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W. Garczyński

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* Permanent address: Institute of Theoretical Physics, University of Wroclaw, Poland

1. Introduotion

In this paper we shall be concerned with some applications of the so-called relativistic Markovian pseudoprocesses [1] to the description of the τ functions of the Minkowski Quantum Field Theory (MQFT). Thus, the paper is a further continuation of our formal studies of the stochastio pseudoprocesses [2,3] (or the quantum stochastio prooesses) based on the assumption of existence of a pseudomeasure needed for the definition of basio objects of the theory like, e.g., Feynman path integrals [4]. We shall remark that among various attempts toward the rigorous definition of the Feynman path integral the nearest to our needs is that by C.De Witt-Morette [5].

In the paper we follow closely the known Ginibre's [6] and Symanzik's [7] works on the Euclidean Quantum Field Theory, (EQFT), in which they used extensively the theory of stochastic processes . One knows that EQFT deals with quantities not directly physically interpretable and the discussion is restricted to a dimension less than four of underlying space-time manifold. We shall replace the Wiener process appearing in the Symanzik paper by the corresponding pseudoprocess and to get the formulae for τ-functions of realistic MQFT. Slightly more general Lagrangians than those treated by Ginibre and Symanzik are considered here, however, the renormalization problems are left untouched. We believe, and it motivates our work, that formulae for τ - functions given here provide the basic formal expressions which should be at hand when trying to generalize them in order to overcome the ultraviolet divergences.

2. The τ - Functions in Terms of the Winer Pseudoprocess. Scalar Neutral Case

Let $a(x)$ be a free neutral scalar field with a mass m. In order to fix the notation we shall write the relevant formulae in some details

$$(\Box - \varkappa^2) a(x) \equiv K a(x) = 0, \quad \Box = -g^{\mu\mu}\partial_\mu \partial_\mu, \quad g^{00} = -g^{kk} = 1, \quad \varkappa = \frac{mc}{\hbar}$$

$$[a(x), a(y)] = -i\Delta(x-y;\varkappa) \tag{2.1}$$

$$\Delta(x;\varkappa) = -i(2\pi)^{-3}\int d^4p\, \epsilon(p_0)\delta(p^2-\varkappa^2)\cdot \exp(ipx).$$

For the τ - functions of this field we may write formally [8]

$$\tau(x_1,\dots,x_n) = \frac{\langle 0|T^* a(x_1)\cdots a(x_n) S|0\rangle}{\langle 0|S|0\rangle}, \tag{2.2}$$

where

$$S = T^* \exp(i L_{int}[a]) \tag{2.3}$$

is the S-matrix of a theory specified by the interaction Lagrangian

$$L_{int}[a] = \int d^4x\, L_{int}[a(x)]. \tag{2.4}$$

A generating functional $J[p]$ for the τ - functions has a form

$$\begin{aligned} J[p] &= 1 + \sum_{n=1}^{\infty} \frac{i^n}{n!} \int \tau(x_n) e[p;x_n] d^4x_n \\ &= \frac{\langle 0|T^* \exp(i a\cdot p) S|0\rangle}{\langle 0|S|0\rangle}, \end{aligned} \tag{2.5}$$

where the following abbreviations for convenience are used

$$\begin{aligned} &\tau(x_m) = \tau(x_1,\dots,x_m) \\ &e[p;x_m] = p(x_1)\cdots p(x_m) \\ &d^4x_m = d^4x_1\cdots d^4x_m \\ &a\cdot p = \int d^4x\, p(x)a(x). \end{aligned} \tag{2.6}$$

The symmetric τ – functions may be recovered from $J[p]$ in the usual way

$$\tau(x_m) = e\left[-i\frac{\delta}{\delta p};x_m\right]\cdot J[p]\Big|_{p=0} \quad . \tag{2.7}$$

It is well known that the generating functional may be expressed in terms of the Feynman functional integrals as follows/9/ *)

$$J[p] = N^{-1}\cdot\int \exp\left(\frac{i}{2}qKq + iL_{int}[q] + iqp\right)dq \tag{2.8}$$

$$= N^{-1}\exp\left(iL_{int}\left[-i\frac{\delta}{\delta p}\right]\right)\exp\left(-\frac{i}{2}pK^{-1}p\right),$$

where

$$K^{-1} = -\Delta^c$$

$$K^{-1}(x,y) = -\Delta^c(x-y) = -\frac{1}{(2\pi)^4}\int\frac{\exp ip(x-y)}{\varkappa^2-p^2-i\epsilon}d^4p \quad . \tag{2.9}$$

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*) We refer the reader to this book also for proofs of all functional identities appearing in the sequel.

The choice of the causal propagator for K^{-1} is a consequence of the presence of the regularizing term $-i\epsilon q^2$ under the exponential sign in the functional integral which is always tacitly assumed.

Now a part of out next considerations will be valid for Lagrangians of the form

$$L_{int}[q(x)] = \mathcal{J}[q^2(x)] \tag{2.10}$$

which includes as a special case the Lagrangian

$$L_{int}[q(x)] = -\frac{g}{4}q^4(x) + \frac{\alpha}{2}q^2(x) \tag{2.11}$$

considered by Symanzik in his Euclidean Quantum Field Theory. In this case we will have for the generating functional

$$\begin{aligned} \mathcal{T}[p] &= N^{-1}\exp\left(i\mathcal{J}\left[-\frac{\delta^2}{\delta p^2}\right]\right)\exp\left(-\frac{i}{2}pK^{-1}p\right) \\ &= N^{-1}\exp\left(i\mathcal{J}\left[-2i\frac{\delta}{\delta u}\right]\right)\exp\left(-\frac{i}{2}u\cdot\frac{\delta^2}{\delta p^2}\right)\Big|_{u=0}\exp\left(-\frac{i}{2}pK^{-1}p\right), \end{aligned} \tag{2.12}$$

where we have used the identity

$$F\left[\frac{\delta}{\delta u}\right]\exp(u\cdot v) = \exp(u\cdot v)F\left[v+\frac{\delta}{\delta u}\right] \tag{2.13}$$

which is valid for any functional expandable into the Volterra series.

Now using another identity valid for any symmetric operator R

$$\exp\left(-\frac{i}{2}\frac{\delta}{\delta p}R\frac{\delta}{\delta p}\right)\exp\left(-\frac{i}{2}pK^{-1}p\right) = \tag{2.14}$$

$$= \exp\left[-\frac{i}{2}p(K+R)^{-1}p\right]\exp\left[-\frac{1}{2}\mathrm{Tr}\ln(1+K^{-1}R)\right],$$

where we put $R = u$, i.e.,

$$R(x,y) = u(x)\cdot\delta(x-y) \tag{2.15}$$

we may write for the generating functional the formula

$$J[p] = N^{-1}\exp\left(i\,J\left[-2i\frac{\delta}{\delta u}\right]\right)\exp\left[-\frac{i}{2}p(K+u)^{-1}p\right] \tag{2.16}$$

$$\cdot\exp\left[\frac{1}{2}\mathrm{Tr}\ln(K+u)^{-1}-\frac{1}{2}\mathrm{Tr}\ln K^{-1}\right]\Big|_{u=0}.$$

Furthermore, using the "proper time" parametric representations/10-12/

$$a^{-1} = -i\int_0^\infty dt\,\exp(ita) \tag{2.17}$$

$$\ln a^{-1} - \ln b^{-1} = \int_0^\infty \frac{dt}{t}\left[\exp(ita)-\exp(itb)\right] \tag{2.18}$$

which hold for a, b having added small imaginary parts $i\epsilon$, $\epsilon>0$ we will get the formulae

$$(K+u)^{-1}_{y,x} = \frac{\gamma}{i}\int_0^\infty dt\,\exp(-i\gamma x^2 t)\left[\exp i\gamma t(\Box+u)\right]_{y,x} \tag{2.19}$$

$$\mathrm{Tr}\ln(K+u)^{-1} - \mathrm{Tr}\ln K^{-1} = \int_0^\infty\frac{dt}{t}\exp(-i\gamma x^2 t)\int dz\left[\exp i\gamma t(\Box+u)\,-\right. \tag{2.20}$$

$$\left. -\exp(i\gamma t\Box)\right]_{z,z}.$$

Here γ is some number which is going to be suitably chosen. We notice that the function

$$f(t;y,x)=\left[\exp i\gamma t(\Box+u)\right]_{y,x} \tag{2.21}$$

solves the following Cauchy problem

$$\left[\partial_t - i\gamma(\Box_x+u)\right]f(t;y,x)=0 \tag{2.22}$$

$$\lim_{t\downarrow 0} f(t;y,x)=\delta(y-x)\ .$$

One sees from it that for $\gamma=\frac{\hbar}{2m}$ we obtain the relativistic Schrödinger equation for the transition amplitude describing spinless particle influenced by the potential $-\frac{\hbar^2}{2m}u$.
Thus, according to a general theory /1/ we will get

$$f(t;y,x)=Q_{0,y}^{t,x}\left\{\exp\frac{\lambda}{2}\int_0^t u[z(\tau)]d\tau\right\}$$

$$=\int_\Omega M_{0,y}^{t,x}(d\omega)\exp\frac{\lambda}{2}\int_0^t u[z(\tau)]d\tau, \tag{2.23}$$

where $\lambda=\frac{i\hbar}{m}$, $z(\tau,\omega)$ is the Wiener pseudoprocess with values in the Minkowski space and $M_{0,y}^{t,x}(d\omega)$ is a conditional pseudomeasure on the space Ω of all trajectories of the pseudoprocess $z(\tau)$.

Notice that the functional $\int_0^t u[z(\tau)]d\tau$ on the pseudoprocess may be written as /7/

$$\int_0^t u[z(\tau)]d\tau=\int d^4x\,u(x)\mathfrak{z}(x,t)=u\cdot\mathfrak{z}(t)\ , \tag{2.24}$$

where the random variable $\mathfrak{z}(x,t,\omega)$ – called the local time of the pseudoprocess $Z(\tau)$ – denotes an accupation time of the state x during the period $[0,t]$ /7,13/

$$\mathfrak{z}(x,t,\omega)=\int_0^t \delta[x-Z(\tau,\omega)]d\tau . \qquad (2.25)$$

Therefore we may write instead of (2.20) and (2.21) the formulae

$$(K+u)^{-1}_{y,x}=-\frac{\lambda}{2}\int_0^\infty dt\,\exp\left(-\frac{\lambda\varkappa^2}{2}t\right)\int_\Omega M^{t,x}_{0,y}(d\omega)\exp\frac{\lambda}{2}u\cdot\mathfrak{z}(t,\omega) \qquad (2.26)$$

and

$$\operatorname{Tr}\ln(K+u)^{-1}-\operatorname{Tr}\ln K^{-1}= \qquad (2.27)$$

$$=\int_0^\infty\frac{dt}{t}\exp\left(-\frac{\lambda\varkappa^2}{2}t\right)\int dz\int_\Omega M^{t,z}_{0,z}(d\bar\omega)\left[\exp\frac{\lambda}{2}u\cdot\mathfrak{z}(t,\bar\omega)-1\right].$$

Hence, for the generating functional $J[p]$ we will get

$$J[p]=\mathrm{Const.}\,\exp\left(i\,\mathfrak{I}\left[-2i\frac{\delta}{\delta u}\right]\right)\exp\left(-\frac{i}{2}pA[u]p\right)B[u]\Big|_{u=0}, \qquad (2.28)$$

where

$$A[y,x;u]=(K+u)^{-1}_{y,x} \qquad (2.29)$$

$$B[u]=\exp\left\{\frac{1}{2}\int_0^\infty\frac{dt}{t}\exp\left(-\frac{\lambda\varkappa^2}{2}t\right)\int dz\int_\Omega M^{t,z}_{0,z}(d\bar\omega)\exp\frac{\lambda}{2}u\cdot\mathfrak{z}(t,\bar\omega)\right\} \qquad (2.30)$$

and the constant factor is determined by the condition $J[0]=1$.

The $\mathcal{T}$ – functions may now be easily calculated using the formula (2.7) and the identity

$$e\left[-i\frac{\delta}{\delta p};x_n\right]\exp\left(-\frac{i}{2}pA[u]p\right)\Big|_{p=0} = \begin{cases} 0 \;, & n=2m+1 \\ \sum\limits_{j_k<j_{k+1}} iA[x_{j_1},x_{j_2};u]\cdots iA[x_{j_{n-1}},x_{j_n};u] \;. & n=2m \end{cases} \tag{2.31}$$

Thus we will get for $\mathcal{T}(x_1,\ldots,x_n)$

$$\mathcal{T}(x_1,\ldots,x_n) = \begin{cases} 0 \;, & n=2m+1 \\ \sum\limits_{j_k<j_{k+1}} \phi(x_{j_1},x_{j_2},\ldots,x_{j_{n-1}},x_{j_n}) \;, & n=2m \;, \end{cases} \tag{2.32}$$

where we denoted

$$\phi(x_1,x_2,\ldots,x_{n-1},x_n) =$$

$$= \mathrm{const.}\,\exp\left(i\,\mathbb{J}\left[-2i\frac{\delta}{\delta u}\right]\right)B[u]\prod_{k=1}^{n} iA[x_k,x_{k+1};u]\Big|_{u=0} = \tag{2.33}$$

$$= \prod_{k=1}^{n}\frac{-i\lambda}{2}\int_0^\infty ds_k \exp\left(-\frac{\lambda\varkappa^2}{2}s_k\right)\int_\Omega M_{0,\,x_k}^{s_k,\,x_{k+1}}(d\omega_k)\,\varphi\left[\mathfrak{z}(s_1,\omega_1),\ldots,\mathfrak{z}(s_m,\omega_m)\right]$$

and auxiliary functions

$$\varphi[z(s_1,\omega_1),\dots,z(s_m,\omega_m)] = \tilde{\varphi}[u\cdot z(s_1,\omega_1),\dots,u\cdot z(s_m,\omega_m)] =$$

$$= \text{const.}\exp\left(i\,\mathcal{J}[-2i\tfrac{\delta}{\delta u}]\right)B[u]\exp\left[\frac{\lambda}{2}\sum_{\ell=1}^{n}u\cdot z(s_\ell,\omega_\ell)\right]\Big|_{u=0} \qquad (2.34)$$

Expanding $B[u]$ onto the Taylor series and using the identity (2.13) we will get for auxiliary functions the final formula

$$\varphi[z(s_1,\omega_1),\dots,z(s_m,\omega_m)] = \sum_{m=0}^{\infty}\frac{\left(\frac{1}{2}\right)^m}{m!}\prod_{k=1}^{m}\int_0^\infty\frac{dt_k}{t_k}\exp\left(-\frac{\lambda x^2}{2}t_k\right)\int dz_k$$

$$\int_\Omega M_{0,z_k}^{t_k,z_k}(d\bar{\omega}_k)\,\text{const.}\exp\left(i\,\mathcal{J}[-i\lambda\sum_{j=1}^{m}z(t_j,\omega_j)-i\lambda\sum_{\ell=1}^{n}z(s_\ell,\omega_\ell)]\right), \qquad (2.35)$$

$(n - \text{even})$.

Thus we expressed the basic functions of a theory, τ - functions, via formulae (2.33),(2.36) by the local times of the Wiener pseudoprocess.

3. The Kirkwood-Salsburg and Meyer-Montroll Type Equations for τ - Functions. Scalar Neutral Case

Let us consider now a model defined by the Lagrangian (2.11). In this case the $\mathcal{J}[q]$ functional is

$$\mathcal{J}[q] = \int d^4x\left[-\frac{g}{4}q^2(x) + \frac{\alpha}{2}q(x)\right] = -\frac{g}{4}q^2 + \frac{\alpha}{2}q\,. \qquad (3.1)$$

According to the formula (2. 34) we will have for the auxiliary functions

$$\varphi(z_1,\dots,z_n) = \mathrm{Const.}\exp\left(i g_0 \frac{\delta^2}{\delta u^2}\right) B[u+\alpha] \exp\left[\frac{\lambda}{2}\sum_{\ell=1}^{n}(u+\alpha)\cdot z_\ell\right]\Bigg|_{u=0} , \quad (3.2)$$

where we denoted for shortness

$$z_\ell(x) = z(x, s_\ell, \omega_\ell) \ , \quad z(x, t_j, \bar{\omega}_j) = z_j(x) \quad (3.3)$$

and performed the translation

$$\exp\left(\alpha\frac{\delta}{\delta u}\right)F[u] = F[u+\alpha] \ . \quad (3.4)$$

Furthermore if we denote

$$\frac{1}{2}\int_0^\infty \frac{dt}{t}\exp\left(-\frac{\lambda x^2}{2}t\right)\int dz \int_\Omega M_{0,z}^{t,z}(d\bar{\omega})\cdots \equiv \int R(d\bar{z})\cdots \quad (3.5)$$

then the functional $B[u+\alpha]$ becomes

$$B[u+\alpha] = \exp\left\{\int R(d\bar{z})\exp\left[\frac{\lambda}{2}(u+\alpha)\cdot\bar{z}\right]\right\} \ . \quad (3.6)$$

Following the idea of Symanzik the $\varphi-$ functions may be recoverd from their generating functional as follows

$$\varphi(z_1,\dots,z_n) = e\left[\frac{\delta}{\delta J}; z_n\right]\Phi\{J\}\Big|_{J=0} = \Phi_{z_1\dots z_n}\{J\}\Big|_{J=0} , \quad (3.7)$$

where

$$\Phi\{J\} = \sum_{n=0}^{\infty}\frac{1}{n!}\int\widetilde{R}(dz_1)\cdots\widetilde{R}(dz_n)\varphi(z_1,\dots,z_n)J(z_1)\cdots J(z_n) \quad (3.8)$$

and the "measure" $\widetilde{R}(dz)$ is restricted by the condition

$$\int\widetilde{R}(dz')\delta(z'-z)F(z') = F(z) \ . \quad (3.9)$$

From the formulae (3.2) and (3.6) we will have for this generating functional

$$\Phi\{J\} = \mathrm{const.}\, \exp\left(ig\frac{\delta^2}{\delta u^2}\right)\exp\Big\{\int R(d\bar{z})\exp\Big[\frac{\lambda}{2}(u+\alpha)\cdot\bar{z}\Big] +$$

$$+\int \widetilde{R}(dz)J(z)\exp\Big[\frac{\lambda}{2}(u+\alpha)\cdot z\Big]\Big\}\Big|_{u=0} \quad . \tag{3.10}$$

Using the identity (2.13) and also the formula

$$\exp\Big\{\int R(dz)f(z)\frac{\delta}{\delta J(z)}\Big\}\exp\Big\{\int \widetilde{R}(dz')J(z')g(z')\Big\} = \tag{3.11}$$

$$\exp\Big\{\int\widetilde{R}(dz')J(z')g(z') + \int R(dz)f(z)g(z)\Big\}\exp\Big\{\int R(dz)f(z)\frac{\delta}{\delta J(z)}\Big\}$$

we may find for the first derivative of $\Phi\{J\}$ the expression

$$\Phi_z\{J\} = \exp\Big[\frac{\lambda}{2}\alpha\cdot z + \frac{ig\lambda^2}{4}z^2\Big]\exp\Big\{-\int R(d\bar{z})K(z;\bar{z})\frac{\delta}{\delta J_z(\bar{z})}\Big\}\Phi\{J_z\}, \tag{3.12}$$

where

$$K(z;\bar{z}) = 1-\exp\left(\frac{ig\lambda^2}{2}z\cdot\bar{z}\right), \tag{3.13}$$

$$J_z(z') = J(z')\exp\left(\frac{ig\lambda^2}{2}z\cdot z'\right) \quad . \tag{3.14}$$

Now, if we expand the last exponential and perform needed functional differentiations according to the formula (3.7) we will get the set of equations of the Kirkwood-Salsburg type

$$\varphi(z_1,\dots,z_n)=\exp\Big[\frac{\lambda}{2}\alpha\cdot z_1+\frac{ig\lambda^2}{4}z_1^2+\frac{ig\lambda^2}{2}\sum_{k=2}^{n}z_1\cdot z_k\Big]\cdot$$

$$\sum_{\ell=0}^{\infty}\frac{(-1)^\ell}{\ell!}\prod_{j=1}^{\ell}\int R(d\bar z_j)K(z_1;\bar z_j)\varphi(z_2,\dots,z_n,\bar z_1,\dots,\bar z_\ell)\,. \tag{3.15}$$

However , after iterations of the formula (3.12) we will end up with the expression

$$\Phi_{z_1\cdots z_n}\{J\}=\exp\Big[\frac{\lambda}{2}\alpha\cdot\sum_{k=1}^{n}z_k+\frac{ig\lambda^2}{4}\sum_{k=1}^{n}z_k^2+\frac{ig\lambda^2}{2}\sum_{1\le k\le k'\le n}z_k\cdot z_{k'}\Big]\cdot \tag{3.16}$$

$$\cdot\exp\Big\{-\int R(d\bar z)\,K(z_1,\dots,z_n;\bar z)\frac{\delta}{\delta J_{z_1\cdots z_n}(\bar z)}\Big\}\Phi\{J_{z_1\cdots z_n}\}\,,$$

where

$$K(z_1,\dots,z_n;\bar z)=1-\exp\Big[\frac{ig\lambda^2}{2}(z_1+\dots+z_n)\cdot\bar z\Big] \tag{3.17}$$

and

$$J_{z_1\cdots z_n}(\bar z)=J(\bar z)\cdot\exp\Big[\frac{ig\lambda^2}{2}(z_1+\dots+z_n)\cdot\bar z\Big]\,. \tag{3.18}$$

Expanding now the last exponential in (3.16) and putting $J=0$ we obtain the set of equations of the Meyer-Montroll type

$$\varphi(z_1,\dots,z_n)=\exp\Big[\frac{\lambda}{2}\alpha\cdot\sum_{k=1}^{n}z_k+\frac{ig\lambda^2}{4}\sum_{k=1}^{n}z_k^2+\frac{ig\lambda^2}{2}\sum_{1\le k\le k'\le n}z_k\cdot z_{k'}\Big]\cdot$$

$$\sum_{\ell=0}^{\infty}\frac{(-1)^\ell}{\ell!}\prod_{j=1}^{\ell}\int R(d\bar z_j)K(z_1,\dots,z_n;\bar z_j)\varphi(\bar z_1,\dots,\bar z_\ell) \tag{3.19}$$

for n- even.

We shall not discuss these equations and their consequences here since it is a separate subject.

4. The τ – Functions in Terms of the Wiener Pseudoprocess, K-S and M-M Equations for φ – Functions. Scalar Charged Case

All the previous considerations may be easily translated into the case of complex scalar selfinteracting field with the Lagrangian

$$L_{int}[a,a^{+}] = \int d^4x\, \mathcal{J}[a^{+}(x),a(x)]$$

$$K a(x) = K \overset{*}{a}(x) = 0 \tag{4.1}$$

$$[a(x), a^{+}(y)] = -i\Delta(x-y;\varkappa)\,.$$

The generating functional for the τ – functions

$$\tau(x_1,\dots,x_m,y_1,\dots,y_n) = \frac{\langle 0|\overset{*}{T} a(x_1)\cdots a(x_m)a^{+}(y_1)\cdots a^{+}(y_n) S|0\rangle}{\langle 0|S|0\rangle} \tag{4.2}$$

may be written in one of the following forms

$$\mathcal{T}[p,\overset{*}{p}] = \frac{\langle 0|\overset{*}{T}\exp i(a\cdot\overset{*}{p} + a^{+}\cdot p)S|0\rangle}{\langle 0|S|0\rangle} = \tag{4.3}$$

$$= \sum_{m,n=0}^{\infty} \frac{i^{m+n}}{m!\,n!} \int dx^4_m \int d^4y_n\, \tau(x_m,y_n)\, e[\overset{*}{p};x_m]\, e[p;y_n] \tag{4.4}$$

$$= N^{-1} \int \exp\{i\overset{*}{q}Kq + iL_{int}[q,\overset{*}{q}] + i\overset{*}{p}q + ip\overset{*}{q}\}\, dq\, d\overset{*}{q} \tag{4.5}$$

$$= N^{-1} \exp\left(i L_{int}\left[-i\frac{\delta}{\delta p^*}, -i\frac{\delta}{\delta p}\right]\right) \exp\left(-i p^* K^{-1} p\right) \tag{4.6}$$

$$= N^{-1} \exp\left(i J\left[-i\frac{\delta}{\delta u}\right]\right) \exp\left[-i p^*(K+u)^{-1} p\right] \exp\left[\mathrm{Tr}\ln(K+u)^{-1} - \mathrm{Tr}\ln K^{-1}\right] \tag{4.7}$$

$$= \text{const.} \exp\left(i J\left[-i\frac{\delta}{\delta u}\right]\right) \exp\left(-i p^* A[u] p\right) B^2[u]\Big|_{u=0} , \tag{4.8}$$

where $A[u]$, $B[u]$ are given by the formulae (2.29) and (2.30) respectively while the constant factors are determined by the normalization condition

$$J[0,0] = 1 \ . \tag{4.9}$$

The τ - functions may now be recovered from $J[p, p^*]$ using the formula (4.6) and

$$\tau(x_m, y_n) = e\left[-i\frac{\delta}{\delta p^*}; x_m\right] e\left[-i\frac{\delta}{\delta p}; y_n\right] J[p, p^*]\Big|_{p=p^*=0} \tag{4.10}$$

and also the identity

$$e\left[-i\frac{\delta}{\delta p^*}; x_m\right] e\left[-i\frac{\delta}{\delta p}; y_n\right] \exp\left(-i p^* A[u] p\right)\Big|_{p=p^*=0} =$$

$$= \begin{cases} 0 \ , & m \neq n \\ \sum\limits_{\pi \in S_m} i A[x_1, y_{\pi(1)}; u] \cdots i A[x_m, y_{\pi(m)}; u] \ , & m = n . \end{cases} \tag{4.11}$$

From this we infer for the τ - functions

$$\tau(x_m, y_n) = \begin{cases} 0 & , m \neq n \\ \sum_{\pi \in S_n} \phi(x_1, y_{\pi(1)}, \ldots, x_n, y_{\pi(n)}) & , m = n , \end{cases} \tag{4.12}$$

where we denoted

$$\phi(x_1, y_1, \ldots, x_n, y_n) = \tag{4.13}$$

$$\mathrm{const.} \exp\left(i \mathcal{J}\left[-i \frac{\delta}{\delta u}\right]\right) B^2[u]\, i A[x_1, y_1; u] \cdots i A[x_n, y_n; u] \Big|_{u=0} .$$

Introducing new auxiliary functions (different from those of the previous section)

$$\varphi[z(s_1, \omega_1), \ldots, z(s_n, \omega_n)] = \tag{4.14}$$

$$= \mathrm{const.} \exp\left(i \mathcal{J}\left[-i \frac{\delta}{\delta u}\right]\right) B^2[u] \exp\left[\frac{\lambda}{2} \sum_{\ell=1}^{n} u \cdot z(s_\ell, \omega_\ell)\right] \Big|_{u=0}$$

we will get using (2.26) the result

$$\phi(x_1, y_1, \ldots, x_n, y_n) = \tag{4.15}$$

$$= \prod_{k=1}^{n} \frac{-i\lambda}{2} \int_0^\infty ds_k \exp\left(-\frac{\lambda \varkappa^2}{2} s_k\right) \int_\Omega M_{0, x_k}^{s_k, y_k}(d\omega_k)\, \varphi[z(s_1, \omega_1), \ldots z(s_m, \omega_m)] .$$

Finally, expanding $B^2[u]$ in the Taylor series and using the formula (2.13) we will get for the auxiliary functions

$$\varphi[\mathfrak{z}(s_1,\omega_1),\dots,\mathfrak{z}(s_n,\omega_n)]=\sum_{m=0}^{\infty}\frac{1}{m!}\prod_{k=1}^{m}\int_0^{\infty}\frac{dt_k}{t_k}\exp\left(-\frac{\lambda\varkappa^2}{2}t_k\right)\int dz_k$$

$$\int_{\Omega} M_{0,z_k}^{t_k,z_k}(d\bar{\omega}_k)\,\mathrm{Const.}\exp\left(i\,\mathcal{J}\left[\frac{-i\lambda}{2}\sum_{l=1}^{n}\mathfrak{z}(s_l,\omega_l)-\frac{i\lambda}{2}\sum_{j=1}^{m}\mathfrak{z}(t_j,\bar{\omega}_j)\right]\right) \tag{4.16}$$

Furthermore calculations will follow closely the Symanzik work valid for the Lagrangian

$$L_{int}[q,q^*]=-\frac{g}{2}(q^*q)^2+\alpha\, q^*q=\mathcal{J}[q^*q]\ . \tag{4.17}$$

Using the previous notataion (3.3) we will get now

$$\varphi(\mathfrak{z}_1,\dots,\mathfrak{z}_n)=\mathrm{Const.}\exp\left(\frac{ig}{2}\frac{\delta^2}{\delta u^2}\right)\exp\left[\frac{\lambda}{2}\sum_{l=1}^{n}(u+\alpha)\cdot\mathfrak{z}_l\right]\cdot \tag{4.18}$$

$$\cdot\exp\left\{\int_0^{\infty}\frac{dt}{t}\exp\left(-\frac{\lambda\varkappa^2}{2}t\right)\int dz\int_{\Omega}M_{0,z}^{t,z}(d\bar{\omega})\exp\frac{\lambda}{2}(u+\alpha)\bar{\mathfrak{z}}\right\}\Big|_{u=0}.$$

Comparing this formula with the corresponding formula (3.2) for auxiliary functions in the case of neutral field one notices that they agree after the substitution

$$\begin{aligned} g &\to \frac{g}{2} \\ R &\to 2R \end{aligned} \tag{4.19}$$

in the neutral case formulae.

Therefore we may get immediately relevant Kirkwood-Salsburg and Meyer-Montroll type equations for our new functions performing the above substitution in the formulae (3.15) and (3.19) respectively. Namely, we will get the $K-S$ type integral equations

$$\varphi(z_1,\dots,z_n)=\exp\left[\frac{\lambda}{2}\alpha\cdot z_1+\frac{ig\lambda^2}{8}z_1^2+\frac{ig\lambda^2}{4}\sum_{k=2}^{n} z_1 z_k\right]$$

$$\cdot\sum_{\ell=0}^{\infty}\frac{(-2)^\ell}{\ell!}\prod_{j=1}^{\ell}\int R(d\bar{z}_j)\,K(z_1,\bar{z}_j)\,\varphi(z_2,\dots,z_n,\bar{z}_1,\dots,\bar{z}_\ell) \tag{4.20}$$

and M-M type equations

$$\varphi(z_1,\dots,z_n)=\exp\left[\frac{\lambda}{2}\alpha\cdot\sum_{k=1}^{n} z_k+\frac{ig\lambda^2}{8}\sum_{k=1}^{n} z_k^2+\frac{ig\lambda^2}{4}\sum_{1\le k<k'\le n} z_k z_{k'}\right]\cdot \tag{4.21}$$

$$\cdot\sum_{\ell=0}^{\infty}\frac{(-2)^\ell}{\ell!}\prod_{j=1}^{\ell}\int R(d\bar{z}_j)\,K(z_1,\dots,z_n;\bar{z}_j)\,\varphi(\bar{z}_1,\dots,\bar{z}_\ell)\ ,$$

where notation are the same as in (3.3) and (3.17).

We consider derived formulae for τ- functions as a starting point of a battle with the divergencies which should be undertaken from the probabilistic standpoint.

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APPENDIX

In order to arrive at the usual perturbation expansion for the τ- functions one has to perform the following typical operations

$$\Delta_1 = \frac{-i\lambda}{2}\int_0^\infty ds \exp\left(-\frac{\lambda x^2}{2}s\right) Q_{0,x}^{s,y}\{\mathfrak{z}(x_1,s)\dots\mathfrak{z}(x_m,s)\} \tag{A.1}$$

and

$$\Delta_2 = \int_0^\infty \frac{dt}{t} \exp\left(-\frac{\lambda x^2}{2}t\right)\int dz\, Q_{0,z}^{t,z}\{\mathfrak{z}(x_1,t)\dots\mathfrak{z}(x_m,t)\} \tag{A.2}$$

in the basic formulae (2.33), (2.35) and (4.15), (4.16).

According to the main formulae of the Markovian relativistic pseudoprocesses /1/ we may write

$$Q_{0,x}^{s,y}\{\mathfrak{z}(x_1,s)\dots\mathfrak{z}(x_m,s)\} = \tag{A.3}$$

$$= \int_0^s d\tau_m \int_0^{\tau_m} d\tau_{m-1}\dots\int_0^{\tau_2} d\tau_1\, (0,x;\tau_1,x_{\pi(1)})\dots(\tau_m,x_{\pi(m)};s,y) \; ,$$

where the transition amplitude is

$$(\tau_k, x_{\pi(k)};\tau_{k+1},x_{\pi(k+1)}) = (2\pi)^{-4}\int d^4p\, \exp\left\{\frac{\lambda}{2}p^2(\tau_{k+1}-\tau_k) - ip\left(x_{\pi(k+1)}-x_{\pi(k)}\right)\right\} . \tag{A.4}$$

The time integrations may be easily performed by changing the variables $\tau_{k+1}-\tau_k = \sigma_k$ and taking into account that $\varkappa^2$ has a small negative imaginary part. As a result, we will get for Δ_1

$$\Delta_1 = -i\left(\frac{2}{\lambda}\right)^n \sum_{\pi \in S_n} \Delta^c(x - x_{\pi(1)})\,\Delta^c(x_{\pi(1)} - x_{\pi(2)}) \cdots \Delta^c(x_{\pi(n)} - y)\ . \tag{A.5}$$

In the calculations of Δ_2 one should take into account that $S_n = \mathrm{cyd}\,(1, \ldots, n) \times S_{n-1}$ and that it is easier to find first the derivative of Δ_2 with respect to $\varkappa^2$ and then to solve elementary Cauchy problem. We find in this way the result

$$\Delta_2 = \left(\frac{2}{\lambda}\right)^n \sum_{\pi \in S_{n-1}} \Delta^c(x_n - x_{\pi(1)})\,\Delta^c(x_{\pi(1)} - x_{\pi(2)}) \cdots \Delta^c(x_{\pi(n-1)} - x_n)\ . \tag{A.6}$$

It is clear that the all troubles of the Quantum Field Theory will appear when some of the points x_k, x_j coincide and it is just the case when interaction Lagrangian is a local one.

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