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THE K_S, K_L MASS DIFFERENCE

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1. Introduction

Studying of nonpolynomial quantum field theories of the chiral type proves to be rather promising from the aspect of the use in description of concrete physical phenomena. Employing the superpropagator methods (SP method) has made it possible to get, within these theories, correct results allowing for the loop-diagram contributions. The calculations for the low-energy pion scattering [1-3] and for the pion form factor in threshold region [4] are in good agreement with experimental data.

As further elaboration of the above approach, we consider an attempt to describe the mass difference of various particles in the chiral-type interactions to be rather intriguing. Along this line the mass difference between K_L and K_S mesons is the most available for our considerations since the process defining this difference can be easily handled by the SP method. Indeed, as we shall see below, this difference, Δm_{K_0}, is mostly defined by the difference of the two superpropagators, one of them contains only even numbers of pions, and the other only odd numbers. These superpropagators can be calculated without difficulties, by allowing for nonpolynomial kaon-pion interaction at every vertex and making use of our method [5].

The next, second, section is devoted to examining the nonpolynomial Lagrangian of interaction of neutral kaons with pions, having nonlinearity in pion fields of the chiral type. This Lagrangian is taken to correspond to the rule $|\Delta T| = 1/2$.

The third section demonstrates a possible way to calculate the two-pion loop-diagram contribution to the neutral-kaon mass difference Δm_{K_0} by the SP method and using the above-seggested Lagrangian.

In the fourth section an estimate is made for relative contributions to Δm_{K_0} from the loop diagrams with even and odd number of virtual pions. It is shown, in particular, that the three-pion diagram contribution is negligibly small as compared to that from the two-pion diagram.

And finally in the fifth section the linear diagram contribution to the neutral kaon mass difference is discussed.

2. The Chiral Type Interaction Lagrangian

Assuming the rule $|\Delta T| = \frac{1}{2}$ being strictly fulfilled and supposing the interaction Lagrangian being without derivatives we arrive at the following form for $K\pi$ interaction Lagrangian /6/

$$\mathcal{L}(x) = a\, U(\vec{\tau}\vec{\pi}/F_\pi)_a^{\beta}\, \Psi_\beta + \text{H.C.}, \tag{1}$$

where $\vec{\pi}(x)$ are the pion fields, $\vec{\tau}$ is the isotopic matrix, $\Psi = \begin{pmatrix} K^+ \\ K_0 \end{pmatrix}$ are the kaon fields, $F_\pi = 92$ MeV is the pion-decay constant and $U(\vec{\tau}\vec{\pi}/F_\pi)$ is two x two matrix given by the chiral group. In particular, in the exponential parametrization it has the form *

$$U_{\exp}\left(\frac{\vec{\tau}\vec{\pi}}{F_\pi}\right) = \frac{1}{\sqrt{2}}\left[\exp\left(i\,\frac{\vec{\tau}\vec{\pi}(x)}{F_\pi}\right) - 1\right]. \tag{2}$$

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* Any choice of Lagrangian different from (2) results in necessary account of all possible reductions (contractions of pion lines) in diagrams with any arbitrary number of pion vertices, when constructing superpropagator. And only this account will provide the physical result which will not depend on a choice of Lagrangian, in accordance with the general theorem by Haag-Ruelle /7/.

From eqs. (1) and (2) one can easily derive for the interaction Lagrangian of neutral kaons with pions the following expression

$$\mathcal{L}_{Int}(x) = a : \{ K_S(x)[\cos z - 1] + K_L(x)\frac{\pi^0(x)}{F_\pi}\frac{\sin z}{z} \} :$$
$$z = \sqrt{\vec{\pi}(x)^2 / F_\pi^2}\,, \tag{3}$$

where

$$K_S = \frac{K_0 + \tilde{K}_0}{\sqrt{2}} \quad ; \quad K_L = i\,\frac{\tilde{K}_0 - K_0}{\sqrt{2}}.$$

As is known /8/, this Lagrangian reproduces, in Born approximation, the low-energy theorems of current algebra concerning the nonlepton decays of K-mesons into two and three pions.

The coupling constant a can be found from the probability of decay, $K_S \to 2\pi$, $(\omega^{(2\pi)})$. As a result one can easily get the following formula:

$$a^2 = \frac{2\pi m_K (2F_\pi)^4}{3[1 - 4m_\pi^2/m_K^2]^{1/2}} \cdot \omega^{(2\pi)} \tag{4}$$

Now we can pass over the evaluation of the neutral-kaon mass difference.

3. The Mass Difference $\Delta m = m_{K_L} - m_{K_S}$

The mass difference for K_L and K_S mesons does exist just due to those distinct virtual states to which these mesons may pass in accordance with their different CP parity. Keeping this point, we can write for the mass difference, Δm, the following formula

$$\Delta m = 2[f_S - f_L], \tag{5}$$

where f_S is the sum of matrix elements corresponding to an infinite set of diagrams drawn in Fig. 1 (on the left), and f_L the same for diagrams plotted in Fig. 1 (on the right)* :

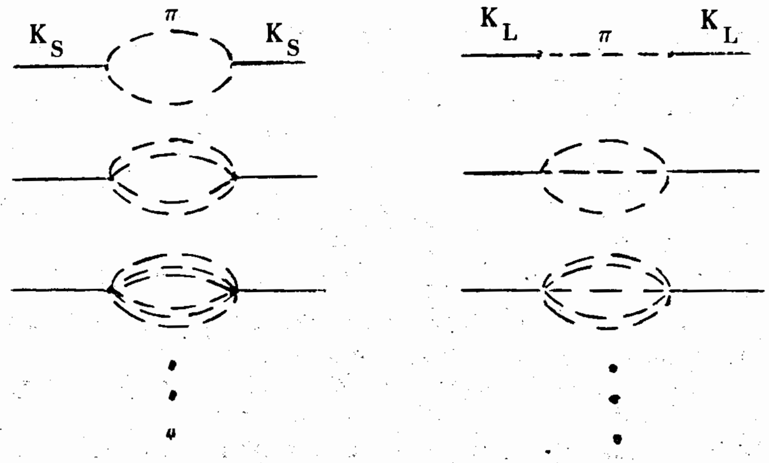

Fig. 1

The set of diagrams in Fig. 1 can be written, in terms of superpropagator, in the very simple form (see Fig. 2):

Fig. 2

Here on the left the superpropagator which consists of even number of pions is drawn, and on the right the same of odd number.

As in the momentum space for these superpropagators finite expressions can be obtained (see our papers [3-5]), then the mass difference for neutral kaons is immediately

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* Here we do not consider the linear diagram. We shall discuss this problem in the fifth section.

defined, too. In this section that part of superpropagator will be calculated which corresponds to the two-massive-pion loop. In the next section the total expression for both superpropagators will be found for the case of massless pions in virtue of their most simplicity. It will be shown there that all loop contributions to Δm, but the two-pion one, can be neglected.

By the use of Lagrangian (3) for the quantity f_S we get the following expression

$$i(2\pi)^4\,\delta^{(4)}(p-p')f_S = -\frac{a^2}{2!}\langle p_{K_S}|\iint d^4x_1\, d^4x_2\, K_S(x_1)\, K_S(x_2)\times$$

$$\times T[:(\cos z(x_1)-1):x:(\cos z(x_2)-1):]|p'_{K_S}\rangle = \tag{6}$$

$$= i(2\pi)^4\,\delta^4(p-p')\,\frac{a^2}{4m_K}\,\sigma(p),$$

where

$$\sigma(p) = i\int d^4x\, e^{ipx}\sum_{1}^{\infty}\frac{2n+1}{(2n)!}\left[-\frac{i\Lambda^c(x)}{F_\pi^2}\right]^{2n}, \tag{7}$$

Λ^c is the propagator of a free scalar particle.

Since here we are interested only in the part of superpropagator connected with the two-pion diagram, we will consider, for simplicity (as in papers [3-5]) only two pions to be massive. All the other, massless, pions serve to regularize this cheif two-pion diagram (see ref. [5]). As a result for the matrix element corresponding to this diagram we get

$$f_S^{(2)} = \frac{3a^2}{4m_k(4\pi F_\pi^2)^2}\,[J(m_k)+\gamma], \tag{8}$$

where

$$J(m_k) = \frac{y}{2}\ln\frac{1-y}{1+y} + i\frac{\pi}{2}y\ ;\quad y = \left(1-4\frac{m_\pi^2}{m_K^2}\right)^{1/2} \tag{9}$$

$$\gamma = \ln \frac{4\pi F_\pi}{m_\pi} - \frac{3}{2} c + \frac{13}{12} ,$$

c is the Euler constant (c = 0.577)..

Inserting formulae (4) and (8) into eq. (5) we obtain for the two-pion diagram contribution to the mass difference the following expression

$$\mathrm{Re}\, \Delta m_{(2\pi)} = 0{,}52\, \omega^{(2\pi)} . \tag{10}$$

This formula reproduces, with rather good accuracy, the mass difference of K_L and K_S mesons, known presently (see ref. /9/).

4. Contribution from Diagrams with Three and More Pion Lines

To evaluate relative contributions to Δm from diagrams with two, three and more lines, we consider the more simple case of massless pions. In this case both superpropagators are calculated explicitly and the quantities f_S and f_L are written in the form of rapidly convergent infinite series.

For the quantity f_S the following expression is found:

$$f_S = \frac{a^2}{4 m_k (4\pi F_\pi^2)^2} \sum_{n=1}^{\infty} \left(\frac{m_k}{4\pi F_\pi}\right)^{4(n-1)} \frac{(2n+1)!}{(2n-2)!(2n-1)!\,[(2n)!]^2} \times \tag{11}$$

$$\times \{ \ln [(\frac{4\pi F_\pi}{m})^2 e^{i\pi}] + \psi(2n-1) + \psi(2n) + 2\psi(2n+1) - \psi(2n+2) \}.$$

Here $\psi(n)$ is the Euler ψ -function, $(m_k/ 4\pi F_\pi)^4 \sim 0.034$. One can easily observe that apart from the small parameter $(m_k/4\pi F_\pi)^4$ all higher orders in (F_π^{-1}) are very suppressed due to three Γ -functions in the denominator. Therefore, the main contribution to f_S comes from the lowest order in F_π^{-1} , i.e., from the two-pion diagram:

$$f_S^{(2)} \Big|_{m_\pi = 0} = 0.2\, \omega^{(2\pi)} . \tag{12}$$

The matrix elements corresponding to the loop diagrams with odd number of pion lines are expressed as follows:

$$f_L = \frac{a^2}{12 m_k (4\pi F_\pi^2)^2} \sum_{n=1}^{\infty} \left(\frac{m_k}{4\pi F_\pi}\right)^{4n-2} \frac{2n+3}{\Gamma(2n)\,\Gamma(2n+1)\,\Gamma(2n+2)} \times \tag{13}$$

$$\times \{ \ln [(\frac{4\pi F_\pi}{m_k})^2 e^{i\pi}] + \psi(2n) + \psi(2n+1) + \psi(2n+2) - 1 \}$$

From (13) we obtain for the three-pion diagram

$$f_L^{(3)} = 0.0017\, \omega^{(2\pi)} < 1\%\, f_S^{(2\pi)} . \tag{14}$$

Hence it is evident that the three-pion contributions to the neutral-kaon mass difference can be neglected. The same concerns the diagrams with four and more pion lines.

5. The Linear Diagram Contribution

In addition to the two-pion loop diagrams, also that with one virtual pion can contribute significanlty to the mass difference Δm.This diagram, however, should be calculated together with that with one virtual η -meson. Because of that the masses of K– and η -mesons are almost the same, the latter diagram possesses a resonance character and one should take the η meson into account in the given approximation. To pions loops the η -meson contributes much less essentially as approximate estimates indicate, and one may neglect its consideration.

The contribution to Δm from diagrams of the above-metnioned type was calculated earlier by various authors (see, e.g., ref. /10/).Within the framework of SU(3) octet scheme the contribution from the sum of these diagrams is found to be proportional to the quantity $(4m^2-3\ m_\eta^2-m_\pi^2)$ which equals zero if the first order of Gell-Mann-Okubo mass formula is used. If one employs the relation of vertices:

$$\sqrt{3}\, f_{K_L \eta} = f_{K_L \pi^\circ} \qquad (15)$$

which follows from SU(3) theory, and for the masses the experimental values are taken, then one gets the following contribution to Δm from the linear diagrams:

$$\Delta m_{(\pi^\circ,\eta)} = -0{,}3\omega^{(2\pi)}. \qquad (16)$$

This diminishes more than twice the value of Δm obtained only with the account of two-pion diagram (formula (10)). Nevertheless, it is well known that the SU(3) octet scheme holds but approximately. At the same time it suffices to change only slightly the relation of coupling constants $f_{K_L\eta}$ and $f_{K_L\pi^\circ}$, e.g., to take

$$2 f_{K_L\eta} = f_{K_L\pi^\circ} \qquad (17)$$

in order to achieve good agreement with experimental data on Δm within our approach. Relation (17) does not contradict that accuracy within which the SU(3) scheme corresponds to experimental data.

6. Conclusion

The calculations performed show that the K_L-,K_S -meson mass difference is mainly defined by the linear and two-pion diagrams. The contributions from diagrams with more pions are negligibly small. As far as K -mesons are nonstable particles the mass difference Δm should be a complex quantity. In this sense the account of two-pion diagrams is of fundamental importance.

Even the account of only two-pion diagram provides the correct value and correct sign for Δm. As to the linear diagrams one can only say that their account most probably results in diminishing of the value of Δm calculated with the two-pion diagrams. The presently existing approximate theoretical schemes and experimental data do not allow one to estimate with required accuracy the value of total contribution from linear diagrams.

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