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In the recent Brookhaven $K^- p \to \Lambda X^0$ data at 2.18 GeV/c /1, 2/ the X^0-meson production and decay correlations have been observed; this fact can be interpreted as a strong argument in favour of the X^0-meson spin-parity hypothesis 2^- /3, 4/. However, in refs. /1, 2/ not all the most sensitive to the X^0-meson spin distributions have been obtained. Henceforth, we'll discuss the X^0-meson production and decay correlations in more detail assuming $J^P(X^0) = 2^-$. From vectors in the X^0-meson production process we construct a vector $\vec{K}$ - X^0-meson spin analyzer in the production process, and from vectors in the X^0-meson decay we construct a vector $\vec{v}$ - X^0-meson spin analyzer in the decay. The distribution over the angle θ between the production and decay X^0-meson spin analyzers depends on the Legendre polynomial $P_2(x)$ and $P_4(x)$ only ($x = \cos\theta$) ; it can be written in the form (see Appendix)

$$W(x) = 1/2 \left[1 + 10/7\, c_2\, d_2\, P_2(x) + 18/7\, c_4\, d_4\, P_4(x)\right], \quad (1)$$

where the quantities $c_{2,4}$ depend on the production mechanism only ; they can be expressed through the diagonal elements ρ_{mm} of the normalized X^0-meson spin density matrix ($Sp\,\hat{\rho} = 1$) determined in a coordinate system x, y, z ($z \parallel \vec{K}$)*)

$$\begin{aligned} c_2 &= \rho_{00} + \tfrac{1}{2}(\rho_{11} + \rho_{-1-1}) - (\rho_{22} + \rho_{-2-2}) \\ c_4 &= \rho_{00} - \tfrac{2}{3}(\rho_{11} + \rho_{-1-1}) + \tfrac{1}{6}(\rho_{22} + \rho_{-2-2}) \end{aligned} \quad (2)$$

The quantities $d_{2,4}$ depend on the X^0-meson decay mechanism only ; they can be expressed through the diagonal elements $r_{\mu\mu}$

*) If parity in the X^0-meson production process is conserved, there takes place the relation $\rho_{mm'} = (-1)^{m-m'} \rho_{-m-m'}$ between density matrix elements.

of the spin density matrix in the X^0-meson decay determined through the decay amplitudes $A_{\{\lambda\}}(J_\zeta = \mu)$ in a coordinate system ξ, η, ζ ($\zeta \parallel \vec{v}$)

$$d_\ell = \overline{a_{\ell 0}} / \overline{S_p \hat{r}} \tag{3}$$

$$a_{20} = r_{00} + \frac{1}{2}(r_{11} + r_{-1-1}) - (r_{22} + r_{-2-2}), \quad a_{40} = r_{00} - \frac{2}{3}(r_{11} + r_{-1-1}) + \frac{1}{6}(r_{22} + r_{-2-2}) \tag{4}$$

$$r_{\mu'\mu} = \sum_{\{\lambda\}} A^*_{\{\lambda\}}(\mu') A_{\{\lambda\}}(\mu) \tag{5}$$

where the line means the averaging over the decay phase space ; $\{\lambda\}$ are the helicities of the decay particles.

The X^0-meson spin will most clearly manifest itself in the distribution (1) if the X^0-meson production and decay spin analyzers correspond to maximal (by moduli) values of the quantities c_1 and d_1 . Note that these quantities are limited by definition

$$-1 \le c_2, d_2 \le 1, \quad -\frac{2}{3} \le c_4, d_4 \le 1 . \tag{6}$$

Let us discuss the question of the "best" spin analyzer in the X^0-meson decay. If we change the analyzer $\vec{v}$ by another analyzer $\vec{v}'$ connected with a coordinate system ξ', η', ζ' ($\zeta' \parallel \vec{v}'$), the decay amplitudes should be transformed with the aid of the D-functions

$$A'_{\{\lambda\}}(J_{\zeta'} = \mu') = \sum_{\mu} A_{\{\lambda\}}(J_\zeta = \mu) D^J_{\mu\mu'}(\alpha, \beta, \gamma), \tag{7}$$

where α, β, γ are the Euler angles of the rotation

$\xi, \eta, \zeta \to \xi', \eta', \zeta'$ [x]. From (7) we get the transformation of the quantities a_{l0}

$$a'_{\ell 0} = \sum_{k=0}^{\ell} Re\left(e^{ik\alpha} a_{\ell k}\right) P_\ell^k(\cos\beta), \tag{8}$$

where P_l^k are the joint Legendre polynomials, and the quantities a_{lk} for $k \neq 0$ are expressed through the nondiagonal elements of the $\hat{r}$-matrix

$$a_{21} = r_{21} - r_{-1-2} + \frac{1}{\sqrt{6}}(r_{10} + r_{0-1}), \quad a_{22} = -\frac{1}{2} r_{1-1} - \frac{1}{\sqrt{6}}(r_{20} + r_{0-2})$$
$$a_{41} = -\frac{1}{6}(r_{21} - r_{-1-2}) + \frac{1}{\sqrt{6}}(r_{10} + r_{0-1}), \quad a_{42} = -\frac{1}{9} r_{1-1} + \frac{1}{6\sqrt{6}}(r_{20} + r_{0-2}) \tag{9}$$
$$a_{43} = \frac{1}{36}(r_{1-2} - r_{2-1}), \quad a_{44} = \frac{1}{72} r_{2-2} .$$

The transformation (8) does not depend on the angle γ since the quantities a_{l0} are expressed only through the diagonal $\hat{r}$-matrix elements independent of the rotation around the ζ'-axis.

For the two-particle decay it is natural to direct the ζ-axis along the relative momentum $\vec{k}$ of the decaying particles in their c.m.s. Since the decay amplitude cannot depend on the rotation around this axis, all the nondiagonal $\hat{r}$-matrix elements should be equal to zero and transformation (8) takes the form

$$a'_{\ell 0} = P_\ell(\cos\beta)\, a_{\ell 0}, \quad |a'_{\ell 0}| \le |a_{\ell 0}|, \tag{10}$$

[x] D-functions are determined here in the form $D^J_{\mu\mu'}(\alpha, \beta, \gamma) = e^{-i\mu\alpha} D^J_{\mu\mu'}(\beta)\, e^{-i\mu'\gamma}$.

where the inequality is a consequence of the inequality $|P_1(\cos\beta)| \le P_1(1) = 1$. Therefore the relative momentum $\vec{k}$ is the best spin analyzer in the two-particle decay (the moduli of the corresponding d_1-values are maximal).

The two-particle decay $X^0 \to \gamma\gamma$ is established well enough. The amplitude of this decay is determined unambiguously. This makes it possible to calculate the $\hat{r}$-matrix elements. It is shown in Appendix that only the r_{00} element differs from zero in the ξ, η, ζ system ($\zeta \parallel \vec{k}$). Thus, the quantities $d_{2,4}$ are equal to their maximal values $d_2 = d_4 = 1$ which makes the $X^0 \to \gamma\gamma$ decay especially attractive.

In the three-particle X^0-meson decays $X^0 \to \eta\pi\pi$ and $X^0 \to \gamma\pi^+\pi^-$ it is already impossible to choose the analyzer $\vec{v}$ so that the moduli of the quantities d_2 and d_4 should be both maximal. We'll now determine analyzers corresponding to the extreme d_2-values*). Let us choose the ζ-axis along the normal $\vec{n}$ to the decay plane and ξ-axis along the η-meson (photon) momentum $\vec{k}$ in the X^0-meson rest frame. In this system (indifferent of the η-axis direction) the consequences of parity conservation in the decay can be written in the particularly simple form /5/, i.e., $a_{1k} = 0$ for odd k, which essentially simplifies the transformation (8). The first derivative $\partial_\alpha a'_{20}$ is then equal to zero on condition that

*) The quantity d_2 is of particular interest in the X^0-meson forward production or at the threshold of the reactions $\pi^- p \to X^0 n$ and $K^- p \to X^0 \Lambda$ where the quantity $c_2 = 1/2(1+\rho_{00})$ cannot become zero in contrast to the quantity $c_4 = 1/3\,(5\rho_{00} - 2)$.

$$e^{-2i\alpha} = \frac{a_{22}}{R}\,, \qquad R = \pm|a_{22}| \tag{11}$$

The second derivative at the stationary point (11) is equal to $\partial_\alpha^2 a'_{20} = -12\,R\sin^2\beta$, i.e., $R = \pm|a_{22}|$ corresponds to the maximal (minimal) a'_{20} values. Besides, the derivative $\partial_{\cos^2\beta} a'_{20}$ does not depend on the angle β, at the stationary point (11) it is equal to

$$\partial_{\cos^2\beta} a'_{20}\Big|_{e^{-2i\alpha} = \frac{a_{22}}{R}} = \frac{3}{2}a_{20} + 3R\,. \tag{12}$$

Therefore the X^0-meson spin analyzer corresponding to the extreme a_{20} values lies either in the decay plane or coincides with the normal to the decay plane ($\cos^2\beta = 0$ or 1)

$$a'_{20}\Big|_{\substack{\min\\ \max}} = \begin{cases} -\frac{1}{2}a_{20} \mp 3|a_{22}| & \text{if } \mp a_{20} < 2|a_{22}|,\ \vec{v} = (\cos\alpha, \sin\alpha, 0) \\ a_{20} & \text{if } \mp a_{20} \ge 2|a_{22}|,\ \vec{v} = \vec{n}, \end{cases} \tag{13}$$

where the angle α is determined by the condition (11)

$$\alpha = \begin{cases} \frac{1}{2}\arccos\frac{\mathrm{Re}\,a_{22}}{R} & \text{if } \frac{\mathrm{Im}\,a_{22}}{R} < 0 \\ \pi - \frac{1}{2}\arccos\frac{\mathrm{Re}\,a_{22}}{R} & \text{if } \frac{\mathrm{Im}\,a_{22}}{R} \ge 0\,. \end{cases} \tag{14}$$

For the $X^0 \to \eta\pi\pi$ decay the quantities a_{20} and a_{22} can be expressed through the complex mixing parameter (w) of the amplitudes with $l_\eta = 0$, $l_{\pi\pi} = 2$ and $l_\eta = 2$, $l_{\pi\pi} = 0$ (see (A.11) in the Appendix). If $\mathrm{Re}\,w \le 0$, the derivative (12) is nonpositive and, according to (13), the quantity a'_{20} is maximal for the analyzer $\vec{v}_0 = (\cos\alpha, \sin\alpha, 0)$ lying in the decay plane and minimal for the analyzer $\vec{v} = \vec{n}$. Using $w = -3i$ *), we get the d_2 values (see (A.12-14) in the Appendix) corresponding to the analyzers $\vec{n}$, $\vec{v}_0$, $\vec{k}$ and $\vec{q}$ ($\vec{q}$ is the momentum one of the pions in the dipion rest frame) :

$$d_2^{max} = d_2^{(n)} = -0.5 \ , \ d_2^{max} = 0.86 \ , \ d_2^{(k)} = 0.42 \ , \ d_2^{(q)} = 0.58. \qquad (15)$$

The negative $d_2^{(n)}$ and positive $d_2^{(k)}$ values are both in agreement with the anisotropies observed in /2/.

For the $X^0 \to \gamma \pi^+ \pi^-$ decay the quantities a_{20} and a_{22} can be expressed through the real mixing parameter (g) of the E2 and M1 transition amplitudes (see (A.18) in the Appendix). The g dependence of the quantities d_2^{min}, d_2^{max}, $d_2^{(n)}$, $d_2^{(k)}$ and $d_2^{(q)}$ is shown in fig. 1. The experimental data on the ρ^0-meson polarization in the $X^0 \to \gamma \rho^0$ decay give for the parameter g the following estimates : $g = -3.5^{+1.4}_{-\infty}$ and $g = 2.0^{+\infty}_{-1.3}$ (see fig. 1). Small negative values of the parameter g are probably excluded by the anisotropy observed /1, 2/ in the $\overleftrightarrow{Kk}$ distribution ($d_2^{(k)} < 0$). Therefore we analyze the two limiting cases : $g = 1$ and $|g| \gg 1$.

a) $g = 1$. The derivative (12) is nonnegative (nonpositive) for $R = \mp |a_{22}|$ and therefore, in accordance with (13), the a'_{20} value is minimal ($R = -|a_{22}|$) and maximal ($R = +|a_{22}|$) for the analyzers $\vec{v}_{min}$ and $\vec{v}_{max}$ both lying in the decay plane. For the quantities d_2 we get the values

$$d_2^{min} = -0.98, \ d_2^{max} = 0.56, \ d_2^{(n)} = 0.42, \ d_2^{(k)} = -0.84, \ d_2^{(q)} = 0.47. \qquad (16)$$

b) $|g| \gg 1$. In this case $|\mathrm{Im}\ a_{22}| \ll |\mathrm{Re}\ a_{22}|$, i.e., $\alpha = 0$ or $\pi/2$. Analyzing the derivative (12) we find the quantity a'_{20} is minimal (maximal) for the analyzer $\vec{v} = \vec{k}$ ($\vec{v} = \vec{n}$). For the d_2 values we get the following estimates :

*) The recent Brookhaven result /2/ is $w^{-1} = -0.02 \pm 0.05 + (0.35 \pm 0.02)i$.

$$d_2^{min} = d_2^{(k)} = -0.714, \ d_2^{max} = d_2^{(n)} = 0.786, \ d_2^{(q)} = -0.2 \ . \qquad (17)$$

It should be stressed that the use of the X^0-meson spin analyzer in the decay corresponding to the extreme d_2 values could essentially increase the confidence level of the arguments in favour of or against the hypothesis 2^-.

The author is much grateful to A.N. Zaslavsky for initiating this work.

Appendix

The differential probability of the decay $X^0 \to 1, 2, \ldots \alpha$ can be expressed through the X^0-meson spin density matrix elements in the production process ($\rho_{mm'}$) and in the decay($r_{m'm}$) determined in a system x, y, z (m, m' are the X^0-meson spin projections on the z-axis)

$$dW = \sum_{m,m'} r_{m'm}\, \rho_{mm'}\, d_\alpha(X; 1, 2, \ldots \alpha), \qquad (A.1)$$

where the decay phase space element takes the form

$$d_\alpha(X; 1, 2, \ldots \alpha) = \prod_{j=1}^{\alpha} \frac{d\vec{p}_j}{2\omega_j}\, \delta^{(4)}\left(p_X - \sum_{i=1}^{\alpha} p_i\right), \qquad (A.2)$$

$p_j = (\vec{p}_j, i\omega_j)$ is the 4-momentum of the particle j. With the aid of the vectors in the X^0-meson decay a coordinate system ξ, η, ζ can be fixed. Let us denote the Euler angles of the rotation x, y, z $\to \xi, \eta, \zeta$ by φ, θ, ψ. The phase space elements in the two- and three-particle decays can be then written in the form

$$d_2(X; 1, 2) = \frac{k}{4m_X}\, d\varphi\, dx\,, \quad d_3(X; 1, 2, 3) = \frac{kq}{8m_X}\, dm_{23}\, d\cos\delta\, d\varphi\, dx\, d\psi, \qquad (A.3)$$

where $x = \cos\theta$; $\vec{k} = \vec{p}_1^{(X^0)}$ is the momentum of particle 1 in the X^0-meson rest frame; $\vec{q} = \vec{p}_2^{(23)}$ is the momentum of particle 2 in the c.m.s. of particles 2, 3; m_{23} is the effective mass of particles 2, 3, and δ is the angle between the momenta $\vec{k}$ and $\vec{q}$.

In order to get the φ, θ, ψ dependence of the distribution (A.1), the $\hat{r}$-matrix elements in the x, y, z system should be expressed through the $\hat{r}$-matrix elements in the ξ, η, ζ system. For this reason the transformation (7) of the decay amplitudes can be used. It is clear that the angle φ enters into the distribtuion (A.1) in the form $e^{i(m-m')\varphi}$. Therefore the distribution integrated over the angle φ can depend on the diagonal $\hat{\rho}$-matrix elements only; it can be written in the form:

$$\int_0^{2\pi} dW = \sum_m r_{mm}\, \rho_{mm} \int_0^{2\pi} d_\alpha(X; 1, 2, \ldots \alpha) = \frac{1}{5}\left(Sp\,\hat{\rho}\cdot Sp\,\hat{r} + \frac{10}{7} c_2 a'_{20} + \frac{18}{7} c_4 a'_{40}\right) \int_0^{2\pi} d_\alpha(X, 1, 2, \ldots \alpha), \qquad (A.4)$$

where $Sp\,\hat{\rho} = 1$ and the quantities c_l and a'_{l0} are determined in the x,y,z system by the formulae (2) and (4); the θ, ψ dependence of the quantities a'_{l0} is given by the formula (8) where we should set $\alpha = -\psi$ and $\beta = -\theta$, i.e.,

$$\int_0^{2\pi} dW = \frac{1}{5}\left[Sp\,\hat{r} + \frac{2}{7}\sum_{l=2,4}\sum_{k=0}^{l}(2l+1)c_l\, Re\left(e^{-ik\psi} a_{lk}\right) P_l^k(x)\right] \cdot \int_0^{2\pi} d_\alpha(X; 1, 2, \ldots \alpha), \qquad (A.5)$$

where the quantities a_{lk} are determined in the ξ, η, ζ system by the formulae (4) and (9). Integrating over the phase space we get the formula (1) for the W(x) distribution.

Let us calculate the quantities a_{lk} for the known X^0-meson decays $X^0 \to \gamma\gamma$, $X^0 \to \eta\pi\pi$ and $X^0 \to \gamma\pi^+\pi^-$. In the two-particle decay we choose the ζ-axis along the momentum $\vec{k}$ of particle 1 in the X^0-meson rest frame. In the three-particle decays we direct the ζ-axis along the normal to the decay plane and the ξ-axis along the momentum $\vec{k}$ of particle 1 (η-meson or γ-quantum).

The $X^0 \to \gamma\gamma$ decay. The amplitude of this decay is unambiguously determined by the requirement for the symmetry under the change of γ -quanta and by the γ -quantum transversality

$$A_{ij} = k_i [\vec{e}^{(1)} \times \vec{e}^{(2)}]_j , \qquad (A.6)$$

where $\vec{e}^{(1,2)}$ are the γ -quantum polarization vectors and $\vec{k} = k(0,0,1)$. The vector representation is connected with the representation of the X^0-meson spin projections on the ζ -axis by the following relations :

$$\begin{aligned} A(\pm 2) &= \tfrac{1}{2}(A_{11} - A_{22}) \pm \tfrac{i}{2}(A_{12} + A_{21}) \\ A(\pm 1) &= \mp \tfrac{1}{2}(A_{13} + A_{31}) - \tfrac{i}{2}(A_{23} + A_{32}) \qquad (A.7) \\ A(0) &= \tfrac{1}{\sqrt{6}}(2A_{33} - A_{11} - A_{22}) . \end{aligned}$$

These relations automatically pick out the symmetric and zero trace part of the amplitude A_{ij}. For the amplitude (A.6) only $A_{33} \neq 0$, i.e., only the r_{00} element differs from zero. Acccording to the formulae (4) and (3), we have

$$a_{20} = a_{40} = Sp\hat{r} = r_{00} \quad , \quad d_2 = d_4 = 1 . \qquad (A.8)$$

The $X^0 \to \eta\pi\pi$ decay. In the lowest orbital moment approximation $l_\eta = 2$, $l_{\pi\pi} = 0$ and $l_\eta = 0$, $l_{\pi\pi} = 2$ the decay amplitude takes the form :

$$A_{ij} = k_i k_j + w\, q_i q_j , \qquad (A.9)$$

where $\vec{k} = k(1, 0, 0)$, $\vec{q} = q(\cos\delta, \sin\delta, 0)$ in the ξ, η, ζ system. According to (A.7), in the representation of the X^0-meson spin projections on the ζ -axis we obtain

$$\begin{aligned} A(\pm 2) &= \tfrac{1}{2}(k^2 + w q^2 \cos 2\delta) \pm \tfrac{i}{2} w q^2 \sin 2\delta \\ A(\pm 1) &= 0 \quad , \quad A(0) = -\tfrac{1}{\sqrt{6}}(k^2 + w q^2) \qquad (A.10) \end{aligned}$$

From the formulae (4,), (5) and (9) we get

$$\begin{aligned} Sp\hat{r} &= \tfrac{2}{3}\left[k^4 + |w|^2 q^4 + 2\,\mathrm{Re}\,w \cdot k^2 q^2 P_2(\cos\delta)\right] \\ a_{20} &= -\tfrac{1}{3}\left[k^4 + |w|^2 q^4 - 2\,\mathrm{Re}\,w \cdot k^2 q^2 + 4\,\mathrm{Re}\,w \cdot k^2 q^2 P_2(\cos\delta)\right] \\ a_{22} &= \tfrac{1}{6}\left[k^4 + \mathrm{Re}\,w \cdot k^2 q^2 + (\mathrm{Re}\,w \cdot k^2 q^2 + |w|^2 q^4)\, e^{-2i\delta}\right] \\ a_{40} &= \tfrac{1}{4}(k^4 + |w|^2 q^4) + \tfrac{5}{18}\,\mathrm{Re}\,w \cdot k^2 q^2 + \tfrac{2}{9}\,\mathrm{Re}\,w \cdot k^2 q^2 P_2(\cos\delta) \qquad (A.11) \\ a_{42} &= -\tfrac{1}{6}\, a_{22} \\ a_{44} &= \tfrac{1}{8 \cdot 36}\left[k^4 + 2\,\mathrm{Re}\,w \cdot k^2 q^2 e^{-2i\delta} + |w|^2 q^4 e^{-4i\delta}\right] \end{aligned}$$

For the quantities d_l determined by the formula (3) we get *)

$$d_2^{(n)} = -\frac{1}{2} + \mathrm{Re}\,w \frac{\alpha_3}{\alpha_1 + |w|^2 \alpha_2} \quad ; \quad d_4^{(n)} = \frac{3}{8} + \frac{5}{12}\,\mathrm{Re}\,w \frac{\alpha_3}{\alpha_1 + |w|^2 \alpha_2} , \qquad (A.12)$$

where α_1, α_2 and α_3 are the phase space integrals over the quantities k^4, q^4 and k^2q^2 ; $\alpha_1 : \alpha_2 : \alpha_3 = 6.6 : 1 : 1.5$. The quantities a_{lo} in another coordinate system ξ', η', ζ' can be calculated with the aid of the transformation (8). For example, the Euler angles $\alpha = 0$, $\beta = \pi/2$ or $\alpha = \delta$, $\beta = \pi/2$ correspond to the choice of the ζ'-axis along the momentum $\vec{k}$ or $\vec{q}$. For the

*) We neglect a possible dependence of the parameter w on the dipion mass.

quantities $d_1^{(k)}$ and $d_1^{(q)}$ we have

$$d_2^{(k)} = d_4^{(k)} = \frac{\alpha_1}{\alpha_1 + |w|^2 \alpha_2} \quad , \quad d_2^{(q)} = d_4^{(q)} = \frac{|w|^2 \alpha_2}{\alpha_1 + |w|^2 \alpha_2} \; . \tag{A.13}$$

If Re $w \leq 0$, the extreme d_2 values are equal to

$$d_2^{min} = d_2^{(n)} \quad , \quad d_2^{max} = d_2^{(v_0)} = \frac{5}{8} + \frac{1}{4} \mathrm{Re}\, w \frac{\alpha_3}{\alpha_1 + |w|^2 \alpha_2} + \frac{3}{8} \frac{\alpha_4}{\alpha_1 + |w|^2 \alpha_2} \; , \tag{A.14}$$

where $\vec{v}_0 = (\cos\alpha, \sin\alpha, 0)$; the angle α is determined by the formula (14) ; α_4 is the phase space integral over the quantity

$$u = \begin{cases} \dfrac{(a-b)^2}{\sqrt{-4ab}} \arcsin \dfrac{\sqrt{-4ab}}{|a-b|} & \text{for } a \leq 0 \text{ or } a > \dfrac{(\mathrm{Im}\, w)^2}{-\mathrm{Re}\, w} k^2 q^2 \\ \dfrac{(a-b)^2}{\sqrt{4ab}} \log \dfrac{\sqrt{4ab} + |a+b|}{|a-b|} & \text{for } 0 < a \leq \dfrac{(\mathrm{Im}\, w)^2}{-\mathrm{Re}\, w} k^2 q^2 \; , \end{cases} \tag{A.15}$$

where $a = \mathrm{Re}\, w\, k^2 q^2 + k^4$, $b = \mathrm{Re}\, w\, k^2 q^2 + |w|^2 q^4$. For the value $w = -3i$ $\alpha_4 : \alpha_2 = 9{,}9 : 1$.

<u>The $X^0 \to \gamma \pi^+ \pi^-$ decay.</u> Taking into account only the M1 and E2 transition amplitudes in the dominating $X^0 \to \gamma \rho^0$ decay, we can write the decay amplitude in the form

$$A_{ij} = \{ q_i [\vec{k} \times \vec{e}]_j + g\, e_i [\vec{k} \times \vec{q}]_j \} f(m_{\pi\pi}) \; , \tag{A.16}$$

where $f(m_{\pi\pi})$ is the ρ^0-meson propagator. Omitting the inessential factor $kq\, f(m_{\pi\pi})$ we get the following expressions for the decay amplitudes in the representation of the X^0-meson spin projections on the γ-axis :

$$\begin{aligned} A(\pm 2) &= \frac{1}{2} (\sin\delta \mp i \cos\delta)\, e_3 \\ A(\pm 1) &= \mp \frac{1}{2} (\cos\delta\, e_2 + g \sin\delta\, e_1) - \frac{i}{2} (g+1) \sin\delta\, e_2 \\ A(0) &= \frac{1}{\sqrt{6}} (2g+1) \sin\delta\, e_3 \; . \end{aligned} \tag{A.17}$$

From the formulae (4), (5) and (9) we then obtain

$$\begin{aligned} Sp\, \hat{r} &= \frac{1}{9} \left[10 + 10g + 7g^2 - (1 + 10g + 7g^2) P_2(\cos\delta) \right] \\ a_{20} &= \frac{1}{18} \left[-\frac{5}{2} + 14g + 11g^2 - (2 + 14g + 11g^2) P_2(\cos\delta) \right] \\ a_{22} &= -\frac{1}{24} \left[2 + 7g + 3g^2 \sin^2\delta - (5 + 7g)\, e^{-2i\delta} \right] \\ a_{40} &= \frac{1}{9} \left[-\frac{5}{4} + 2g^2 - (1 + 2g^2) P_2(\cos\delta) \right] \\ a_{42} &= \frac{1}{72} \left[1 - 2g^2 \sin^2\delta + e^{-2i\delta} \right] \\ a_{44} &= -\frac{1}{8 \cdot 36} e^{-2i\delta} \; . \end{aligned} \tag{A.18}$$

For the quantities d_1 corresponding to the analyzers $\vec{n}$, $\vec{k}$ and $\vec{q}$ we get the following expressions :

$$\begin{aligned} d_2^{(n)} &= \frac{1}{4} \frac{-0{,}5 + 2{,}8g + 2{,}2g^2}{1 + g + 0{,}7g^2} \; , \quad d_4^{(n)} = \frac{1}{4} \frac{-0{,}5 + 0{,}8g^2}{1 + g + 0{,}7g^2} \\ d_2^{(k)} &= -\frac{1}{2} \frac{0{,}7 + 2{,}8g + g^2}{1 + g + 0{,}7g^2} \; , \quad d_4^{(k)} = \frac{0{,}2g^2}{1 + g + 0{,}7g^2} \\ d_4^{(q)} &= 0{,}7 \frac{1 + g - 0{,}2g^2}{1 + g + 0{,}7g^2} \; , \quad d_4^{(q)} = 0 \; . \end{aligned} \tag{A.19}$$

The g dependence of the quantities d_2 is shown in fig. 1 together with the extreme d_2 values. In fig. 1 we also present the g dependence of the ρ_{00} density matrix element of the ρ^0-meson produced in the $X^0 \to \gamma \rho^0$ decay which (in the helicity frame) takes the form

$$\rho_{00}^{H} = 0.3/(1 + g + 0.7g^2) \; . \tag{A.20}$$

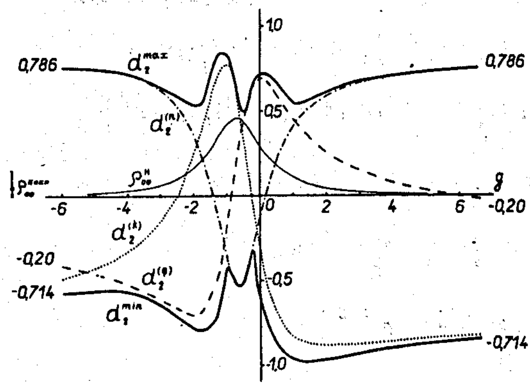

Fig. 1. The decay coefficients d_2 vs the mixing parameter g of the E2 and M1 transition amplitudes in the $X^o \to \gamma \rho^o$ decay. The g dependence of the ρ_{oo} density matrix element of the ρ^o-meson produced in this decay is presented as well.

References

1. G.R.Kalbfleisch et al., Phys.Rev.Lett., 31, 333 (1973).
2. J.S.Danburg et al., preprint BNL-17997, NG-261 (1973).
3. V.I.Ogievetsky, W.Tybor, A.N.Zaslavsky, Phys.Lett., 35B, 69 (1971).
4. S.Giler, I.Klosinski, W.Lefik, W.Tybor, Acta Phys.Pol., A37, 475 (1970).
5. S.M.Berman, M.Jacob, Phys.Rev., 139, B1023 (1965).

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