q. To take account of all the poles, we should examine all possibilities of scale and nonscale regimes for all the s-blocks. Owing to Rule III all the scaled blocks (i.e., blocks in scale regime) are wholly one inside the other.

Consider a diagram with (n+n'+1) kernels (Fig.1). Let the senior scaled block begins at m-th kernel and ends at m'-th one (the union $\{m,m'\}$). By Rule II, this situation gives the term $C_{n-m}(p^2,g^2)v_m^{m'}(j,g_0^2)C_{n'-m'}(p'^2,g^2)$, where $v_m^{m'}$ is the contribution from the scaled block m,m' which is some polynomial in $(j+2+2\varepsilon_q)^{-1}$, and C_{n-m}, $C_{n'-m'}$ are contributions from the nonscaled ends. Allowing for all the possibilities, we can write

$$\Phi_n^{n'}(j,p^2,p'^2)=\sum_{m,m'=0}^{n,n'} C_{n-m}(p^2,g)v_m^{m'}(j,g_0^2)C_{n'-m'}(j,g^2)+R_n^{n'}, \qquad (A.1)$$

where $C_0\equiv 1$ and R is regular at the point $j=-2-2\varepsilon_q$.

Now let us examine the quantity $v_m^{m'}$. According to Rule I, it has the pole $(j+2+2\varepsilon_q)^{-1}$ the coefficient for which, by Rule II, is either the product of contributions from the subsequent scaled s-block, say $v_s^{s'}$, and from nonscaled ends k_{m-s} and $k_{m'-s'}$, remaining in $\{m,m'\}$ after contracting of $\{s,s'\}$, or that from the contracted blocks v_m^o, $v_o^{m'}$ (Fig.1b). Examining all the possibilities leads to

$$(j+2+2\varepsilon_q)v_m^{m'}(j,g_0^2)=\sum_s k_{m-s}(g_0^2)v_s^{s'}(j,g_0^2)k_{m'-s'}(g_0^2)+r_m^{m'}(g_0^2)- \qquad (A.2)$$
$$-v_m^{m'}(j,g_0^2)+(j+2+2\varepsilon_q)v_o^{m'}(j,g_0^2)v_m^o(j,g_0^2),$$

where $r_m^{m'}$ corresponds to the absence of internal contractions, $r_o^o\equiv 0$, $k_o\equiv 1$, and the quantity $v_m^{m'}$ should be subtracted in order to avoid the double counting of contraction of the whole $\{m,m'\}$ block.

Summing eqs.(A.1),(A.2) over the number of kernels gives the equations

$$\Phi(j,p^2,p'^2)= C(p^2,g^2)v(j,g_0^2)C(p'^2,g^2)$$

$$(j+2+2\varepsilon_q)v(j,g_0^2)= k^2(g_0^2)v(j,g_0^2)+r(g_0^2)-v(j,g_0^2)+(j+2+2\varepsilon_q)v_o(j)v^o(j)$$

$$(j+2+2\varepsilon_q)v^0(j,q_0^2) = k(q_0^2)v^0 + r^0(q_0^2) - v^0(j,q_0^2)$$

the solution of which is

$$\Phi(j,p^2,p'^2) = C(p^2,q^2)\,\frac{r(q_0^2) + \dfrac{r_0^2(j+2+2\varepsilon_q)}{(j+2+2\varepsilon_q+(1-k))^2}}{j+2+2\varepsilon_q+(1-k^2(q_0^2))}\,C'(p'^2,q) + R. \qquad (A.3)$$

Substituting (A.3) into (5) and integrating over j around the poles immediately gives result (8).

The other results can be derived in a similar way.

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