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I. The method exploited now for describing the high-energy inclusive processes is just the phenomenological Regge analysis suggested by Mueller [1,2]. Also investigations of these processes are undoubtfully of current interest within the framework of Lagrangian field theory [3].

In the present paper an attempt is made to apply to these processes the approach which is known now as the straight-line path approximation in quantum field theory [4]. As has been shown [4,5] this approximation results in the eikonal representation of elastic scattering amplitude at high energies and small momentum transfers.

The calculation scheme is rather simple. One starts with constructing the three-particle elastic forward scattering amplitude (in the above approximation). The discontinuity of the amplitude in the appropriate energy variable gives the inclusive cross section [1,6].

For simplicity, we consider scalar ''nucleons'' interacting with neutral vector mesons: $\mathcal{L}_{int} = -ig\,\psi^* \overleftrightarrow{\partial}_\mu \psi A^\mu$. Just in this model the eikonal approximation provides the correct high energy behaviour for the sum of all possible ladder diagrams [4].

We have shown that in the above approximation the inclusive cross section appears to be independent of energy variables s, t, u in both regions - fragmentation one and that of pionization, and it is only a function of $\vec{q}_\perp^{\,2}$ ($\vec{q}_\perp$ is the transversal component of momentum of the detected particle).

II. Consider now the inclusive reaction $a + b \to c + \ldots$ (Fig. 1).

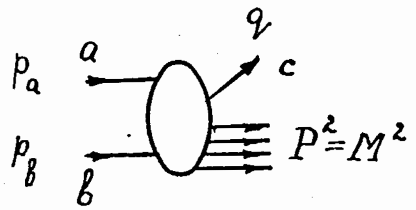

Fig. 1.

Its invariant cross section when all the three particles a, b, c are scalar (nucleons, ψ) can be represented as follows *

$$(2\pi)^3\, 2\omega(\vec{q})\, \frac{d\sigma}{d^3q} = f(s,t,u) =$$

$$= \frac{1}{4\sqrt{(p_a p_b)^2 - m_a^2 m_b^2}} \int dy\, e^{-iqy} \langle a, b(+) \,|\, j(y)\, j(0) \,|\, a, b(+) \rangle, \tag{1}$$

where $j(x)$ is the current corresponding to c -particle. This cross section is, in general, a function of three variables. The latter can be chosen from four variables s, t, u and M^2 where

$$s = (p_a + p_b)^2, \quad t = (p_a - q), \quad u = (p_b - q)^2,$$

$$s + t + u = M^2 + 3m^2.$$

The inclusive cross section (1) relates to the elastic scattering amplitude T for the process $a+b+\bar{c} \to a+b+\bar{c}$ in the following way /1, 2, 6/

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* The invariant normalization for one-particle states $\langle p | p' \rangle = (2\pi)^3\, 2\omega(\vec{p})\, \delta^{(3)}(\vec{p} - \vec{p}\,')$ is used here.

$$f(s,t,u) = \frac{\Delta_{M^2} T(a+b+\bar{c} \to a+b+\bar{c})}{4\sqrt{(p_a p_b)^2 - m_a^2 m_b^2}} \underset{s\to\infty}{=} \frac{\Delta_{M^2} T(a+b+\bar{c} \to a+b+\bar{c})}{2s}, \tag{2}$$

where $\Delta_{M^2} T$ stands for the discontunuity of the amplitude T in the variable $M^2 = (p_a + p_b - q)^2$

$$\Delta_{M^2} T = T(s,t,u,M^2 + i\epsilon) - T(s,t,u,M^2 - i\epsilon).$$

The three-particle elastic scattering amplitude $T(p_1 + p_2 + p_3 \to q_1 + q_2 + q_3)$ can be found in the eikonal approximation using a method developed in papers /7/ for the two-body problems. If in the amplitude $T(p_1 + p_2 + p_3 \to q_1 + q_2 + q_3)$ one puts

$$p_1 = q_1 = p_a, \; p_2 = q_2 = p_b, \; p_3 = q_3 = -q,$$

then one gets the amplitude for the process $a+b+\bar{c} \to a+b+\bar{c}$, connected with the inclusive cross section via eqs. (1), (2).

When constructing the three-particle amplitude we proceed, following papers /7/, from the corresponding Green function which use allows one to define the S -matrix element through the reduction formula

$$\langle q_1, q_2, q_3(-) \,|\, p_1, p_2, p_3(+) \rangle =$$

$$= i(2\pi)^4\, \delta\Big(\sum_{i=1}^{3} p_i - \sum_{i=1}^{3} q_i\Big)\, T(p_1 + p_2 + p_3 \to q_1 + q_2 + q_3) =$$

$$= i^6 Z^{-3} \int \prod_{k=1}^{3} dx_k\, dy_k\, e^{-iy_k p_k + ix_k q_k} (\Box_{x_k} + m^2)(\Box_{y_k} + m^2).$$

$$\langle T(\psi(x_1)\,\psi(x_2)\,\psi(x_3)\,\psi^*(y_1)\,\psi^*(y_2)\,\psi^*(y_3)) \rangle_0 . \tag{3}$$

To simplify our consideration, all the three particles a, b, c are treated to be nonidentical and the vacuum polarization effects are not taken into account. In this case the three-particle Green function in the r.h.s. of eq. (3) can be written as:

$$G(x_1,x_2,x_3;y_1,y_2,y_3) \equiv \langle T(\psi_a(x_1)\psi_b(x_2)\psi_c(x_3) \times$$

$$\times \psi^*_a(y_1)\psi^*_b(y_2)\psi^*_c(y_3))\rangle = \tag{4}$$

$$=(i)^3 \exp\{\frac{i}{2}\int\int d\xi_1 d\xi_2 \frac{\delta}{\delta A_\sigma(\xi_1)} D^c_{\sigma\rho} \frac{\delta}{\delta A_\rho(\xi_2)}\} \prod_{i=1}^{3} G(x_i,y_i|A)|_{A=0},$$

where $G(x,y|A)$ is the Green function of a "nucleon" in the external field $A_\mu(x)$ obeying the equation

$$[(i\partial_\mu - gA_\mu(x))^2 - m^2]\,G(x,y|A) = -\delta(x-y).$$

A solution to this equation can be written in the form of Feynman path integral /8/

$$G(x,y|A) = i\int_0^\infty d\tau\, e^{-im^2\tau} \int [\delta^4\nu]_0^\tau \exp\{-ig\int d^4z\, j^\lambda(z)\, A_\lambda(z)\}.$$

$$\delta^{(4)}(x-y-2\int_0^\tau \nu(\eta)\,d\eta), \tag{5}$$

where the classic current $j^\lambda(z)$ of nucleon ψ has the form

$$j^\lambda(z) = 2\int_0^\tau d\xi\,\nu^\lambda(\xi)\,\delta^{(4)}(z-x+2\int_\xi^\tau \nu(\eta)\,d\eta);$$

$$[\delta^4\nu]_0^\tau \equiv \frac{\exp\{-i\int_0^\tau \nu^2(\eta)\,d\eta\}\,\prod_\eta d^4\nu(\eta)}{\int \exp\{-i\int_0^\tau \nu^2(\eta)\,d\eta\}\,\prod_\eta d^4\nu(\eta)}.$$

On substituting (5) into (4) the variational differentiation is easily performed and the Green function achieves the following form

$$G(x_1,x_2,x_3;y_1,y_2,y_3) = \prod_{k=1}^{3}(i\int_0^\infty d\tau_k\, e^{-im^2\tau_k}\int[\delta^4\nu_k]_0^{\tau_k} \times$$

$$\times\, \delta^{(4)}(x_k-y_k-2\int_0^{\tau_k}\nu_k(\eta)\,d\eta))\ \exp\{-i\frac{g^2}{2}\sum_1^3 j_n D^c j_m\}. \tag{6}$$

Here the conventions are used:

$$j_n\cdot D^c\cdot j_m \equiv \int\int dz_1 dz_2\, j_n^\sigma(z_1)\, D^c_{\sigma\rho}(z_1-z_2)\, j_m^\rho(z_2).$$

When going over to the mass shell in eq. (3) it is convenient to replace the Green function G by the following expression:

$$\bar G(x_1,x_2,x_3;y_1,y_2,y_3) = G(x_1,x_2,x_3;y_1,y_2,y_3) -$$

$$-\sum_{i\neq k\neq \ell}^{3}(-i)\,G(x_i,y_i)\,G(x_k,x_\ell;y_k,y_\ell) + 2\prod_{k=1}^{3}(-i)\,G(x_k,y_k).$$

Here we employed that the difference

$$G(x_1,x_2,x_3;y_1,y_2,y_3) - \bar G(x_1,x_2,x_3;y_1,y_2,y_3)$$

is zero on the mass shell, $p_i^2 = q_i^2 = m^2 (i=1,2,3)$. This can be easily demonstrated if one keeps in mind that the one-particle Green function $G(p,q)$ near the mass shell is of the form

$$G_{p^2,q^2\to m^2}(p,q) \approx \frac{Z(2\pi)^4\,\delta^{(4)}(p-q)}{m^2-p^2}$$

that gives

$$\lim_{p^2,q^2\to m^2}(p^2-m^2)(q^2-m^2)\,G(p,q) = 0.$$

To obtain $\bar G$ without contribution from the vacuum polarization it sufficies to change in (6) the expression

$$\exp\{-ig^2 \sum_1^3 j_n \cdot D^c \cdot j_m\}$$

by the following sum

$$\prod_{\substack{1\\ k<\ell}}^{3} \left(-ig^2 j_k \cdot D^c \cdot j_\ell \int_0^1 d\lambda \exp\{-i\lambda g^2 j_k \cdot D^c \cdot j_\ell\}\right) +$$

$$+ \prod_{k=2,3} \left(-ig^2 j_1 \cdot D^c \cdot j_k \int_0^1 d\lambda \exp\{-i\lambda g^2 j_1 \cdot D^c \cdot j_k\}\right) +$$

$$+ \prod_{k=1,3} \left(-ig^2 j_2 \cdot D^c \cdot j_k \int_0^1 d\lambda \exp\{-i\lambda g^2 j_2 \cdot D^c \cdot j_k\}\right) +$$

$$+ \prod_{k=1,2} \left(-ig^2 j_3 \cdot D^c \cdot j_k \int_0^1 d\lambda \exp\{-i\lambda g^2 j_3 \cdot D^c \cdot j_k\}\right).$$

Passing over onto the mass shell is realized then by the standard method /7/. As a result the three-particle forward elastic scattering amplitude appears to consist of four terms. First of all, a term is present describing interactions of pairs of all the three particles a, b and c:

$$T(1\leftrightarrow 2,\ 1\leftrightarrow 3,\ 2\leftrightarrow 3) = (2g)^6 \int d\overset{\approx}{x}\, d\bar{x} \prod_{k=1}^{3} \left(\int_{-\infty}^{+\infty} d\xi_k \int [\delta^4 \nu_k]_{-\infty}^{+\infty} \right) \times$$

$$\times [\nu_1(\xi_1) - p_1] D^c [-\overset{\approx}{x} - \frac{1}{2}\bar{x} + 2\int_0^{\xi_1} \nu_1(\eta)\, d\eta - 2p_1\xi_1] \times$$

$$\times [\nu_2(0) - p_2][\nu_1(0) - p_1] D^c [-\overset{\approx}{x} + \frac{1}{2}\bar{x} - 2\int_0^{\xi_3} \nu_3(\eta)\, d\eta + 2p_3\xi_3] \times$$

$$\times [\nu_3(\xi_3) - p_3][\nu_2(\xi_2) - p_2] D^c [\bar{x} + 2\int_0^{\xi_2} \nu_2(\eta)\, d\eta - 2p_2\xi_2] \times$$

$$\times [\nu_3(0) - p_3] \int_0^1 d\lambda_1 d\lambda_2 d\lambda_3 \exp\{-ig^2 \lambda_1 j_1 \cdot D^c \cdot j_2 -$$

$$- ig^2 \lambda_2 j_1 \cdot D^c \cdot j_3 - ig^2 \lambda_3 j_2 \cdot D^c \cdot j_3\}, \tag{7}$$

where the nucleon currents are as follows:

$$j_i(z) = 2\int_{-\infty}^{+\infty} d\alpha [\nu_i(\alpha) - p_i\theta(-\alpha) - q_i\theta(\alpha)] \times$$

$$\times \delta^{(4)}\{z - x_i - 2\int_0^{\alpha} \nu_i(\eta)\, d\eta + 2\alpha[p_i\theta(-\alpha) + q_i\theta(\alpha)]\},$$

$$(i = 1, 2, 3), \tag{8}$$

and

$$x_1 - x_2 = -\overset{\approx}{x} - \frac{1}{2}\bar{x}, \quad x_1 - x_3 = -\overset{\approx}{x} + \frac{1}{2}\bar{x},$$

$$x_2 - x_3 = \bar{x}. \tag{9}$$

The Feynman graphs corresponding to (7) are drawn in Fig. 2a.

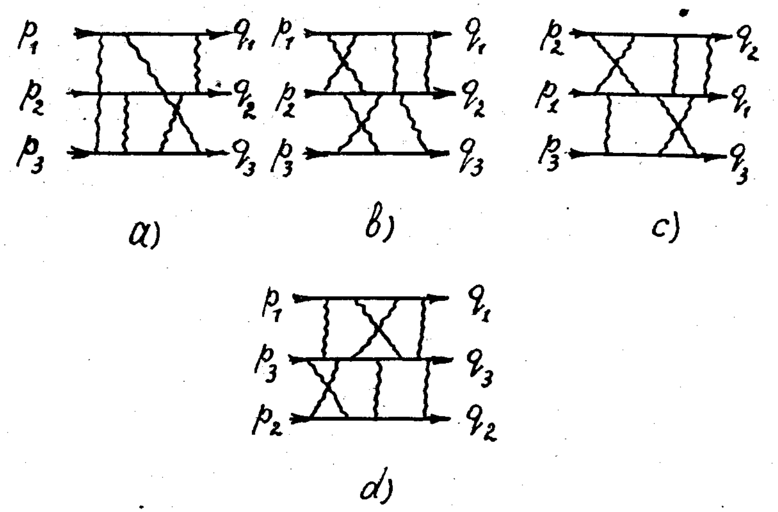

Fig. 2.

The remaining three terms of the scattering amplitude describe the diagrams without meson exchange between any one of pairs of particles a, b, c (Figs. 2b, 2c, 2d):

$$T_{\substack{i \neq k \neq \ell \\ i,k,\ell = 1,2,3}} (i \leftrightarrow k,\ k \leftrightarrow \ell) = i(2g)^4 \int d\tilde{\tilde{x}}\, d\bar{x} \prod_{m=1}^{3} \int [\delta^4 \nu_m] \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} d\xi$$

$$[\nu_i(0) - p_i] D^c(x_i - x_k)[\nu_k(0) - p_k] \times$$

$$\times [\nu_k(\xi) - p_k] D^c[x_k - x_\ell + 2\int_0^{\xi} \nu_k(\eta)\, d\eta - 2p_k \xi][\nu_\ell(0) - p_\ell] \times$$

$$\times \int_0^1 d\lambda_1 \int_0^1 d\lambda_2 \exp\{ig^2 \lambda\, j_\ell \cdot D^c \cdot j_k - i g^2 \lambda_2 j_i \cdot D^c \cdot j_k\}, \qquad (10)$$

where the currents $j_i(z)$ are given by (8) and x_i, x_k are expressed via $\tilde{\tilde{x}}, \bar{x}$ by (9).

In eqs. (7) and (10) the forward scattering is considered, i.e., $p_i = q_i$ ($i = 1,2,3$) is put and the radiation corrections to the lines of ψ-field are omitted.

Note also should be made that among the diagrams of Figs. 2b, 2c, 2d such diagrams are present, as well, which do not contribute to the three-particle scattering amplitude /9/. In these diagrams one particle resulting from an interaction of any pair of the initial particles, is incoming for the process of scattering where a third particle participates (see Fig. 3).

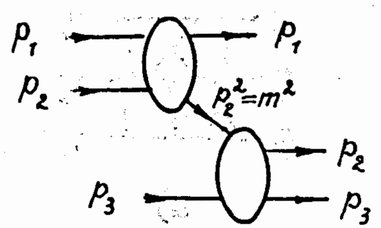

Fig. 3

As a result, additional divergences of the type of $\frac{1}{p^2 - m^2}(p^2 = m^2)$ do appear which are not present in the two-particle reactions.

Nevertheless, in what follows it will not be required to subtract the contributions of these diagrams from eqs. (10) because, as will be shown, the diagrams of this type in the eikonal approximation do not contribute into the inclusive cross section.

To calculate exactly the functional integrals over paths in formulae (7), (10) is not possible, thus we employ the straight-line path approximation /4/ or the eikonal approximation in one of the most simple variants: we omit the functional variables $\nu_i(\eta)$ and $\nu_k(\eta)$ in the expressions $j_i \cdot D^c \cdot j_k$, as was made in investigations of the Green functions and two-particle scattering amplitudes /5,7/.

Let us take the c.m.s. of colliding particles, a and b: $\vec{p}_a = -\vec{p}_b = \vec{p}$ and direct the axis z along the momentum $\vec{p}$. In this system the variables s, t and u are, correspondingly, equal to:

$$s = (p_a + p_b)^2 = 4p_0^2,$$

$$t = (p_a - q)^2 = 2m^2 + 2p_0\left(\frac{p_z}{p_0} q_z - q_0\right) \approx -2p_0(-q_z + q_0),$$

$$u = (p_b - q)^2 = 2m^2 + 2p_0\left(-\frac{p_z}{p_0} q_z - q_0\right) \approx -2p_0(q_z + q_0),$$

$$q_0 > 0, \quad p_z = |\vec{p}|. \qquad (11)$$

Consider now the fragmentation region for the particle b, i.e., the region where $s, -t \to \infty$ and $u = \text{const}$ (or, more exactly, $s \to \infty$, s/M^2 and u fixed, i.e., the so-called one-reggeon limit). This means that $q_0 - q_z \approx \sqrt{s}$, $q_0 + q_z = \text{const}$, i.e., $q_z < 0$ (the particle c is emitted along the direction of motion of the particle b).

First we examine equation (7) in this region.

Omitting the functional variables ν_i $(i = 1,2,3)$ in all the expressions $j_i \cdot D^c \cdot j_k$ we get:

$$-ig^2 j_1 \cdot D^c \cdot j_2 = -ig^2 4p_a \cdot p_b \int \frac{d^4k}{(2\pi)^4} \frac{e^{ikb_1}}{\mu^2 - k^2} \times$$

$$\times \int_{-\infty}^{+\infty} d\xi_1 d\xi_2\, e^{2ik(p_a\xi_1 - p_b\xi_2)} =$$

$$= -ig^2 4p_a p_b \int \frac{d^4k}{(2\pi)^2} \frac{e^{ikb_1}}{\mu^2 - k^2} \delta(2p_a k)\delta(2p_b k) =$$

$$= -\frac{g^2}{2\pi} K_0(\mu b_{1\perp}),$$

$$b_1 = \approx{x} + \frac{1}{2}\bar{x}. \qquad (12)$$

For $j_1 \cdot D^c \cdot j_3$ we have

$$-ig^2 j_1 \cdot D^c \cdot j_3 = +ig^2 4p_a q \int \frac{d^4k}{(2\pi)^4} \frac{e^{ikb_2}}{\mu^2 - k^2} \times$$

$$\times \int_{-\infty}^{+\infty} d\xi_1 d\xi_2\, e^{2ik(p_a\xi_1 + q\xi_2)} =$$

$$= ig^2 4p_a q \int \frac{d^4k}{(2\pi)^2} \frac{e^{ikb_2}}{\mu^2 - k^2} \delta(2kp_a)\delta(2kq),$$

$$b_2 = \approx{x} - \frac{1}{2}\bar{x}.$$

Using δ -functions the integration over k_0 and k_z can be performed:

$$k_0 = k_z = \frac{\vec{q}_\perp \vec{k}_\perp}{q_0 - q_z}.$$

Since in the region under consideration $q_0 - q_z \approx \sqrt{s}$, then one may put $k_0 = k_z = 0$. This gives the following result

$$-ig^2 j_1 \cdot D^c \cdot j_3 = +i\frac{g^2}{2\pi} K_0(\mu b_{2\perp}). \qquad (13)$$

The expression $j_1 \cdot D^c \cdot j_3$ with p_3 replaced by $-q$ describes the scattering of a -particle on c -antiparticle, therefore formulae (12) and (13) differ in sign.

Analogously, for $j_2 \cdot D^c \cdot j_3$ we obtain

$$-ig^2 j_2 \cdot D^c \cdot j_3 = ig^2 4p_b q \int \frac{d^4k}{(2\pi)^2} \frac{e^{ik(b_1 - b_2)}}{\mu^2 - k^2} \times$$

$$\times \delta(2p_b k)\delta(2qk) = ig^2 \int \frac{d^2k_\perp}{(2\pi)^2} \frac{e^{ik_\perp(b_1 - b_2)_\perp}}{\mu^2 + \vec{k}_\perp^2} \times$$

$$\times \exp\{i \frac{\vec{k}_\perp \vec{q}_\perp}{q_0 + q_z}(b_0 + b_z)\}. \qquad (14)$$

In the integral of eq. (14) there is the fast oscillating function $\exp\{i \frac{k_\perp q_\perp}{q_0 + q_z}(b_0 + b_z)\}$, as $q_0 + q_z$ is the fixed quantity and one integrates over b_0 and b_z in formula (7) from $-\infty$ to $+\infty$.

Therefore in the approximation under consideration one may set:

$$-ig^2 j_2 \cdot D^c \cdot j_3 \approx 0.$$

Now we examine the expression before the integral over $d\lambda_i$ in eq. (7):

$$(2g^2)^3 (2p_a p_b)(2p_a q)(2p_b q) \int d^4b_1 \int d^4b_2 \prod_{j=1}^{3} \int_{-\infty}^{+\infty} d\xi_j \times$$

$$\times \int \frac{d^4k_j}{(2\pi)^4} (\mu^2 - k_j^2)^{-1} \exp\{ik_1 b_1 + ik_2 b_2 + ik_3(b_1 - b_2) +$$

$$+ 2ip_a k_1 \xi_1 + 2iqk_2\xi_2 - 2ip_b k_3 \xi_3\}.$$

Integrating over $d\xi_i$ $(i = 1,2,3)$ and over db_k° and db_k^z $(k = 1,2)$ gives the following δ -functions:

$$\delta(2p_a k_1)\,\delta(2qk_2)\,\delta(2p_b k_3)\,\delta(k_1^\circ + k_3^\circ)\,\delta(k_1^z + k_3^z)\times$$

$$\times\,\delta(k_2^\circ - k_3^\circ)\,\delta(k_2^z - k_3^z)\,,$$

whence it follows that

$k_i^\circ = k_i^z = 0\,,\quad i = 1,2,3\,.$

As a result the considered expression is rewritten in the form

$$2tu\int\frac{d^2k_\perp}{2\pi}\,S(k,q_\perp^2)\int d^2b_{1\perp}\,e^{-ik_\perp b_{1\perp}}\,\frac{g^2}{2\pi}\,K_0(\mu b_{1\perp})\times$$

$$\times\int d^2b_{2\perp}\,e^{+ik_\perp b_{2\perp}}\,\frac{g^2}{2\pi}\,K_0(\mu b_{2\perp})\,,$$

where

$$S(k,q_\perp^2) = g^2\,\frac{\delta(\vec{k}_\perp\vec{q}_\perp)}{k_\perp^2 + \mu^2}\,. \tag{15}$$

After integrating over $d\lambda_i$ $(i = 1,2,3)$ formula (7) can be represented as follows:

$$T(a\leftrightarrow b,\, a\leftrightarrow c,\, b\leftrightarrow c) = 2tu\int\frac{d^2k}{2\pi}\,S(k,q_\perp^2)\,F_{ab}(k)\,F_{a\bar{c}}(k)\,, \tag{16}$$

where

$$F_{ik}(k) = \int d^2b\,e^{ikb}\,(e^{i\chi_{ik}(b)} - 1)\,, \tag{17}$$

χ_{ik} and $S(k,q_\perp^2)$ are defined in eqs. (12), (13) and (15).

According to (1) and (2) the corresponding contribution to the inclusive cross section is equal to:

$$f(s,t,u) = (m^2 + q_\perp^2)\int\frac{d^2k}{2\pi}\,S(k,\vec{q}_\perp^{\,2})\,F_{ab}(k)\,F_{a\bar{c}}(k) = f(q_\perp^2)\,. \tag{18}$$

Here the equality $\frac{t\cdot u}{s} = m^2 + q_\perp^2$ is taken into account.

Thus, in the region under consideration the inclusive cross section does not depend upon variables s, t, u and is a function of $q_\perp^2$ only.

The Regge analysis of inclusive reactions /1, 2/ in the fragmentation region leads in addition to the $q_\perp^2$ dependence of f also to the dependence on the Feynman variable $x = \frac{2q_z}{\sqrt{s}}$ $(0 < |x| < 1)$. That this dependence is not present in eq. (18) is accounted for the approximate calculation of the eikonal phase, eq. (13) (i.e., terms of the order $\frac{k_\perp q_\perp}{2q_z}$ were dropped out and $p_0 = p_z$ was assumed).

In an analogous way one can examine formulae (10) describing the diagrams drawn in Figs. 2b, 2c, 2d. Thus, e.g., for $T(a\leftrightarrow b,\ b\leftrightarrow c)$ we have

$$T(a\leftrightarrow b,\, b\leftrightarrow c) = -2isF_{ab}(0)\,S_1(u)\,, \tag{19}$$

where F is given by eq. (17) and

$$S_1(u) = 2ig^2(-u - 2m^2)\int_{-\infty}^{+\infty}d\xi\int d^4x\prod_{k=1}^{2}\int[\delta^4\nu_k]_{-\infty}^{+\infty}\times$$

$$\times\,D^c[x + 2\int_0^\xi \nu_2(\eta)\,d\eta - 2p_b\xi]\int_0^1 d\lambda\,\exp\{-ig^2\lambda\, j_2\cdot D^c\cdot j_3\}.$$

The currents j_2 and j_3 dependent on the functional variables ν_2 and ν_3 are given by (8).

Additional divergences connected with diagrams presented in Fig. 3 are included in $S_1(u)$. Consequently, the amplitude under consideration is a product of two functions one of which depends only on s and the other upon u only. Therefore, $T(a\leftrightarrow b, b\leftrightarrow c)$ does not depend upon the third variable M^2 and does not contribute to the discontinuity in M^2. Note that such a factorization results from the use of the straight-line path approximation. In general case, the diagrams drawn in Figs. 2b, 2c and 2d depend upon s, t and M^2.

The same answer can be received for the amplitudes $T(a\leftrightarrow c\,,\ b\leftrightarrow c)$ and $T(a\leftrightarrow b\,,\ a\leftrightarrow c)$.

Formulae (16) and (19) obtained for the three-particle scattering amplitude are rather similar to the Glauber representation for the amplitude of high-energy scattering on a weakly connected system, e.g., on deuteron /10/. As is known, in the nonrelativistic limit this representation is as follows:

$$F(\vec q) = S\left(\frac{1}{2}\vec q\right) f_n(\vec q) + S\left(-\frac{1}{2}\vec q\right) f_p(\vec q) +$$

$$+ \frac{i}{2\pi k}\int S(\vec q\,') f_n\left(\frac{1}{2}\vec q + \vec q\,'\right) f\left(\frac{1}{2}\vec q - \vec q\,'\right) d^2\vec q\,',$$

where

$$\sigma_d = \frac{4\pi}{k}\,\mathrm{Im}\,F(0)\,,\quad \sigma_j = \frac{4\pi}{k}\,\mathrm{Im}\,f_j(0)\,,\quad j = n, p;$$

$S(\vec q) = \int e^{i\vec q\vec r}\,|\Phi(\vec r)|^2 d\vec r$ - is the deuteron form factor.

Next we proceed to the pionization region, i.e., that region where all the three variables s, t and u are large, which results in the condition $q_0 \pm q_z \approx \sqrt{s} \to \infty$ in accordance with (11). Now one cannot put $j_2 \cdot D^c \cdot j_3 \approx 0$ and formula (14) gives:

$$-ig^2 j_2 \cdot D^c \cdot j_3 = ig^2\int \frac{d^2k_\perp}{(2\pi)^2}\,\frac{e^{-ik_\perp(b_1-b_2)_\perp}}{\mu^2 + k_\perp^2} =$$

$$= i\frac{g^2}{2\pi} K_0(\mu\,|(b_1 - b_2)_\perp|)\,.$$

Therefore the three-particle scattering amplitude $T(a\leftrightarrow b,\ b\leftrightarrow c,\ a\leftrightarrow c)$, eq. (7), can be represented in the pionization region in the following form:

$$T(a\leftrightarrow b,\ b\leftrightarrow c\,,\ a\leftrightarrow c) = 2tu\int d^2b_1\int d^2b_2\int \frac{d^2k}{2\pi}\,\frac{\delta(\vec k_\perp \vec q_\perp)}{k_\perp^2+\mu^2}\times$$

$$\times\, e^{i\vec k_\perp(\vec b_1-\vec b_2)_\perp}\int_0^1 d\lambda\, e^{i\lambda\chi(b_1-b_2)}(e^{-i\chi(b_1)}-1)(e^{i\chi(b_2)}-1)\,,$$

where $$\chi(b) = \frac{g^2}{2\pi}K_0(\mu b)\,.$$

Unlike (16), in this formula the additional factor $\int_0^1 d\lambda\, e^{i\lambda\chi(b_1-b_2)}$ has appeared and the contribution to the inclusive cross section in this case is as follows:

$$f(q_\perp^2) = (m^2 + q_\perp^2)\int d^2b_1\int d^2b_2\,\frac{d^2k_\perp}{2\pi}\,S(k, q_\perp^2)\,e^{ik_\perp(b_1-b_2)_\perp}\times$$

$$\times\int_0^1 d\lambda\, e^{i\lambda\chi(b_1-b_2)} F_{ab}(k)\, F_{a\bar c}(k)\,. \qquad (20)$$

For the diagrams plotted in Figs. 2b, 2c, 2d in the pionization region formulae analogous to (19) can be derived and it can be shown that these do not contribute to the inclusive cross section in the pionization region as well.

Thus, the eikonal approximation within the framework of quantum field model $\mathfrak{L}_{int} = -ig^2\psi^*\overleftrightarrow{\partial}_\mu\psi A^\mu$ results in the inclusive cross section dependent only upon $q_\perp^2$ both in the fragmentation, eq. (19), and pionization, eq. (20), regions.

To complete the paper we make the following remarks concerning the scheme suggested for studying the inclusive processes.

The type of diagrams, Figs. 2, we have considered corresponds to those inelastic processes where only two particles of the type c (two nucleons) are produced. It would be interesting therefore to apply this approach to such an inclusive reaction in which the detected particle is just a meson A.

Throughout all the calculations, in fact, the discontinuity of the three-particle scattering amplitude in M^2 was not calculated, as it was assumed that the amplitude, being a function of three variables s, t, u (or t, u and $q_\perp^2$ in eq. (16)), has the discontinuity in M^2 of which the high energy behaviour is the same as for the amplitude itself, that, of course, is not proven assertion. The high energy behaviour of the three-particle amplitude and of the corresponding inclusive cross section depends essentially on the choice of interaction Lagrangian $\mathfrak{L}_{int}$. Thus, for instance, if one takes the scalar model $\mathfrak{L}_{int} = g\psi^*\psi\phi$, then all the eikonal phases, χ_{ik}, will turn out to be proportional to $\frac{1}{(p_i+p_k)^2}$, i.e., these will decrease with growing s, t or u.

Unlike this, if one considers the exchange not by virtual particles but via reggeons (such an interaction can be modelled, within the framework of field theory, by the infinite sum of ladder diagrams /11/), then the eikonal phases will be complex and equal to:

$$\chi_{ik}(b,\eta_{ik}) = \text{const} \int \frac{d^2k_\perp}{(2\pi)^2} e^{ikb} (\eta_{ik})^{\alpha(-k_\perp^2)-1} \frac{1+e^{-i\pi\alpha(-k_\perp^2)}}{\sin\pi\alpha(-k_\perp^2)} ,$$

where $\eta_{ik} = (p_i + p_k)^2$, $\alpha(-k_\perp^2)$ - is the Regge trajectory.

In this case, the three-particle scattering amplitude and inclusive cross section possess a different behaviour as depending on the value of $\alpha(0)$ (i.e., $\alpha(0)<1$, $\alpha(0)=1$ or $\alpha(0)>1$).

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