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Let us dwell upon the measuring procedure of distances (lengths) with the direct use of a clock and light signals*.

a) In order to measure the length of a rod at rest **, an observer standing by one of its ends (for example, at the left end point A) sends a light signal at t°_{A} which reflects from the other end (B) of the rod *** and arrives back at A at $t^{\circ\prime}_{A}$ (here t°_{A} and $t^{\circ\prime}_{A}$ are measured by the same clock). In this case the length of the rod at rest is defined by the value

$$\ell^{\circ} = \frac{1}{2} c\,(t^{\circ\prime}_{A} - t^{\circ}_{A}) = \frac{c}{2}\Delta t^{\circ}, \qquad (1)$$

where c is the velocity of light.

b) Let us consider the indicated experiment from the view-point of another frame of reference K uniformly moving (for example, in the direction BA) with the velocity $-v_x = \beta c$ with respect to K°. It is evident that t_A, measured by the clock in K which is near A at the instant of sending a light signal, corresponds to t°_{A} in K°. The return time of the light signal corresponds to t'_A measured by another clock in**** K which is at the left end point A of the moving rod at the instant of coming there the earlier sent light signal. Note that the two clocks in K are spaced $\Delta x = \ell^{\circ}\beta\gamma$ apart, where $\gamma = (1-\beta^2)^{-1/2}$.

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* The radar method can be used for practical realization of this procedure.

** A frame of reference (K°) in which the measured rod is at rest will now be called proper.

*** Say, from the mirror mounted there.

**** This clock is synchronized with the first clock in K by a usual method.

According to the principle of relativity, a complex of physical events used for defining a physical concept (in our case - length) in one inertial system should have an equivalent sense in any other inertial system as well. Therefore we define the length of a moving rod in K according to (1) by the following expression /1/:

$$\ell = \frac{1}{2} c (t'_A - t_A) = \frac{c}{2} \Delta t. \qquad (2)$$

It should be stressed that the doctrine of relativistic length introduced in such a way does not in fact single out any frames of reference since observers in different systems can use the same light signal for measuring the length of a given rod *.

Using the formula of relativistic time retardation we obtain

$$\ell = \ell^\circ \gamma . \qquad (3)$$

It means that the proposed measuring procedure is indicative of extension (not contraction) of longitudinal dimensions of fast-moving objects.

As was mentioned above /1e/, the concept of relativistic length in four-dimensional representation can be interpreted as a spatial part of the half-difference (X) of the two 4-vectors describing the processes of propagation of light in the direct (X_{AB}) and opposite (X_{BA}) directions. Then in the proper system we have

$$X^\circ_{AB} (\ell^\circ, 0, 0, i\ell^\circ), \qquad (4)$$

$$X^\circ_{BA} (-\ell^\circ, 0, 0, i\ell^\circ). \qquad (4a)$$

Hence using special Lorentz transformations we find

$$X_{AB} (\ell^\circ (1+\beta) \gamma, 0, 0, i\ell^\circ (1+\beta) \gamma), \qquad (5)$$

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* As far as the conventional definition is concerned, it does not satisfy the formulated condition.

$$X_{BA} (-\ell^\circ (1-\beta) \gamma, 0, 0, i\ell^\circ (1-\beta) \gamma). \qquad (5a)$$

As a result, for the value

$$X = \frac{1}{2} (X_{AB} - X_{BA}) \qquad (6)$$

we have correspondingly

$$X^\circ (\ell^\circ, 0, 0, 0), \qquad (7)$$

$$X (\ell^\circ \gamma, 0, 0, i\beta \ell^\circ \gamma). \qquad (8)$$

For the squares of intervals we obtain

$$S^{\circ 2} = X^{\circ 2}_1 + X^{\circ 2}_4 = \ell^{\circ 2}$$

$$S = X^2_1 + X^2_4 = \ell^\circ \gamma^2 - \beta^2 \ell^{\circ 2} \gamma^2 = \ell^{\circ 2}.$$

Thus, one can conclude that the above concept of relativistic length is in agreement with the requirement for interval invariance.

Taking into account the fact that the front of a spherical light wave emitted, say, from the K° origing takes the form of ellipsoid stretched in the OX direction with the semi-axes $OX = \ell^\circ \gamma$ and $OY = OZ = \ell^\circ$, it is evident that, for example, the sphere volume in motion will increase by a factor of γ.

At the end of this section I would like to touch upon the method of measuring the length of a fast-moving luminuous rod by means of photographing proposed by Terrell /3/. One can get quite a number of values for the rod length $\ell^\circ (1+\beta) \gamma ..., \ell^\circ \gamma ..., \ell^\circ ..., \ell^\circ \gamma^{-1} ..., \ell^\circ (1-\beta) \gamma$

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* It should be especially noted that when the Michelson-Morley interference experiment is considered from the viewpoint of improper system for the times of propagators of light there and back along the longitudinal interferometer arm, we have $t_{AB} = \ell_{AB}/c$ and $t_{BA} = \ell_{BA}/c$ /2/. Then as well as in the frame of the conventional definition of the length of a moving rod, the specified values are expressed through $t_{AB} = \ell_{AB}/c-v$ and $t_{BA} = \ell_{BA}/c+v$.

depending upon the time of photographing. It is obvious that in this case the above value will correspond to a half-sum of limiting of this series.

II

It should be stressed that the concept of length considered in section I is in close logical accord with that of distance applied in electrodynamics which is based, in particular, on the Liénard-Wiechert potentials [1e]. It might be well to point out the known procedure of introducing the 4-vector distance starting from the world line of the uniformly moving charge * and normal to it whose end point is the field point (see, e.g., [4]). It is not difficult to see that in the proper system the components of the 4-vector are $X^{\circ}(t^{\circ}, 0)$; this is in full agreement with (4).

It should be also emphasized that the approach based on the true transformation of Rohrlich [5] and the asynchronous formulation ** of Cavalleri and Salgarelli [6] (see also [7]) is physically meaningful only in the framework of the above concept of relativistic length.

As is known, recently this approach has been widely used for solving, in particular, some difficulties arising from the consideration of the problem of equilibrium in special relativity and a number of questions on relativistic thermodynamics (see, e.g., [8]) ***. Among the last-mentioned range of questions the following fact deserves especial attention: based on the equations of ideal gas state the transformation formulae for volume and temperature turn out to be closely related. In this case the Ott transformation formulae for temperature [10a]

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* Considering the electromagnetic field created by this charge.

** The direct sense of the last-mentioned name means that $X_4 \neq 0$ (see formula (8)).

*** See also ref. [9].

(see also [10b]) corresponds just to the formula of "extension" (3).

Next let us consider one of the important problems of relativistic electrodynamics concerning the electromagnetic mass of charge.

Many years ago Fermi [11] basing on the analysis of the expression for force gave attention to a serious discrepancy between the Abraham-Lorentz electrodynamic theory, which attributed the rest mass $(4/3)u/c^2$ (u is an electrostatic energy of charge) to the spherical distribution of electric charge, and the theory of relativity according to which the corresponding mass is equal to u/c^2. He overcome this difficulty using the covariant formulation of Hamilton's principle. This led to a required modification of the expression for self-force, whereby the coefficient 4/3 was substituted by 1.

For removal of a similar contradictions in the expressions for momentum, i.e., for covariant definitions of the electromagnetic energy and momentum of a charged particle *, it is necessary to rest upon the above concept of relativistic length. Just in the framework of the indicated approach for the transformation components of the 4-vector of the infinitesimal volume element we have

$$dV_4 = dV_4^{\circ}\,\gamma, \quad dV_1 = \beta\, dV_4^{\circ}\,\gamma. \tag{9}$$

Taking into account the transformation formulas for the components of an electromagnetic energy-momentum tensor T_{ik} we obtain the required expression

$$G_x = \frac{u}{c^2}\beta c \gamma \tag{10}$$

(This equation is different from the known expression $G_x = (4/3)(u/c^2)\beta c \gamma$ which is a direct consequence of the use of the Lorentz contraction formula).

The foregoing and, in particular, (9) show that the procedure of introducing, e.g., the value of "energy density", (see, for example, [8b])

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* In this connection see, in particular, ref. [12].

$$h^t = i T_{4\mu} \frac{v^\mu}{c}.$$

(v^μ is a 4-velocity) can be hardly admitted to be satisfactory. This value represents an amount of energy measured in one system (K) but referred to unity of space volume measured in another system (K°).

In conclusion I would like to add that in the framework of the proposed concept of relativistic length there also disappear the difficulties connected with the strange appearance of charge in moving current-carrying conductors /13/.

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Note added in proof:

1) In addition to ref. /8/ see also: S.Pahor, J.Strnad. Nuovo Cim., 20B, 105, 1974.

2) In his recent paper I.V.Polubarinov (Comm. JINR, P2-7532, Dubna, 1973) actually refuses to favour the above concept of relativistic length (corresponding to measurements on the surfaces orthogonal to the world strip of a rod) over the conventional definition. However, I want to emphasize once again that out of these two concepts only the first one satisfies the principle of relativity as it depends only on the elements of the world strip. At the same time the conventional definition depending on the choice of a frame of reference evidently contradicts the specified principle.