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# THE ISOSPIN- AND ISOSPIN- SPIN-POLARIZATIONS OF INTERACTING HADRONS

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Изоспиновая и изоспин-спиновая поляризация взаимодействующих адронов

В данной работе мы вводим две новых динамических переменных: изоспиновую поляризацию и изоспин-спиновую поляризацию адрона в конечном состоянии данной реакции. В качестве приложения изучается изоспиновая поляризация нуклона в рассеянии пиона на нуклоне. Обсуждаются численные результаты, полученные для поляризации изоспина нуклона из экспериментальных данных по $\pi^- P$ упругому рассеянию и реакциям обмена заряда.

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The Isospin- and Isospin-Spin-Polarizations of Interacting Adrons

In this paper we introduce two new dynamical variables: the isospin-polarization and isospin-spin polarization of a hadron in the final state of a given reaction. As an application the nucleon isospin-polarization in pion-nucleon scattering is studied. The numerical results for nucleon iso-polarization, determined from $\pi^- P$-elastic and charge exchange reactions, are discussed.

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## 1. Introduction

Recently [1] Van Hove have suggested to investigate the correlation between electric charge and some kinematical variables of secondary particles in the multiple production processes at high energies. Starting with this idea some authors [2] have presented charge distributions for various inclusive or exclusive reactions at different energies.

More recently, A.Mihul et al. [3] have introduced the "conditioned average" for the additive quantum numbers such as electric, barion and strange quantum numbers in order to study the leading properties of the incident particles in inclusive reactions.

In this paper we introduce the isospin and isospin-spin polarizations (Sect. 2) of a hadron in a given reaction and in particular we give the explicit expressions for the nucleon isospin and isospin-polarizations in $\pi N \to \pi N$-scattering (Sect. 3). The numerical results on nucleon isospin-polarization obtained from the experimental data of $\pi^- P \to \pi^- P$, $\pi^- P \to \pi^0 n$ reactions are discussed in Sect. 4.

## 2. The Isospin and Spin-Polarizations

In this section we define the isospin-polarization of a hadron in a given reaction as the expectation value of its isospin operator from the direct product of isospin spaces of the final hadrons.

For this we start with a general reaction

$$a + b \to 1 + 2 + \dots + n \qquad (1)$$

where $a, b, 1, \dots, n$ are isospin multiplets with the isospins $T_a$, $T_b$, $T_1, \dots, T_n$. We describe the initial state of the reaction (1) by the isospin density matrix

$$\rho_{in} = \rho_{in}^{(T)} \times \rho_{in}^{(J)} \qquad (2)$$

in the combined isospin-spin space of the initial isomultiplets a and b. $\rho_{in}^{(T)}$ is the isospin density matrix in the direct product of isospin space of the initial isomultiplets while $\rho_{in}^{(J)}$ is the spin density matrix describing their polarization states.

The isospin and spin density matrices may be expanded as

$$\rho_{in}^{(T)} = \frac{1}{(2T_a+1)(2T_b+1)} \sum_{\mu} \langle \zeta_\mu \rangle_{in} \zeta_\mu$$

$$\rho_{in}^{(J)} = \frac{1}{(2J_a+1)(2J_b+1)} \sum_{\lambda} \langle \sigma_\lambda \rangle_{in} \sigma_\lambda , \qquad (3)$$

where $\{\zeta_\mu\}$ and $\{\sigma_\lambda\}$ are the complete sets of basic isospin and spin operators, respectively, satisfying the relations:

$$\mathrm{Tr}(\zeta_\mu \zeta_\nu) = (2T_a+1)(2T_b+1)\delta_{\mu\nu}$$

$$\mathrm{Tr}(\sigma_\lambda \sigma_\eta) = (2J_a+1)(2J_b+1)\delta_{\lambda\eta} \qquad (3a)$$

and $\langle \zeta_\mu \rangle_{in}$, $\langle \sigma_\lambda \rangle_{in}$ defined by

$$\langle \zeta_\mu \rangle_{in} = \mathrm{Tr}(\rho_{in}^{(T)} \zeta_\mu) \qquad (3b)$$

$$\langle \sigma_\lambda \rangle_{in} = \mathrm{Tr}(\rho_{in}^{(J)} \sigma_\lambda) \qquad (3c)$$

are the expectation values of the isospin and spin operators in the initial states of reaction (1).

Therefore we have



$$\rho_{in} = \frac{1}{(2T_a+1)(2T_b+1)(2J_a+1)(2J_b+1)} \sum \langle \zeta_\mu \rangle_{in} \langle \sigma_\lambda \rangle_{in} \zeta_\mu \times \sigma_\lambda \qquad (2a)$$

The final state of the reaction (1) may be completely characterized by a density matrix $\rho_{out}$ in the combined isospin-spin spaces of the final hadrons. The transition amplitude may be written as a matrix T whose rows and columns are characterized by the isospin and spin quantum numbers of the initial and final particles. Now, if $\rho_{in}$ is known as well as the transition matrix T then $\rho_{out}$ is defined as:

$$\rho_{out} = T \rho_{in} T^+ \qquad (4)$$

$$\rho_{out} = \frac{1}{(2T_a+1)(2T_b+1)(2J_a+1)(2J_b+1)} \sum_{\mu,\lambda} \langle \zeta_\mu \rangle_{in} \langle \sigma_\lambda \rangle_{in} \times T(\zeta_\mu \times \sigma_\lambda) T^+ . \qquad (4a)$$

So that the expectation value of any isospin-spin operator $\Lambda^{\alpha\beta}$ from the combined isospin-spin spaces of the final hadrons is

$$\langle \Lambda^{\alpha\beta} \rangle_{out} = \frac{\sum_{\mu,\lambda} \langle \zeta_\mu \rangle_{in} \langle \sigma_\lambda \rangle_{in} \mathrm{Tr}\{T(\zeta_\mu \times \sigma_\lambda) T^+ \Lambda^{\alpha\beta}\}}{\sum_{\mu,\lambda} \langle \zeta_\mu \rangle_{in} \langle \sigma_\lambda \rangle_{in} \mathrm{Tr}\{T(\zeta_\mu \times \sigma_\lambda) T^+\}} . \qquad (5)$$

The relation (5) expresses the expectation value of any isospin-spin operator $\Lambda^{\alpha\beta}$ in the final state of the reaction (1) by the expectation values of the complete sets $\{\zeta_\mu\}$, $\{\sigma_\lambda\}$ of base isospin-spin operators in the initial state of this reaction. The expectation values $\langle \Lambda^{\alpha\beta} \rangle_{out}$ and $\mathrm{Tr}\rho_{out}$

$$\mathrm{Tr}\rho_{out} = \frac{1}{(2T_a+1)(2T_b+1)(2J_a+1)(2J_b+1)} \times \sum_{\mu,\lambda} \langle \zeta_\mu \rangle_{in} \langle \sigma_\lambda \rangle_{in} \mathrm{Tr}\{T(\zeta_\mu \times \sigma_\lambda) T^+\} \qquad (6)$$

give a complete description of all physical observables for the general reaction (1) in terms of the transition matrix. The relations (5) and (6) hold for hadrons with arbitrary isospins and spins and for the beams with arbitrary isospin-spin polarizations.

### 3. Isospin Polarization of the Nucleon in $\pi N$ -Scattering

In this section we give the concrete expression for the constructions of the nucleon isopolarization from the experimental data in the reaction:

$$\pi N \to \pi N. \tag{10}$$

The initial states of this reaction can be described by the isospin-spin density matrix $\rho_{in} = \rho_{in}^{(T)} \times \rho_{in}^{(J)}$. For the isospin density matrix $\rho_{in}^{(T)}$ we choose the matrix

$$\rho_{in}^{(T)} = \frac{1}{6} \sum_i \alpha_i I_i , \tag{11}$$

where $I_i$ are 6 x 6 isospin matrices defined by $(I_i)_{kj} = \delta_{kj}\delta_{ij}$ and $\alpha_i$ are real numbers satisfying the condition $\Sigma \alpha_i = 6$. The initial and final states of reaction (10) are indexed as follows: $1 \equiv \pi^+ p$, $2 \equiv \pi^+ n$, $3 \equiv \pi^\circ p$, $4 \equiv \pi^\circ n$, $5 \equiv \pi^- p$, $6 \equiv \pi^- n$. The reaction (10) is described by the transition matrix T with their elements of the form:

$$T_{fi} = f_{fi} + \vec{\sigma}\vec{n}\, g_{fi} , \tag{12}$$

where $f_{fi}$, $g_{fi}$ are the non-spin-flip and spin-flip scattering amplitudes, $\vec{\sigma}$ are the Pauli's matrices and $\vec{n} = \vec{k}_i \times \vec{k}_f / |\vec{k}_i \times \vec{k}_f|$.

For the isospin-spin density matrix $\rho_{out}$ when

$$\rho_{in} = \frac{1}{6} \sum_1^6 \alpha_i I_i \times \rho_{in}^{(J)}$$



we have the following reduction to the spin density matrices

$$\rho_{11} = \frac{1}{6}\{\alpha_1 T_{11}\rho_{in}^{(J)} T_{11}^{+}\},$$

$$\rho_{22} = \frac{1}{6}\{\alpha_2 T_{22}\rho_{in}^{(J)} T_{22}^{+} + \alpha_3 T_{23}\rho_{in}^{(J)} T_{23}^{+}\},$$

$$\rho_{23} = \frac{1}{6}\{\alpha_2 T_{22}\rho_{in}^{(J)} T_{32}^{+} + \alpha_3 T_{23}\rho_{in}^{(J)} T_{33}^{+}\},$$

$$\rho_{32} = \frac{1}{6}\{\alpha_2 T_{32}\rho_{in}^{(J)} T_{22}^{+} + \alpha_3 T_{33}\rho_{in}^{(J)} T_{23}^{+}\}, \tag{13}$$

$$\rho_{33} = \frac{1}{6}\{\alpha_2 T_{32}\rho_{in}^{(J)} T_{32}^{+} + \alpha_3 T_{33}\rho_{in}^{(J)} T_{33}^{+}\},$$

$$\rho_{44} = \frac{1}{6}\{\alpha_4 T_{44}\rho_{in}^{(J)} T_{44}^{+} + \alpha_5 T_{45}\rho_{in}^{(J)} T_{45}^{+}\},$$

$$\rho_{45} = \frac{1}{6}\{\alpha_4 T_{44}\rho_{in}^{(J)} T_{54}^{+} + \alpha_5 T_{45}\rho_{in}^{(J)} T_{55}^{+}\},$$

$$\rho_{54} = \frac{1}{6}\{\alpha_4 T_{54}\rho_{in}^{(J)} T_{44}^{+} + \alpha_5 T_{55}\rho_{in}^{(J)} T_{45}^{+}\},$$

and

$$\rho_{55} = \frac{1}{6}\{\alpha_4 T_{54}\rho_{in}^{(J)} T_{54}^{+} + \alpha_5 T_{55}\rho_{in}^{(J)} T_{55}^{+}\},$$

$$\rho_{66} = \frac{1}{6}\{\alpha_6 T_{66}\rho_{in}^{(J)} T_{66}^{+}\}. \tag{14}$$

$\rho_{1i} = 0$ for $i \neq 1$; $\rho_{2i} = \rho_{3i} = 0$ for $i \neq 2,3$

$\rho_{4i} = \rho_{5i} = 0$ for $i \neq 4,5$; $\rho_{6i} = 0$ for $i \neq 6$.

We are interested in obtaining the expectation values of the nucleon isospin-spin operators

$$\mathcal{T}_k \equiv 1\times\zeta_k\times 1\,,\quad \vec{\Lambda}_k \equiv 1\times\zeta_k\times\vec{\sigma}\,,\quad \Lambda_0 \equiv 1\times 1\times\vec{\sigma}\,,\ k=1,2,3,$$

where $\zeta_k$ and $\vec{\sigma}$ are the nucleon isospin and spin matrices.

From Eqs. (5) and (13) we obtain:

$$<\mathcal{T}_k>_{out} = \frac{\sum_{i=1}^{6}(-1)^{i+1}\,\mathrm{Tr}\,\rho_{ii}\,\delta_{k3}}{\sum_{i=1}^{6}\mathrm{Tr}\,\rho_{ii}} \qquad (15)$$

$$<\vec{\Lambda}_k>_{out} = \frac{\sum_{1}^{6}(-1)^{i+1}\,\mathrm{Tr}\,(\rho_{ii}\,\vec{\sigma}\,)\,\delta_{k3}}{\sum_{1}^{6}\mathrm{Tr}\,\rho_{ii}}\,, \qquad (16a)$$

$$<\vec{\Lambda}_0>_{out} = \frac{\sum_{1}^{6}\mathrm{Tr}(\rho_{ii}\,\vec{\sigma}\,)}{\sum_{1}^{6}\mathrm{Tr}\,\rho_{ii}}\,. \qquad (16b)$$

where $\rho_{ii}$, $i=1,\dots,6$ are the spin density matrices (13). In the particular case when the incident nucleons are unpolarized, taking into account the relation (12), we obtain

$$\mathrm{Tr}\{\,T_{fi}\,\rho_{in}^{(J)}\,T_{fi}^{+}\,\} = \frac{d\sigma_{fi}}{d\Omega} \qquad (17)$$

and

$$\mathrm{Tr}\{T_{fi}\,\rho_{in}^{(J)}\,T^{+}\vec{\sigma}\} = \vec{P}_{fi}\,\frac{d\sigma_{fi}}{d\Omega} = 2\,\mathrm{Re}\{\,f_{fi}^{*}\,g_{fi}\,\}\,\vec{n}\,. \qquad (18)$$

Therefore, from (15), (16a,b) and (17) and (18) we can define, for the reaction (10), the following physically measurable quantities:

(A) the isospin-unpolarized, spin-unpolarized differential cross section:



$$\frac{d\sigma}{d\Omega} \equiv \mathrm{Tr}\,\rho_{out} = \frac{1}{6}\sum_{i=1}^{6}\alpha_i\,\frac{d\sigma_i}{d\Omega}\,; \qquad (19a)$$

(B) the isospin-polarization: ($P^{(T)} \equiv <\mathcal{T}_k>_{out}$ )

$$P^{(T)}\frac{d\sigma}{d\Omega} = \frac{1}{6}\sum_{i=1}^{6}\alpha_i\,P_i^{(T)}\,\frac{d\sigma_i}{d\Omega}\,; \qquad (19b)$$

(C) the spin-polarization: ( $\vec{P}^{(J)} \equiv <\vec{\Lambda}_0>_{out}$)

$$\vec{P}^{(J)}\,\frac{d\sigma}{d\Omega} = \frac{1}{6}\sum_{i=1}^{6}\alpha_i\,\vec{P}_i\,\frac{d\sigma_i}{d\Omega}\,; \qquad (19c)$$

(D) the isospin-spin-polarization: ($\vec{P}^{(T,J)} \equiv <\vec{\Lambda}_k>_{out}$ )

$$\vec{P}^{(T,J)}\frac{d\sigma}{d\Omega} = \frac{1}{6}\sum_{i=1}^{6}\alpha_i\,\vec{P}_i^{(T,J)}\,\frac{d\sigma_i}{d\Omega} \qquad (19d)$$

where $\frac{d\sigma_i}{d\Omega}$, $P_i^{(T)}$, $\vec{P}_i^{(J)}$, $\vec{P}_i^{(T,J)}$ are defined by the relations (20):

$$\frac{d\sigma_i}{d\Omega} = \frac{d\sigma_{ii}}{d\Omega};\quad P_i^{(T)}\,\frac{d\sigma_i}{d\Omega} = (-1)^{i+1}\,\frac{d\sigma_{ii}}{d\Omega}\,;$$

$$P_i^{(J)}\,\frac{d\sigma_i}{d\Omega} = P_{ii}\,\frac{d\sigma_i}{d\Omega};\quad P_i^{(T,J)}\frac{d\sigma_i}{d\Omega} = (-1)^{i+1}\,P_{ii}\,\frac{d\sigma_{ii}}{d\Omega}\,. \qquad i=1,6 \qquad (20a)$$

$$\frac{d\sigma_i}{d\Omega} = \frac{d\sigma_{ii}}{d\Omega} + \frac{d\sigma_{i+1,i}}{d\Omega}\,;\quad P_i^{(T)}\frac{d\sigma_i}{d\Omega} = \frac{d\sigma_{i+1,i}}{d\Omega} - \frac{d\sigma_{ii}}{d\Omega}\,;$$

$$P_i^{(J)}\,\frac{d\sigma_i}{d\Omega} = P_{ii}\,\frac{d\sigma_{ii}}{d\Omega} + P_{i+1,i}\,\frac{d\sigma_{i+1,i}}{d\Omega}\,; \qquad (20b)$$

$$i=2,4$$

$$P_i^{(T,J)}\,\frac{d\sigma_i}{d\Omega} = P_{i+1,i}\,\frac{d\sigma_{i+1,i}}{d\Omega} - P_{ii}\,\frac{d\sigma_{ii}}{d\Omega}\,.$$

$$\frac{d\sigma_i}{d\Omega} = \frac{d\sigma_{i-1,i}}{d\Omega} + \frac{d\sigma_{ii}}{d\Omega}; \quad P_i^{(T)}\frac{d\sigma_i}{d\Omega} = \frac{d\sigma_{ii}}{d\Omega} - \frac{d\sigma_{i-1,i}}{d\Omega};$$

$$i=3,5$$

$$P_i^{(J)}\frac{d\sigma_i}{d\Omega} = P_{i-1,i}\frac{d\sigma_{i-1,i}}{d\Omega} + P_{ii}\frac{d\sigma_{ii}}{d\Omega}; \tag{20c}$$

$$P_i^{(T,J)}\frac{d\sigma_i}{d\Omega} = P_{ii}\frac{d\sigma_{ii}}{d\Omega} - P_{i-1,i}\frac{d\sigma_{i-1,i}}{d\Omega},$$

The new physical observables $\frac{d\sigma_i}{d\Omega}$, $P_i^{(T)}\frac{d\sigma_i}{d\Omega}$, $P_i^{(J)}\frac{d\sigma_i}{d\Omega}$ and $P_i^{(T,J)}\frac{d\sigma_i}{d\Omega}$ defined by the relations (20a,b,c) correspond to the unpolarized (isospin-spin) differential cross-sections, isospin-polarization, spin-polarization and isospin-spin polarization for the reactions:

$$\pi^+p \to \pi^+p \quad (i=1, f=1) \tag{10a}$$

$$\pi^+n \to \pi^+n \quad (i=2, f=2) \tag{10b}$$
$$\searrow \pi^\circ P \quad (i=2, f=3)$$

$$\pi^\circ P \to \pi^\circ P \quad (i=3, f=3) \tag{10c}$$
$$\searrow \pi^+n \quad (i=3, f=2)$$

$$\pi^\circ n \to \pi^\circ n \quad (i=4, f=4) \tag{10d}$$
$$\searrow \pi^-P \quad (i=4, f=5)$$

$$\pi^-P \to \pi^-P \quad (i=5, f=5) \tag{10e}$$
$$\searrow \pi^\circ n \quad (i=5, f=4)$$

$$\pi^-n \to \pi^-n \quad (i=6, f=6) \tag{10f}$$

Therefore the relations (19 a,b,c,d) give a complete description of the reaction (10) in terms of physical observables of the reactions (10 a,b,c,d,e,f) defined by the relations (20,a,b,c) as linear combinations of $\frac{d\sigma_{fi}}{d\Omega}$ and $P_{fi}\frac{d\sigma_{fi}}{d\Omega}$.

Also, it is interesting to obtain from the experimental data the quantities $\bar{P}_i^{(m)}$, $m=T,J,(T,J)$ defined by the relations:

$$\bar{P}_i^{(m)} = \frac{1}{\sigma_i}\int_{(4\pi)} P_i^{(m)}\frac{d\sigma_i}{d\Omega}d\Omega, \quad m=T,J,(T,J), \tag{20'}$$

where

$$\sigma_i = \int_{(4\pi)} \frac{d\sigma_i}{d\Omega}d\Omega.$$

With these relations we can define

$$\bar{P}^{(m)} = \frac{1}{6\sigma}\sum_{i=1}^{6} a_i \bar{P}_i^{(m)}\sigma_i, \quad \sigma = \frac{1}{6}\sum_{i=1}^{6} a_i\sigma_i. \tag{19'}$$

Let us consider the implications of the isospin invariance on the physical observables (20a,b,c). Let $f^{(T)}$ and $g^{(T)}$ be the $\pi N$ -non-spin-flip and spin-flip scattering amplitudes in the isospin 1/2 and 3/2 state. With the usual relations between these scattering amplitudes and scattering amplitudes $f_{fi}$ and $g_{fi}$ we obtain:

$$\frac{d\sigma_i}{d\Omega} = \sum_{T=1/2,3/2} C_i^{(T)}\{|f^{(T)}|^2 + |g^{(T)}|^2\}; \tag{21a}$$

$$\vec{P}_i^{(J)} \frac{d\sigma_i}{d\Omega} = \sum_{T=1/2,3/2} 2\,C_i^{(T)} \mathrm{Re}\,\{[f^{(T)}]^* g^{(T)}\}\vec{n} \; ; \qquad (21b)$$

$$P_i^{(T)} \frac{d\sigma_i}{d\Omega} = \sum_{T=1/2,3/2} D_i^{(T)} \{|f^{(T)}|^2 + |g^{(T)}|^2\} + \qquad (21c)$$

$$+ 2D_i^{(1/2,3/2)} \mathrm{Re}\,\{[f^{(1/2)}]^* f^{(3/2)} + [g^{(1/2)}]^* g^{(3/2)}\};$$

$$\vec{P}_i^{(T,J)} \frac{d\sigma_i}{d\Omega} = \sum_{T=1/2,3/2} 2D_i^{(T)} \mathrm{Re}\,\{[f^{(T)}]^* g^{(T)}\}\vec{n} + \qquad (21d)$$

$$+ 2D_i^{(1/2,3/2)} \mathrm{Re}\,\{[f^{(1/2)}]^* g^{(3/2)} + [f^{(3/2)}]^* g^{(1/2)}\}\vec{n},$$

where the coefficients $C_i^{(T)}$ , $D_i^{(T)}$ and $D_i^{(1/2,3/2)}$ are given for each reaction in Table 1. Now, if we define the coefficients

$$C^{(1/2)}(\alpha) = \frac{1}{6}\sum_1^6 \alpha_i C_i^{(1/2)} = \frac{1}{18}\{2\alpha_2 + \alpha_3 + \alpha_4 + 2\alpha_5\}; \qquad (22a)$$

$$C^{(3/2)}(\alpha) = \frac{1}{6}\sum_1^6 \alpha_i C_i^{(3/2)} = \frac{1}{18}\{3\alpha_1 + \alpha_2 + 2\alpha_3 + 2\alpha_4 + \alpha_5 + 3\alpha_6\}; \qquad (22b)$$

$$D^{(1/2)}(\alpha) = \frac{1}{6}\sum_1^6 \alpha_i D_i^{(1/2)} = \frac{1}{54}\{\alpha_4 + 2\alpha_5 - 2\alpha_2 - \alpha_3\}; \qquad (22c)$$

$$D^{(3/2)}(\alpha) = \frac{1}{6}\sum_1^6 \alpha_i D_i^{(3/2)} = \frac{1}{54}\{9\alpha_1 + \alpha_2 + 2\alpha_3 - 2\alpha_4 - \alpha_5 - 9\alpha_6\} \qquad (22d)$$



Table I

The coefficients $C_i^{(T)}$ , $D_i^{(T)}$ , $D_i^{(1/2,3/2)}$ and their ratio for the reactions (10 a,b,c,d,e,f).

| Reaction | $C_i^{(1/2)}$ | $C_i^{(3/2)}$ | $D_i^{(1/2)}$ | $D_i^{(3/2)}$ | $D_i^{(1/2,3/2)}$ | $D_i^{(1/2)}/C_i^{(1/2)}$ | $D_i^{(3/2)}/C_i^{(3/2)}$ |
|---|---|---|---|---|---|---|---|
| 10a | 0 | +1 | 0 | +1 | 0 | — | +1 |
| 10b | +2/3 | +1/3 | -2/9 | +1/9 | -4/9 | -1/3 | +1/3 |
| 10c | +1/3 | +2/3 | -1/9 | +2/9 | +4/9 | -1/3 | +1/3 |
| 10d | +1/3 | +2/3 | +1/9 | -2/9 | -4/9 | +1/3 | -1/3 |
| 10e | +2/3 | +1/3 | +2/9 | -1/9 | +4/9 | +1/3 | -1/3 |
| 10f | 0 | +1 | 0 | -1 | 0 | — | -1 |

$$D^{(1/2,3/2)}(\alpha)=\frac{1}{6}\sum_1^6 \alpha_i D_i^{(1/2,3/2)}=\frac{1}{27}\{\alpha_3-\alpha_2+\alpha_5-\alpha_4\}, \quad (22e)$$

then the observables $\frac{d\sigma}{d\Omega}$, $P^{(T)}\frac{d\sigma}{d\Omega}$, $P^{(J)}\frac{d\sigma}{d\Omega}$ and $P^{(T,J)}\frac{d\sigma}{d\Omega}$ can be expressed in terms of the scattering amplitudes $f^{(T)}$, $g^{(T)}$ by the same relations (21 a,b, c,d) with the substitution

$$C_i^{(T)} \to C^{(T)}(\alpha)\ ;\ D_i^{(T)} \to D^{(T)}(\alpha);\ D_i^{(1/2,3/2)} \to D^{(1/2,3/2)}(\alpha).$$

Let us consider now the case when all pion-nucleon initial states have the same weights $\alpha_i$. Therefore when $\alpha_1=\alpha_2=\alpha_3=\alpha_4=\alpha_5=\alpha_6$ or in the general case when $\alpha_1=\alpha_6\ ;\alpha_2=\alpha_5\ ;\ \alpha_3=\alpha_4$ from (22c,d,e)

$$D^{(1/2)}(\alpha)=D^{(3/2)}(\alpha)=D^{(1/2,3/2)}(\alpha)=0, \quad (23)$$

so that

$$P^{(T)}\frac{d\sigma}{d\Omega}=P^{(T,J)}\frac{d\sigma}{d\Omega}=0, \quad (24)$$

when the isospin invariance is valid and different from zero when the isospin invariance is violated in the pion-nucleon scattering. In the resonance region, when the $\pi N$-decay mode is dominant, we have

$$P_i^{(T)} \simeq D_i^{(T)}/C_i^{(T)} \quad (25)$$

for all values of $\cos\theta$ if the isospin invariance is valid. The ratios $D_i/C_i$ for each reaction (10 a,b,c,d,e,f) are given in Table I.



## 4. Determination of the Nucleon Isospin-Polarization from Experimental Data

In this section we present some results on the nucleon isospin-polarization obtained from the experimental data of reactions (10e). Therefore, we restrict our analysis to the case when only $\pi^-$ are present in the incident beam ($\alpha_5=6$ and $\alpha_i=0$ for $i=1,2,3,4,6$). Then, from (19b) and (20c), we have the following definition of the nucleon isospin-polarization parameter:

$$P^{(T)}=(\sigma_- - \sigma_{CE})/\sigma\ ;\ \sigma=\sigma_- + \sigma_{CE} \quad (26)$$

or

$$\sigma_- = \frac{1}{2}(1+P^{(T)})\sigma\ ;\ \sigma_{CE}=\frac{1}{2}(1-P^{(T)})\sigma, \quad (26a)$$

where $\sigma_- \equiv \frac{d\sigma_{55}}{d\Omega}$ ; $\sigma_{CE} \equiv \frac{d\sigma_{45}}{d\Omega}$ are the differential cross sections for $\pi^- P$ -elastic and charge exchange reactions, respectively. If we take in (26) the corresponding integrated cross sections we obtain the definition (20') for $\bar{P}^{(T)}$. The energy dependence of the nucleon isospin-polarization parameter $\bar{P}^{(T)}$ determined from the experimental data /4–7/ is presented in Fig. 1. From this figure we see that $P^{(T)} \approx -0.33$ in the interval of 0.20-0.4 GeV/c which is obviously an effect of $\Delta(1238)$ - resonance (see relation (25) and Table I). Also, we see that the point $P^{(T)} \simeq +0.33$ is passed at $p_{LAB} \approx 0.7$GeV/c and this is a consequence of $N'(1470)$- and $N'(1520)$ -resonances which occur in this momentum region. At $p_{LAB} > 3$ GeV/c $\bar{P}^{(T)} \to +1$ as $\bar{P}^{(T)} \sim 1-0.2 p_{LAB}^{-1}$. In Fig. 2 we give the $t$ -distributions of the nucleon isospin-polarization parameter $P^{(T)}(t)$ determined from the experimental data /4,8/ at incident beam momenta between 3.0 - 18.3 GeV/c, in the $t$ -range $0.1 \le |t| \le 1$ $(\mathrm{GeV}/c^2)^2$. A plateau is observed in the region of small $|t|$ in all presented distributions. The average values of $P^{(T)}(t)$ over the plateau interval $\Delta|t|$ are given in Table II. We see that these are increasing

Table II

The values of the average nucleon iso-polarization $\bar{P}^{(T)}(\Delta|t|)$ at different beam momenta between 3.0 - 18.3 GeV/c.

| $P_{LAB}$ (GeV/c) | $\bar{P}^{(T)}(\Delta\|t\|)$ | $\Delta\|t\|$ $(GeV/c^2)^2$ |
|---|---|---|
| 3.0 | 0.934 ± 0.030 | 0.117 - 0.460 |
| 3.5 | 0.944 ± 0.029 | 0.096 - 0.557 |
| 4.0 | 0.957 ± 0.030 | 0.121 - 0.643 |
| 5.0 | 0.961 ± 0.018 | 0.099 - 0.649 |
| 6.0 | 0.964 ± 0.060 | 0.120 - 0.684 |
| 10.0 | 0.979 ± 0.028 | 0.110 - 0.620 |
| 13.0 | 0.984 ± 0.008 | 0.122 - 0.835 |
| 18.3 | 0.987 ± 0.008 | 0.119 - 0.825 |

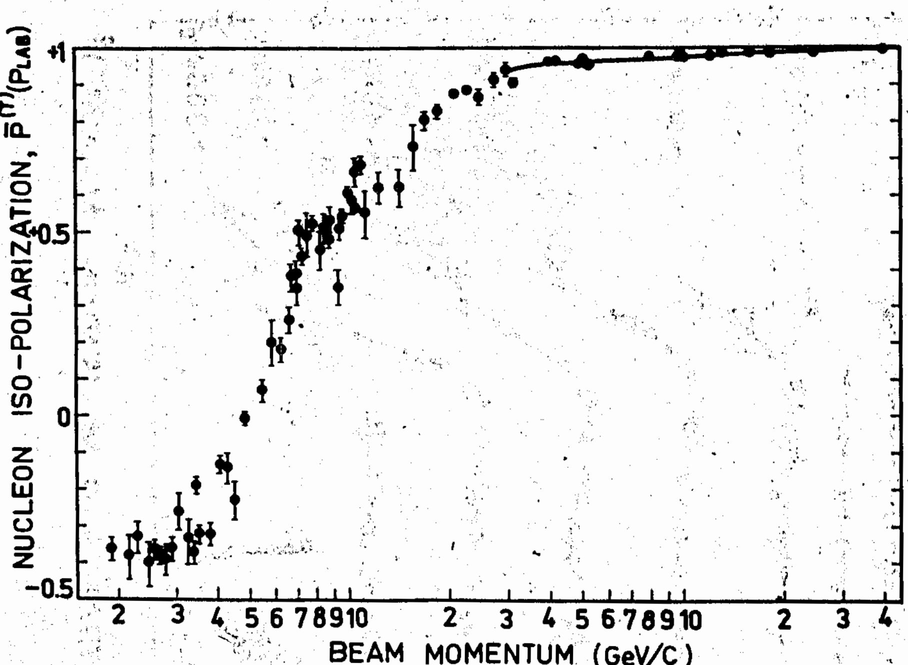


Fig. 1. The energy dependence of the nucleon iso-polarization parameter $\bar{p}^{(T)}$ determined from $\pi^-P$-elastic and charge exchange data /4,5,6,7/. The solid curve is determined from relation $\bar{P}^{(T)} = 1-0.2p^{-1}_{LAB}$.

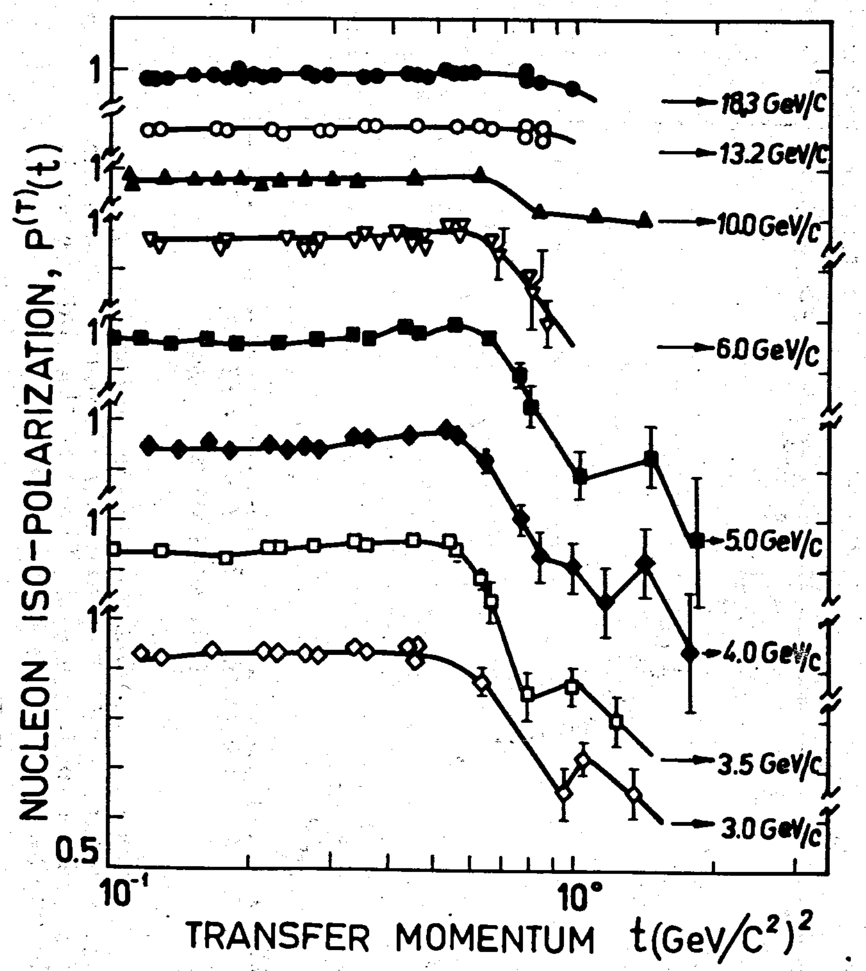


Fig. 2. The $t$ -distribution of the nucleon iso-polarization parameter $P^{(T)}(t)$ obtained from the elastic and charge exchange data /4,8/. The solid curves hand-drawn are curves to guide the eyer.



with $p_{LAB}$, from 0.93 ± 0.03 to 0.99 ± 0.01 for $3 \le P_{LAB} \le 18.3$ GeV/c. A behaviour of the form $\bar{P}^{(T)}(\Delta|t|) \simeq 1 - 0(p_{LAB}^{-1})$ and an extension at high $|t|$ are expected at high energies. The value of $P^{(T)}(0)$, determined from the data /4,9,12/ and presented in Fig. 3, has a shape similar to $\bar{P}^{(T)}$ both at low and high energies. A comparison with $P^{(T)}(0)$ obtained from the dispersion relation data /13/ is given in Fig. 3. Therefore, $<\bar{T}_{3N}> \to +\frac{1}{2}$ at high energies and $<T_{3N}> \to +\frac{1}{2}$ in the large $\Delta|t|$ -interval at high energies. In other words, the incident nucleons (as well as incident pions), in the scattering at high energies, tend to preserve their third isospin components.

The $u$ -distributions of the nucleon isospin-polarization parameter at 5.9 and 10 GeV/c presented in Fig. 4 are determined from the backward differential cross sections for the reactions of $\pi^- P$ -elastic /14/ and $\pi^- P$ -charge exchange /15/. We see that, in contrast to $P^{(T)}(t)$ -distributions, the $P^{(T)}(u)$ -distributions in the backward region $u_{min} \le |u| \le 1\ (GeV/c^2)^2$ have a wave-like character. These distributions, due to their wave-like character, do not contribute essentially to the average values $\bar{P}^{(T)}(p_{LAB})$.

The $P^{(T)}(u)$-values for the extreme backward points ($\theta = 180°$) in the low and intermediate regions are strongly correlated with the distribution of the resonance poles and zeros of the backward amplitudes. This statement is illustrated in Fig. 5 where we present the nucleon isospin-polarization parameter $P^{(T)}(180°)$ as a function of $P_{LAB}$. We see that the actual values of this parameter are near -0.33 for most of $\Delta$ -resonance positions and near + 0.33 for some $N'$-resonances. The point $P^{(T)}(180°) = +1$ is reached at 0.530 GeV/c where $\sigma_{CE}$ is practically zero while the $P^{(T)}(180°)$ is near -1 at

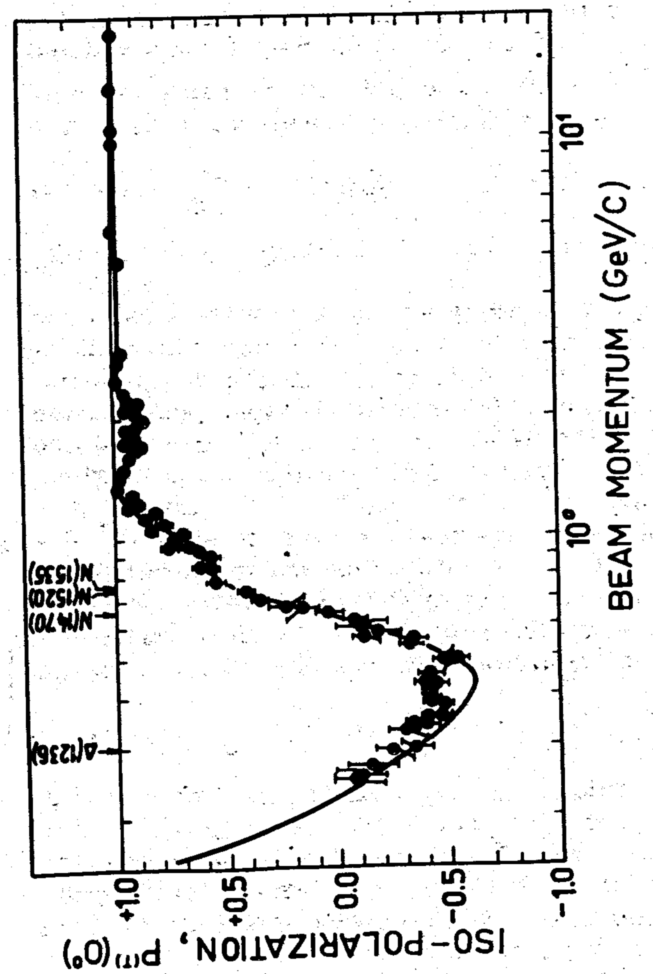


Fig. 3. The $p_{LAB}$-dependence of the forward isospin-polarization parameter determined from the forward $\pi^-P$-elastic and charge exchange data /4,9–12/ and compared with the dispersion relation predictions of Höhler and Strauss/13/ (solid curve).



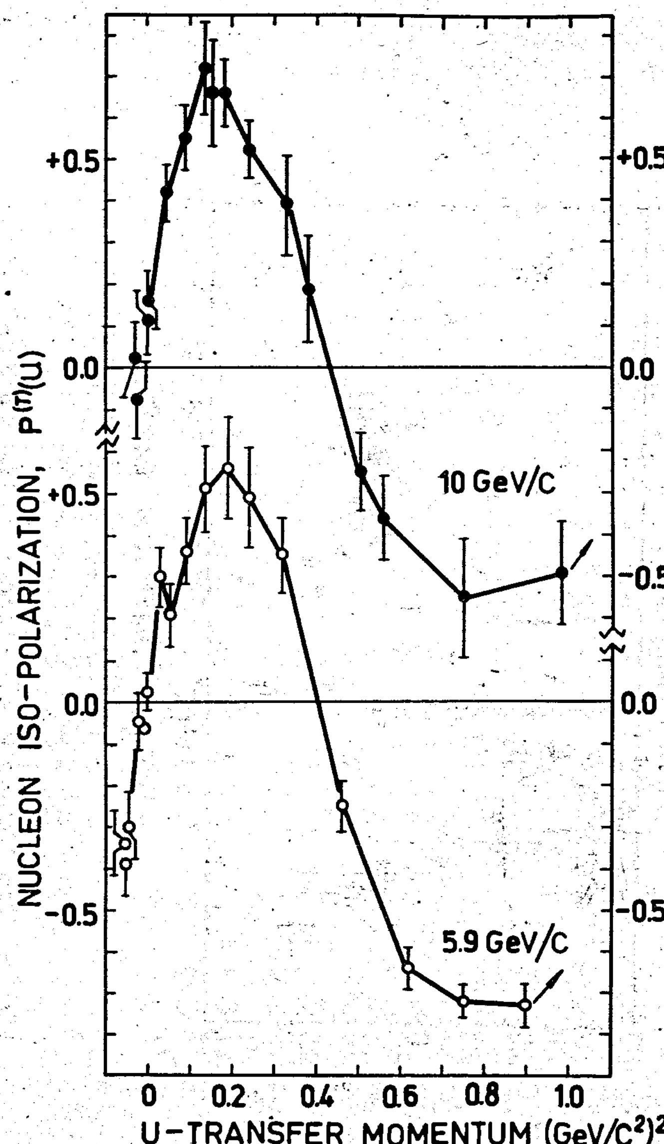


Fig. 4. The u-dependence of the nucleon isospin-polarization parameter $P^{(T)}(u)$ obtained from the data/14,15/ at $p_{LAB}$ = 5.9 and 10 GeV/c. The solid curves are hand-drawn curves to guide the eyer.

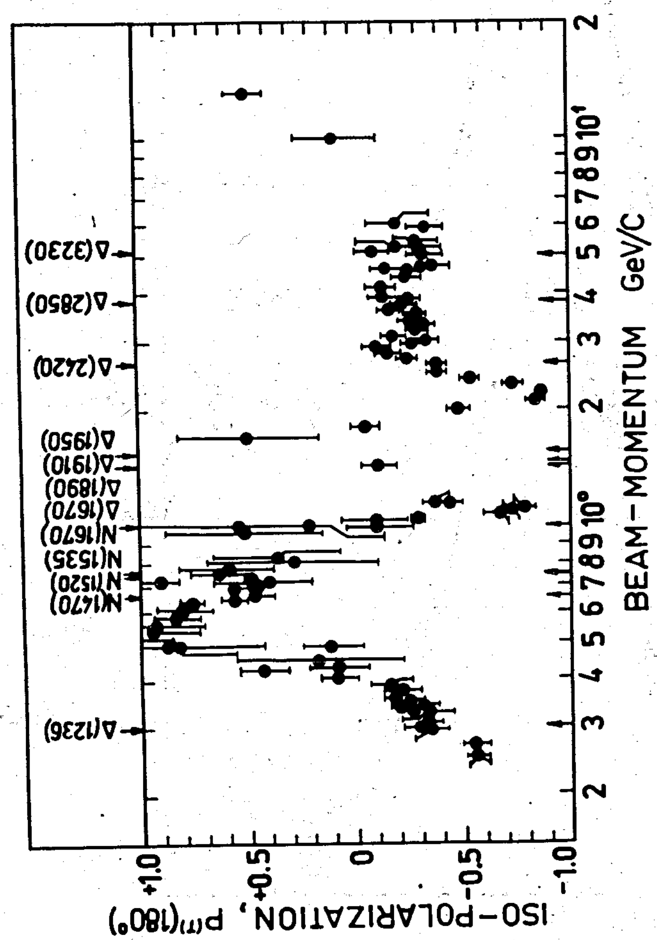


Fig. 5. The $P^{LAB}_{(T)}(180°)$ -dependence of the nucleon isospin parameter $P_{(T)}(180°)$ in the backward direction determined from the backward $\pi^-P$ -elastic and charge exchange cross-sections /16/.



1.1 GeV/c and 2.15 and these are probable consequences of the zeros in the $\pi^-P$-elastic scattering amplitudes near the physical region.

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