

ОБЪЕДИНЕННЫЙ  
ИНСТИТУТ  
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ДУБНА



*24/10-74*

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E2 - 7733

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*2094 / 2-74*

THE NUCLEON ISOSPIN-POLARIZATION  
AND ISOSPIN BOUNDS  
IN PION-NUCLEON SCATTERING

**1974**

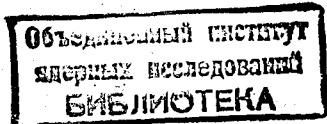
ЛАБОРАТОРИЯ  
ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

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THE NUCLEON ISOSPIN-POLARIZATION  
AND ISOSPIN BOUNDS  
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*Submitted to Annals of Physics*



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<sup>1</sup> On leave of absence from Institute for Atomic Physics Bucharest, Romania.

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E2 - 7733

Поляризация изоспина нуклона и изоспиновые ограничения  
в пион-нуклонном рассеянии

Параметры поляризации изоспина нуклона и его изоспиновые ограничения определяются из экспериментальных данных по  $\pi^-p$ -упругому рассеянию и реакциям зарядового обмена. Насыщение изоспиновых ограничений объясняется с помощью резонансных полюсов и нулей в амплитудах реакций с обменом изоспином в  $s$ -,  $t$ - и  $u$ -каналах.

Препринт Объединенного института ядерных исследований.  
Дубна, 1974

Ion D.B., Mihul A.C.

E2 - 7733

The Nucleon Isospin-Polarization and Isospin  
Bounds in Pion-Nucleon Scattering

The nucleon isospin-polarization parameter and its isospin bounds are determined from the experimental data of  $\pi^\pm p$ -elastic and charge exchange reactions. The saturation of the isospin bounds is explained in terms of the resonance poles and zeros of the isospin exchange amplitudes in the  $s$ -,  $t$ -,  $u$ -channels.

Preprint. Joint Institute for Nuclear Research.  
Dubna, 1974

## 1. Introduction

In a recent paper<sup>/1/</sup> we have introduced the isospin and isospin-spin polarizations of a hadron in the final state of a given reaction. As an application the nucleon isospin-polarization parameter in the pion-nucleon scattering was determined from the experimental data of  $\pi^-p$ -elastic and charge exchange reactions at high energies.

The purposes of this paper are (i) to determine the nucleon isospin polarization parameter at all  $\cos\theta$  in the low and intermediate energy regions and (ii) to investigate the implications following from the isospin invariance in pion-nucleon scattering. For this in sect. 2 we discuss the isospin bounds using the  $s, t, u$ -channel decomposition of  $\pi N$ -scattering amplitudes. The nucleon isospin polarization parameter  $P^{(T)}$  and its isospin bounds  $P_{\min}^{(T)}$ ,  $P_{\max}^{(T)}$  are presented in sect. 3. The saturation of the isospin bounds is discussed, also in sect. 3, in terms of the resonance poles and zero positions of the isospin exchange amplitudes.

## 2. The Isospin Bounds in $\pi N$ -Scattering

In this section we discuss the isospin bounds in  $\pi^\pm p$ -elastic and charge exchange reactions by using the  $s, t, u$ -channel isospin decomposition of the  $\pi N$ -amplitudes.

For this problem it is more convenient to introduce, as in ref. <sup>/2/</sup>, a vector  $\vec{G}$  in the helicity space

$$\vec{G} = G_{++} \vec{\ell} + G_{+-} \vec{m} \quad (1)$$

where  $G_{++}, G_{+-}$  are the  $s$ -channel helicity amplitudes and  $\vec{\ell}, \vec{m}$  are orthogonal unit vectors.

In terms of the complex vectors  $\vec{G}$  the differential cross sections and polarized are expressed as

$$\sigma \equiv \frac{d\sigma}{dx} = \vec{G}^* \vec{G} \quad (2)$$

$$\vec{P} \sigma \equiv i(\vec{G}^* \times \vec{G}) \vec{n}, \quad \vec{n} = \vec{\ell} \times \vec{m}. \quad (3)$$

(x is one of the kinematical variables:  $\cos\theta, t, u$ ). Let us consider  $\vec{G}$  defined for each of the reactions:

$$\pi^\pm P \rightarrow \pi^\pm P : \vec{G}_\pm \quad (4)$$

$$\pi^- P \rightarrow \pi^0 n : \vec{G}_{CE}$$

According to  $s, t, u$ -channel isospin decomposition of the  $\vec{G}_{\pm, CE}$ -amplitudes we obtain:

a)  $s$ -channel relations

$$\begin{aligned} \sigma_+ &= \sigma_{3s}, \quad I_s = 3/2 \\ \sigma_- &= \frac{1}{9} \sigma_+ + \frac{4}{9} \sigma_{1s} + \frac{4}{9} (\sigma_+ \sigma_{1s})^{1/2} \cos \delta_s; \\ \sigma_{CE} &= \frac{2}{9} \sigma_+ + \frac{2}{9} \sigma_{1s} - \frac{4}{9} (\sigma_+ \sigma_{1s})^{1/2} \cos \delta_s, \end{aligned} \quad (5)$$

where we have defined

$$\begin{aligned} \sigma_{1s} &\equiv |\vec{N}_s|^2 = \frac{1}{2} [3\sigma_- + 3\sigma_{CE} - \sigma_+], \quad I_s = \frac{1}{2}, \\ \cos \delta_s &= \frac{1}{2} [\vec{N}_s^* \cdot \vec{\Delta}_s + \vec{N}_s \cdot \vec{\Delta}_s^*] / (|\vec{N}_s| |\vec{\Delta}_s|) \\ &= \frac{3\sigma_- + \sigma_+ - 6\sigma_{CE}}{[8\sigma_+ (3\sigma_- + 3\sigma_{CE} - \sigma_+)]^{1/2}}. \end{aligned} \quad (6)$$

b)  $t$ -channel relations

$$\sigma_+ = \sigma_0 + \sigma_2 - 2(\sigma_0 \sigma_2)^{1/2} \cos \delta_t,$$

$$\sigma_- = \sigma_0 + \sigma_2 + 2(\sigma_0 \sigma_2)^{1/2} \cos \delta_t,$$

$$\sigma_{CE} = 2\sigma_2,$$

where

$$\sigma_0 \equiv |\vec{F}_0|^2 = \frac{1}{2} [\sigma_+ + \sigma_- - \sigma_{CE}], \quad I_t = 0,$$

$$\sigma_2 \equiv |\vec{F}_1|^2 = \frac{1}{2} \sigma_{CE}, \quad I_t = 1,$$

$$\begin{aligned} \cos \delta_t &\equiv \frac{1}{2} [\vec{F}_0^* \cdot \vec{F}_1 + \vec{F}_0 \cdot \vec{F}_1^*] / (|\vec{F}_0| |\vec{F}_1|) \\ &= \frac{1}{2} \frac{\sigma_- - \sigma_+}{[\sigma_{CE} (\sigma_+ + \sigma_- - \sigma_{CE})]^{1/2}} \end{aligned}$$

c)  $u$ -channel relations:

$$\sigma_+ = \frac{1}{9} \sigma_- + \frac{4}{9} \sigma_{1u} + \frac{4}{9} (\sigma_- \sigma_{1u})^{1/2} \cos \delta_u,$$

$$\sigma_- = \sigma_{3u}, \quad I_u = 3/2,$$

$$\sigma_{CE} = \frac{2}{9} \sigma_- + \frac{2}{9} \sigma_{1u} - \frac{4}{9} (\sigma_- \sigma_{1u})^{1/2} \cos \delta_u,$$

where

$$\sigma_{1u} \equiv |\vec{N}_u|^2 = \frac{1}{2} [3\sigma_+ + 3\sigma_{CE} - \sigma_-], \quad I_u = \frac{1}{2},$$

$$\begin{aligned} \cos \delta_u &\equiv \frac{1}{2} [\vec{N}_u^* \cdot \vec{\Delta}_u + \vec{N}_u \cdot \vec{\Delta}_u^*] / (|\vec{N}_u| |\vec{\Delta}_u|) \\ &= \frac{3\sigma_+ + \sigma_- - 6\sigma_{CE}}{[8\sigma_- (3\sigma_+ + 3\sigma_{CE} - \sigma_-)]^{1/2}} \end{aligned}$$

We have denoted by  $\vec{\Delta}_s, \vec{N}_s, \vec{F}_0, \vec{F}_1, \vec{\Delta}_u,$   
 $N_u$  the  $I_s=3/2, 1/2; I_t=0, 1; I_u=3/2, 1/2$  - isospin exchange helicity amplitudes respectively written in the vectorial form (1).

Also, on the basis of the relation:

$$\vec{G}_+ = \vec{G}_- + \sqrt{2} \vec{G}_{CE} \quad (11)$$

we can define, in a similar way:

$$\begin{aligned} (4\sigma_+ \sigma_-)^{1/2} \cos \delta_{+-} &= \sigma_+ + \sigma_- - 2\sigma_{CE} \\ (8\sigma_+ \sigma_{CE})^{1/2} \cos \delta_{+CE} &= \sigma_+ - \sigma_- + 2\sigma_{CE} \end{aligned} \quad (12)$$

$$(8\sigma_- \sigma_{CE}) \cos \delta_{-CE} = \sigma_+ - \sigma_- - 2\sigma_{CE}$$

Now, on the basis of relations (6), (8), (10), (11) and (12) the following isospin bounds can be introduced:

$$\sigma_+ \leq 3(\sigma_+ + \sigma_{CE}); \quad (6a)$$

$$(3\sigma_- + \sigma_+ - 6\sigma_{CE})^2 \leq 8\sigma_+(3\sigma_- + 3\sigma_{CE} - \sigma_+); \quad (6b)$$

$$\sigma_{CE} \leq \sigma_+ + \sigma_-; \quad (8a)$$

$$(\sigma_- - \sigma_+)^2 \leq 4\sigma_{CE}(\sigma_+ + \sigma_- - \sigma_{CE}); \quad (8b)$$

$$\sigma_- \leq 3(\sigma_+ + \sigma_{CE}); \quad (10a)$$

$$(3\sigma_+ + \sigma_- - 6\sigma_{CE})^2 \leq 8\sigma_-(3\sigma_+ + 3\sigma_{CE} - \sigma_-); \quad (10b)$$

$$(\sigma_+)^{1/2} \leq (\sigma_-)^{1/2} + (2\sigma_{CE})^{1/2}; \quad (11a)$$

$$(2\sigma_{CE})^{1/2} \leq (\sigma_+)^{1/2} + (\sigma_-)^{1/2}; \quad (11b)$$

$$(\sigma_+ + \sigma_- - 2\sigma_{CE})^2 \leq 4\sigma_+ \sigma_-; \quad (11c)$$

$$(\sigma_-)^{1/2} \leq (\sigma_+)^{1/2} + (2\sigma_{CE})^{1/2}; \quad (12a)$$

$$(\sigma_+ + 2\sigma_{CE} - \sigma_-)^2 \leq 8\sigma_+ \sigma_{CE}; \quad (12b)$$

$$(\sigma_- + 2\sigma_{CE} - \sigma_+)^2 \leq 8\sigma_- \sigma_{CE}; \quad (12c)$$

The isospin bounds (6b), (8b), (10b), (11a,b,c), (12a,b,c) are all equivalent to

$$\sigma_+^2 + \sigma_-^2 + 4\sigma_{CE}^2 - 2\sigma_+ \sigma_- - 4\sigma_{CE}(\sigma_+ + \sigma_-) \leq 0 \quad (13)$$

and also to

$$(\sigma_+^{1/2} - \sigma_-^{1/2})^2 \leq 2\sigma_{CE} \leq (\sigma_+^{1/2} + \sigma_-^{1/2})^2 \quad (14)$$

All these bounds are valid for differential cross-sections at an arbitrary kinematical value  $x$  as well as for the integrated cross-sections.

The bound (8b) was given recently by Roy<sup>/3/</sup> for the differential cross sections at an arbitrary angle, and by Tornquist<sup>/4/</sup> for the backward cross-sections. Other authors<sup>/5,6/</sup> have introduced stronger bounds, for which the polarization data are needed, and which follow from the above ones if  $\sigma$  is replaced by  $\Sigma^\pm = \frac{1}{2}(1 \pm P)\sigma$ .

Therefore the introduction of the  $\vec{G}$ -amplitudes allows to discuss the isospin bounds at any kinematical value  $x$  by analogy with the well-known case of the backward scattering<sup>/4,7/</sup>.

### 3. The Nucleon Isospin Polarization and Isospin Bound

In this section we discuss the quantitative limits for isospin invariance in  $\pi N$ -scattering at all angles and different energies using the nucleon isospin parameter and its bounds determined from the experimental data.

The nucleon isospin polarization parameter  $P^{(T)}$  in  $\pi$ -P-elastic and charge exchange reactions is expressed in terms of the differential cross-sections as

$$P^{(T)} = \frac{\sigma_- - \sigma_{CE}}{\sigma_- + \sigma_{CE}} \quad (15)$$

Our analysis on the isospin bounds of the nucleon isospin-polarization is restricted to the bounds induced by the relation (14), so that we have

$$P_{\min}^{(T)} \leq P^{(T)} \leq P_{\max}^{(T)}, \quad (16)$$

where

$$P_{\min}^{(T)} = \left[ \sigma_- - \frac{1}{2} (\sigma_+^{1/2} + \sigma_-^{1/2})^2 \right] / \left[ \sigma_- + \frac{1}{2} (\sigma_+^{1/2} + \sigma_-^{1/2})^2 \right], \quad (16a)$$

$$P_{\max}^{(T)} = \left[ \sigma_- - \frac{1}{2} (\sigma_+^{1/2} - \sigma_-^{1/2})^2 \right] / \left[ \sigma_- + \frac{1}{2} (\sigma_+^{1/2} - \sigma_-^{1/2})^2 \right]. \quad (16b)$$

Therefore, if the isospin invariance in pion-nucleon scattering is valid up to conventional electromagnetic corrections, the nucleon isospin polarization parameter

is bounded by  $P_{\min}^{(T)}$  and  $P_{\max}^{(T)}$  defined by the relations (16a,b). In general, the lower and upper isospin

bounds  $P_{\min}^{(T)}$ ,  $P_{\max}^{(T)}$  will be saturated for  $\cos \delta_{ij} = \pm 1$  where  $\cos \delta_{ij}$  are defined by the relations (6), (8), (10) and (12). In particular, if one of the isospin exchange amplitudes:  $\vec{\Delta}_s$ ,  $\vec{N}_s$ ,  $\vec{F}_0$ ,  $\vec{F}_1$ ,  $\vec{\Delta}_u$ ,  $\vec{N}_u$  has a zero in the physical domain, then the isospin polarization param-

eters  $P^{(T)}$ ,  $P_{\min}^{(T)}$ ,  $P_{\max}^{(T)}$  are completely determined according to the s, t, u-channel decomposition (5),

(7), (9). The values of  $P^{(T)}$ ,  $P_{\min}^{(T)}$ ,  $P_{\max}^{(T)}$  so obtained are given in Table I. According to this table the isospin

polarization bounds are degenerated ( $P^{(T)} = P_{\min}^{(T)} = P_{\max}^{(T)}$ )

only if  $|\vec{\Delta}_s| = 0$  or  $|\vec{\Delta}_u|^2 = 0$ . The lower bound is satu-

rated only for  $|\vec{F}_0| = 0$  and the upper bound is saturated

for  $|\vec{N}_s| = 0$  or  $|\vec{F}_1| = 0$  or  $|\vec{N}_u| = 0$ . All the zeros of

the isospin exchange amplitudes lie on the  $\delta = 0, \pi$  -

"phase contours". The phase contours defined here by

$\cos \delta_{ij} = \pm 1$  are different from that studied in ref. <sup>16/</sup>

They agree with that introduced in ref. <sup>17/</sup> only for

$\theta_\pi = 180^\circ$ . Therefore, as a test for the isospin invari-

ance in pion-nucleon scattering at any angle we present

here our estimations of the isospin polarization parameters

$P^{(T)}$ ,  $P_{\min}^{(T)}$ ,  $P_{\max}^{(T)}$  from the experimental data of

$\pi$ -P-elastic and charge exchange reactions. In Figs.(1-3)

we show the distributions of all these parameters at all

angles and different laboratory momenta between 0.3 and

1.5 GeV/c. The backward distributions of the isospin

polarization parameters at 6 and 10 GeV/c are presented

in Figs. (4a,b) and their values for the extreme backward

points  $\theta = 180^\circ$  at the laboratory momenta between 0.25

and 6 GeV/c are presented in Fig. 5.

We have used two ways to estimate the nucleon isospin

polarization parameters:

(i) directly from the experimental data in their original

form <sup>18,9/</sup> (for the data presented in Figs. (2-4), and

Fig. 5 for  $P_{LAB} > 0.6$  GeV/c); and

(ii) using the coefficients from the original polynomial

fits (for the data presented in Figs. (1-3) and Fig. 5 for

$P_{LAB} < 0.6$  GeV/c).

From the presented data we see that in general the

nucleon isospin polarization parameter have a behaviour

in agreement with its isospin bounds (16a,b). In particular,

Table

The values of the nucleon isospin polarization parameters  $P^{(T)}$ ,  $P_{\min}^{(T)}$ ,  $P_{\max}^{(T)}$  corresponding to the zeros of isospin exchange amplitudes.

Zero of	$P_{\min}^{(T)}$	$P^{(T)}$	$P_{\max}^{(T)}$	Saturation of
$ \vec{\Delta}_s $	+1/3	+1/3	+1/3	degenerated bounds
$ \vec{N}_s $	-7/9	-1/3	-1/3	upper bound
$ \vec{F}_0 $	-1/3	-1/3	+1	lower bound
$ \vec{F}_1 $	-1/3	+1	+1	upper bound
$ \vec{\Delta}_u $	-1	-1	-1	degenerated bounds
$ \vec{N}_u $	+1/17	+7/11	+7/11	upper bound

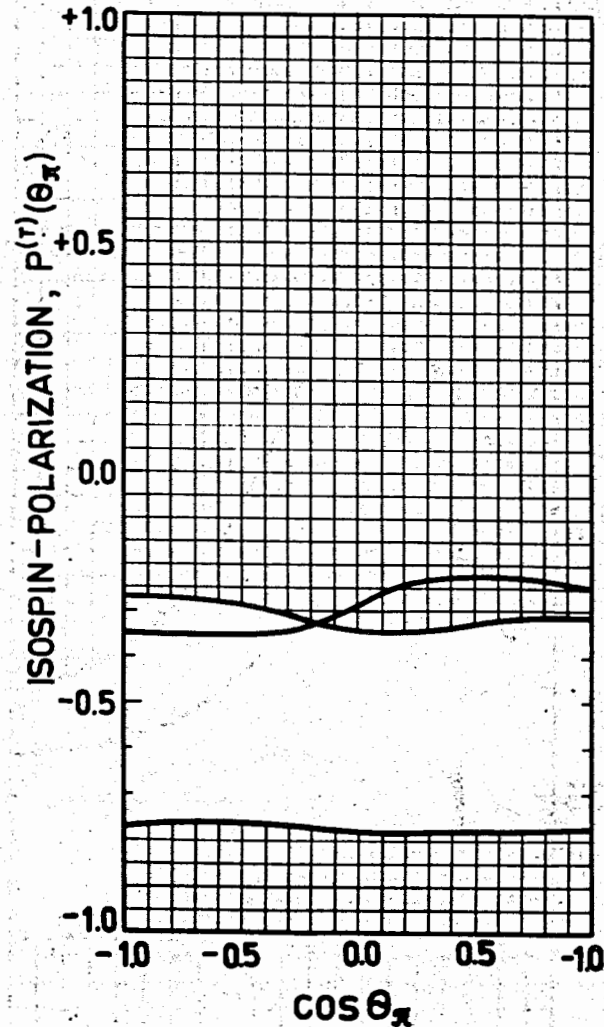


Fig. 1a. The nucleon isospin polarization parameter and its isospin bounds at: a)  $p_{\text{LAB}} = 0.295$  GeV/c determined from the data of Glickman et al. <sup>/8,9/</sup> and Anderson et al. <sup>/8/</sup>; b)  $p_{\text{LAB}} = 0.385$  GeV/c determined from the data of Zinov et al. and Mukhin et al. <sup>/8/</sup>; c)  $p_{\text{LAB}} = 0.490$  GeV/c determined from the data of Ogden et al. and Chen et al. <sup>/8/</sup>; d)  $p_{\text{LAB}} = 0.580$  GeV/c determined from the data of Ogden et al. and Chen et al. <sup>/9/</sup>.

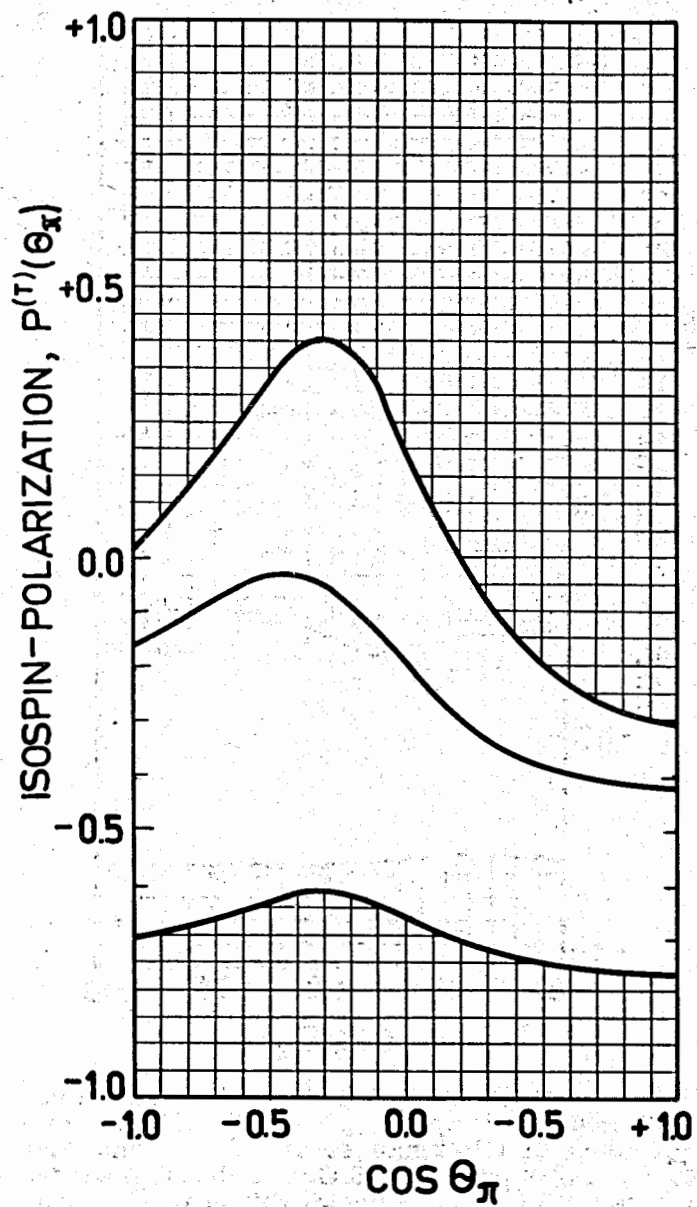


Fig. 1b.

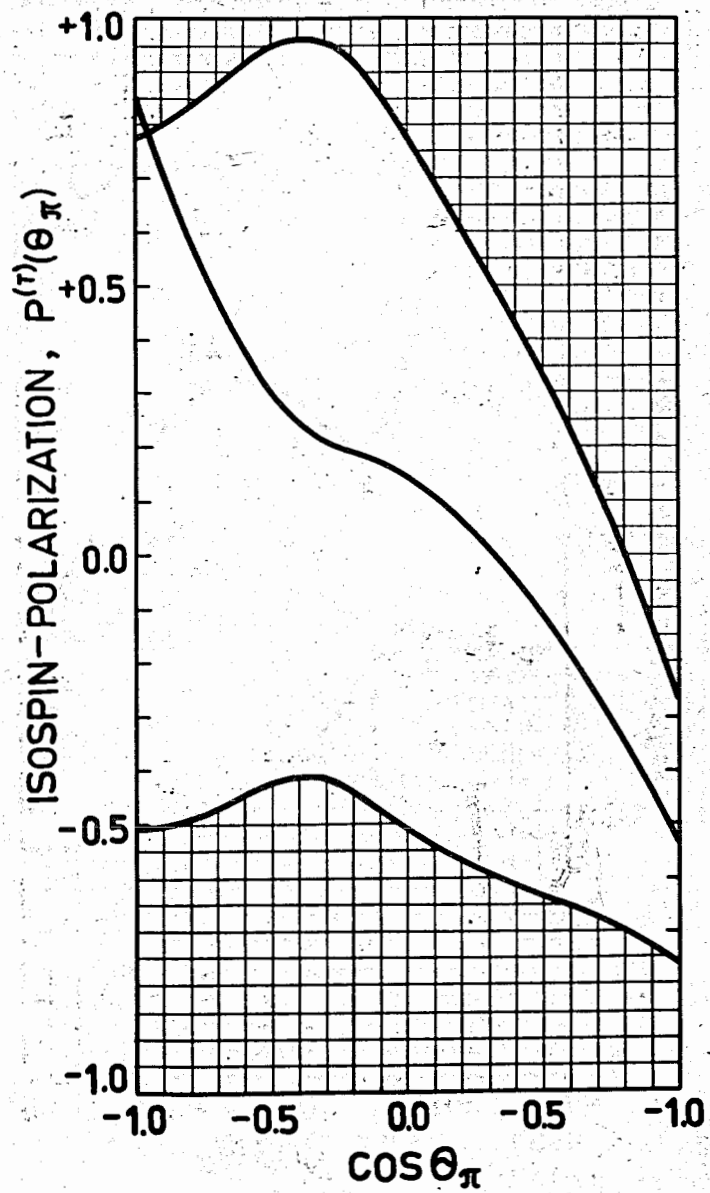


Fig. 1c.



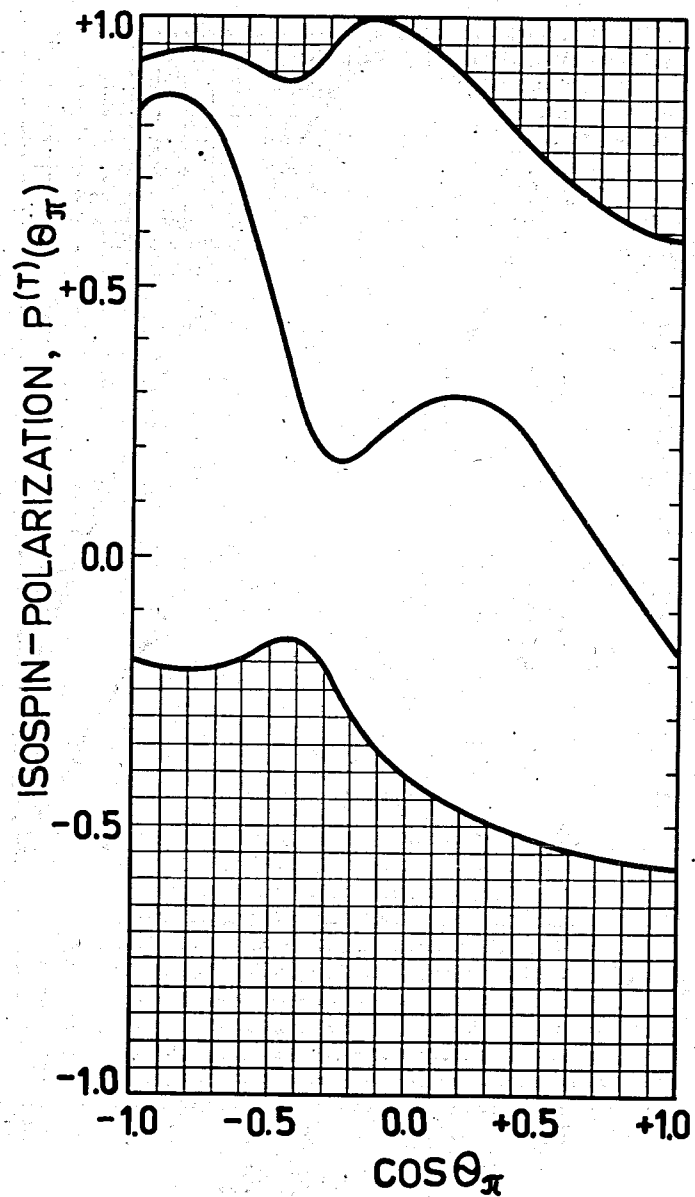


Fig. 1d.

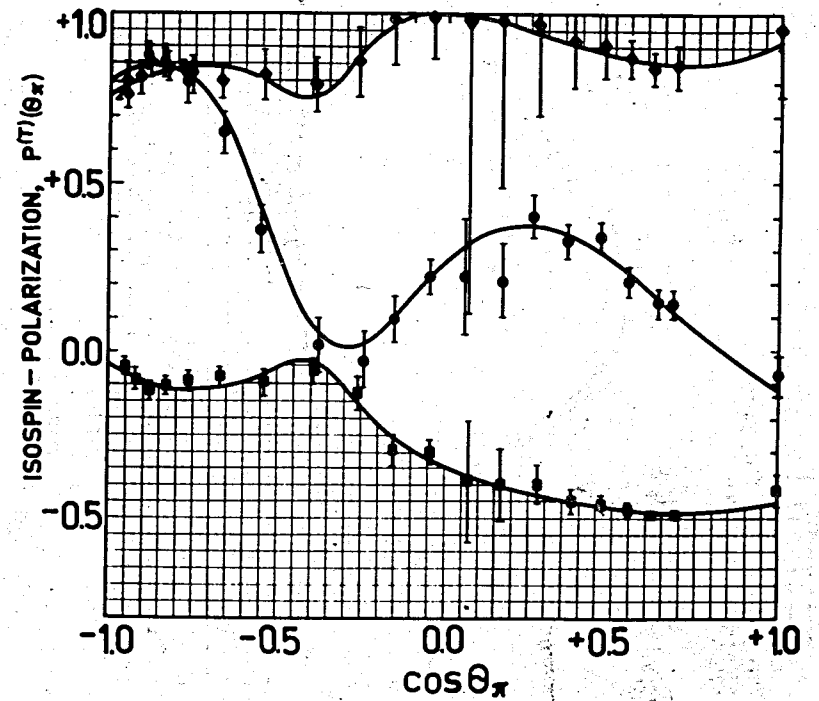


Fig. 2a. The nucleon isospin polarization parameter  $P^{(T)}(\bullet)$  and the isospin bounds  $P_{\min}^{(T)}(\blacksquare)$ ,  $P_{\max}^{(T)}(\blacklozenge)$  at: a)  $P_{\text{LAB}} = 0.614$  GeV/c determined from the data of Ogden et al. <sup>8/</sup> and Chiu et al. <sup>9/</sup>; b)  $P_{\text{LAB}} = 0.658$  GeV/c determined from the data of Helland et al. <sup>8/</sup> and Chiu et al. <sup>9/</sup>; c)  $P_{\text{LAB}} = 0.710$  GeV/c determined from the data of Helland et al. <sup>8/</sup> and Chiu et al. <sup>9/</sup>; d)  $P_{\text{LAB}} = 0.777$  GeV/c determined from the data of Ogden et al. <sup>8/</sup> and Chiu et al. <sup>9/</sup>

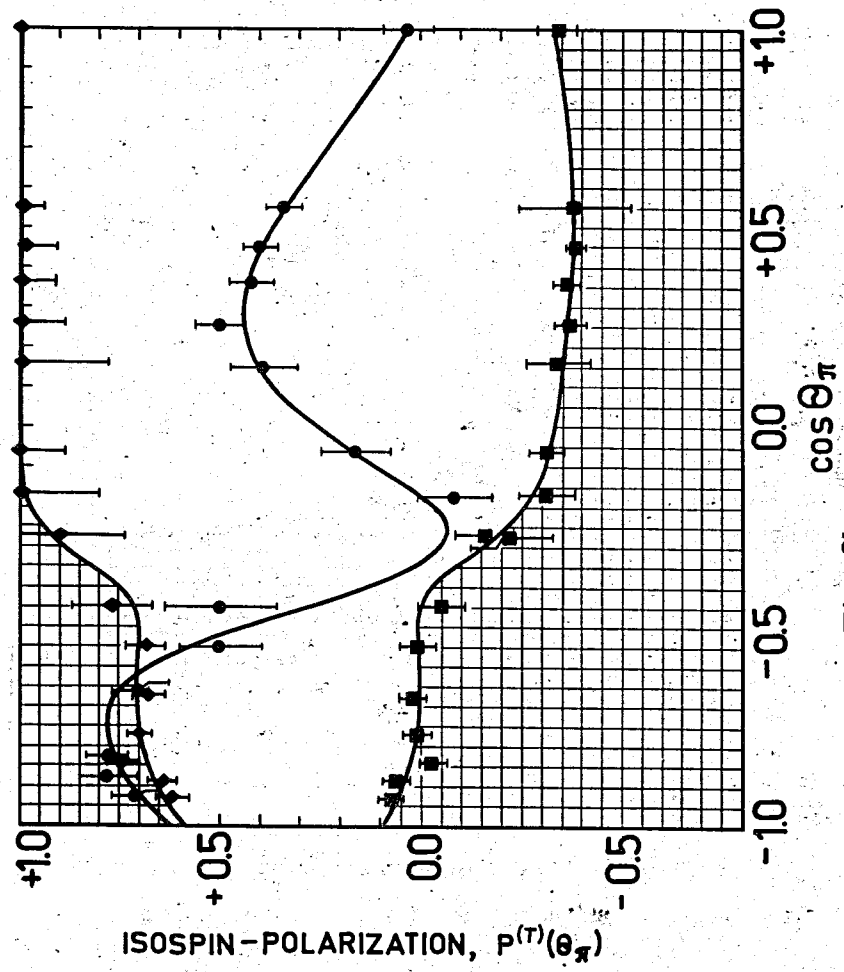


Fig. 2b.

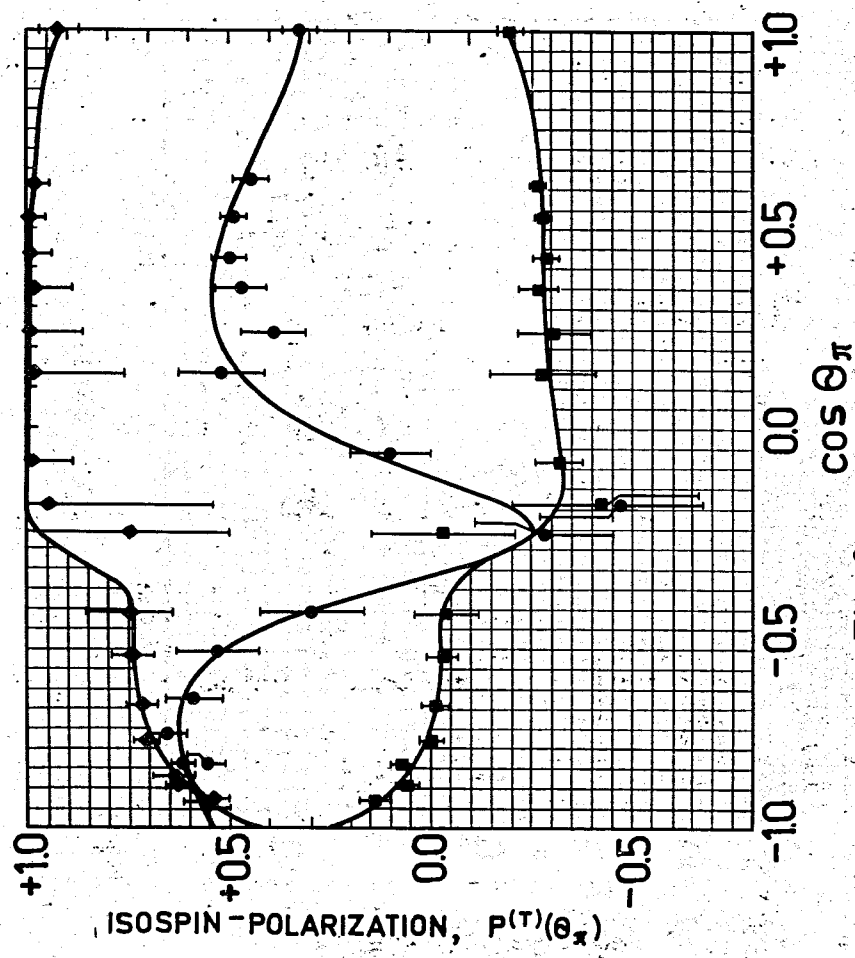


Fig. 2c.

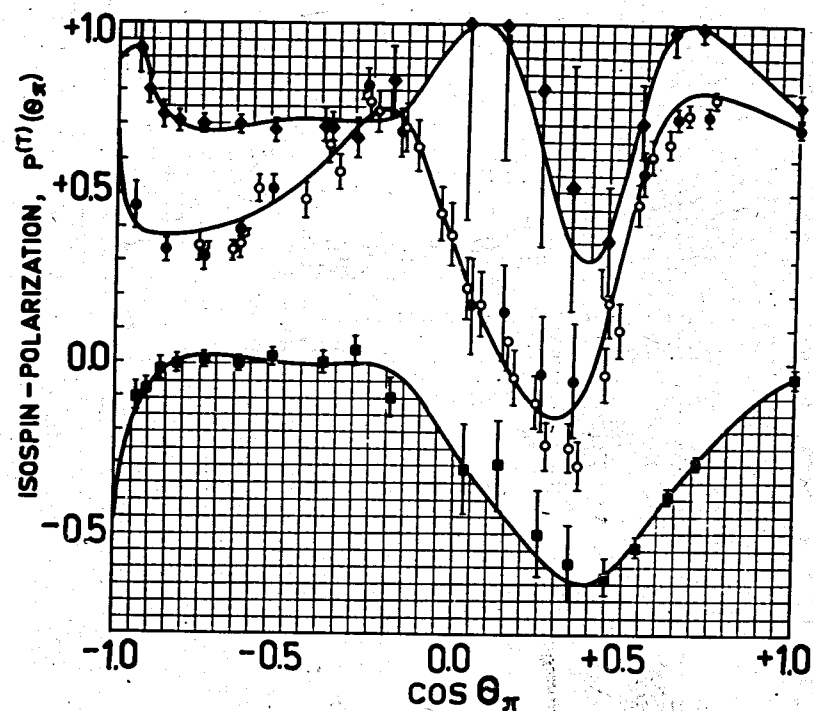
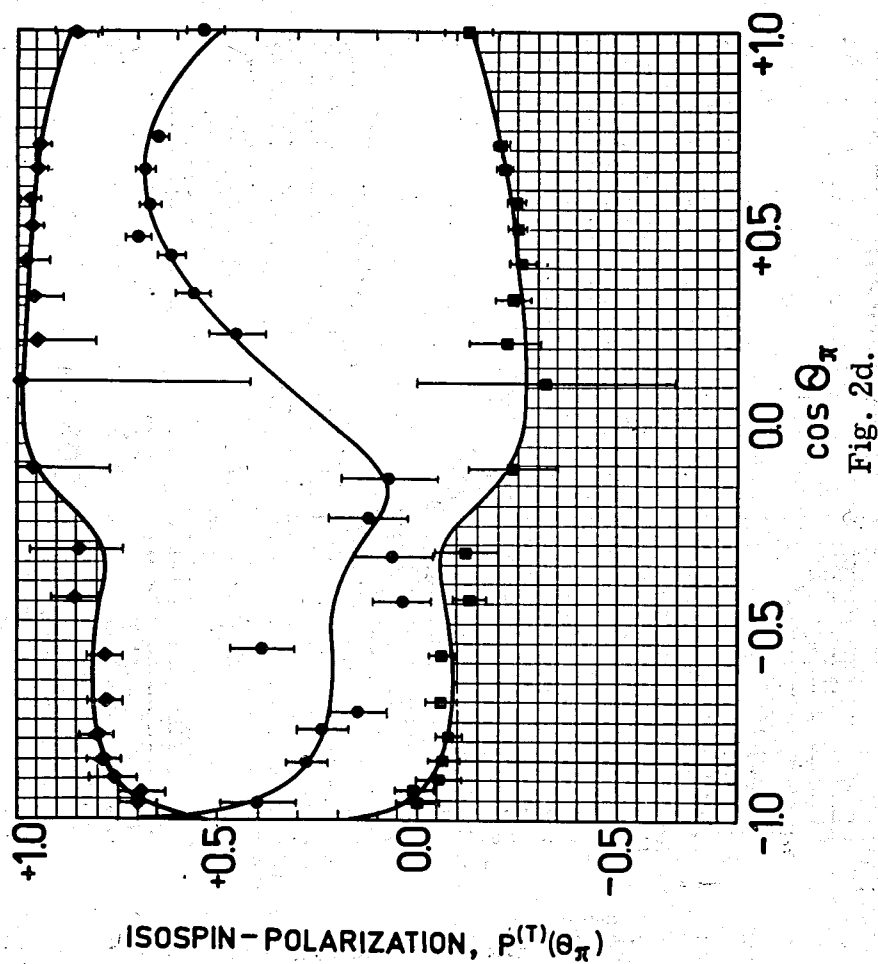


Fig. 3a. The nucleon isospin polarization parameter  $P^{(T)}$  and the isospin bounds  $P_{\min}^{(T)}$ ,  $P_{\max}^{(T)}$  at: a)  $p_{\text{LAB}} = 1.0$  GeV/c determined from the data of (•) Helland et al. /8/, (o) Albrow et al. /8/ and Chiu et al. /9/; b)  $p_{\text{LAB}} = 1.1$  GeV/c determined from the data of (•) Helland et al. /8/, (o) Albrow et al. /9/ and Chiu et al. /9/; c)  $p_{\text{LAB}} = 1.43$  GeV/c determined from the data of Alpin et al. /8/ and Chiu et al. /9/. The interpolation curve for  $P^{(T)}$  was obtained from the coefficients of Brody et al. /8/ and Chiu et al. /9/ while the isospin bounds was calculated with the coefficients given by Helland et al. /8/ ( $\pi^+$ ) and Duke et al. /8/ ( $\pi^-$ ).

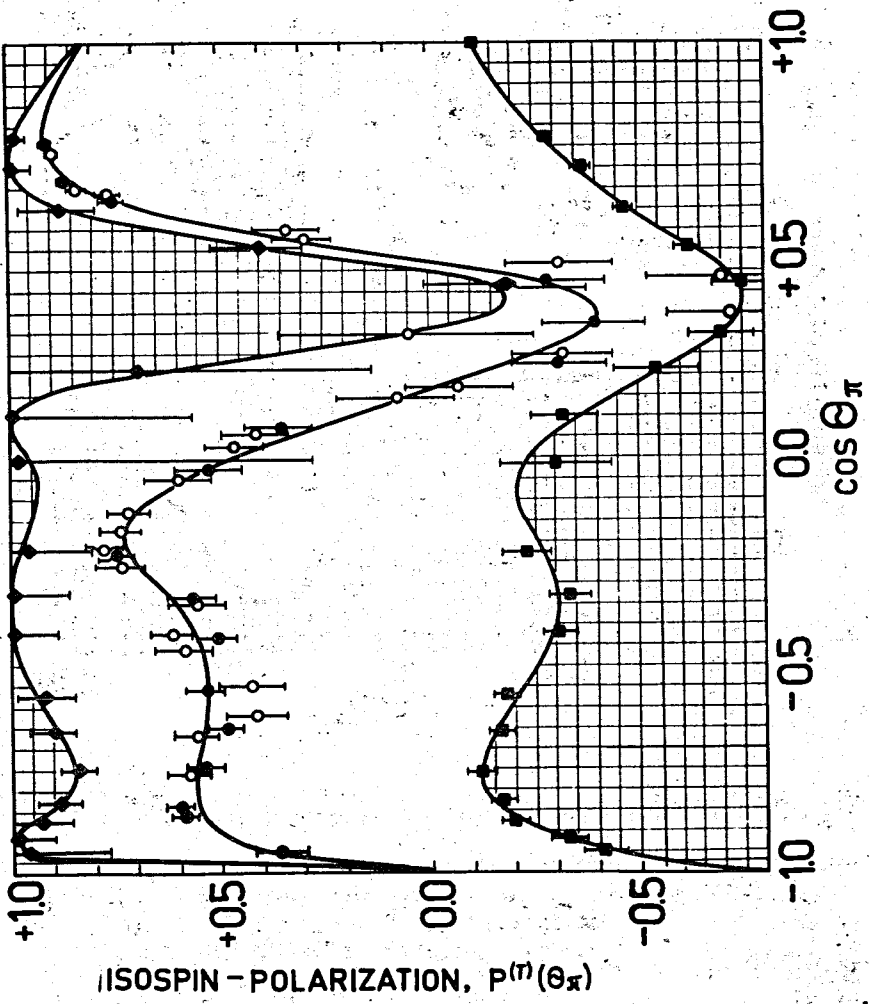


Fig. 3b.

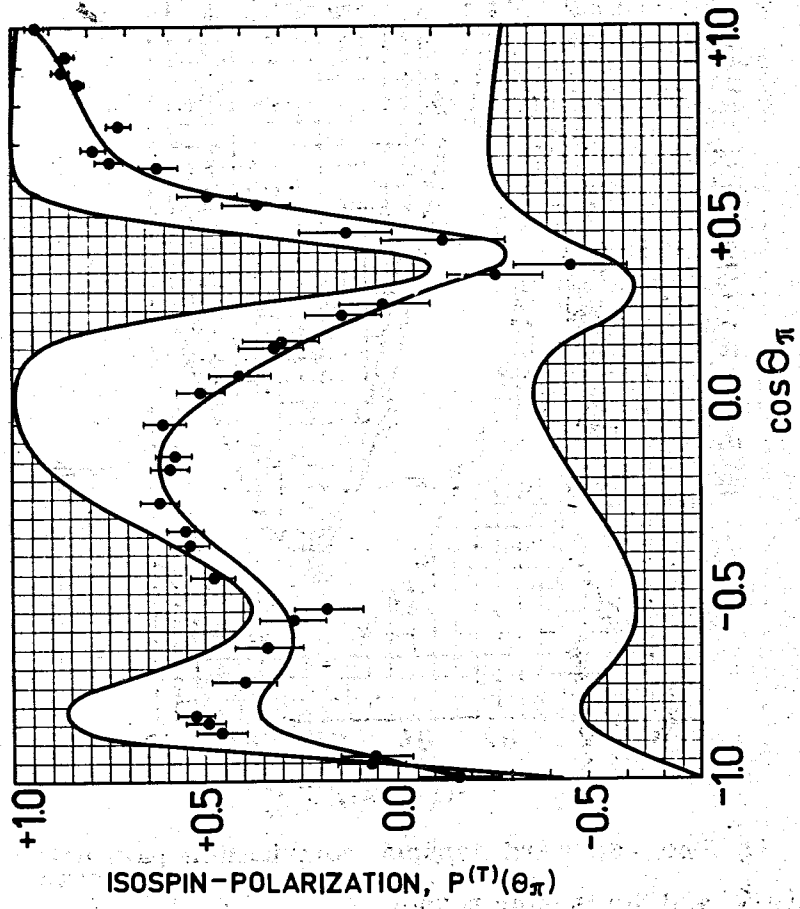


Fig. 3c.

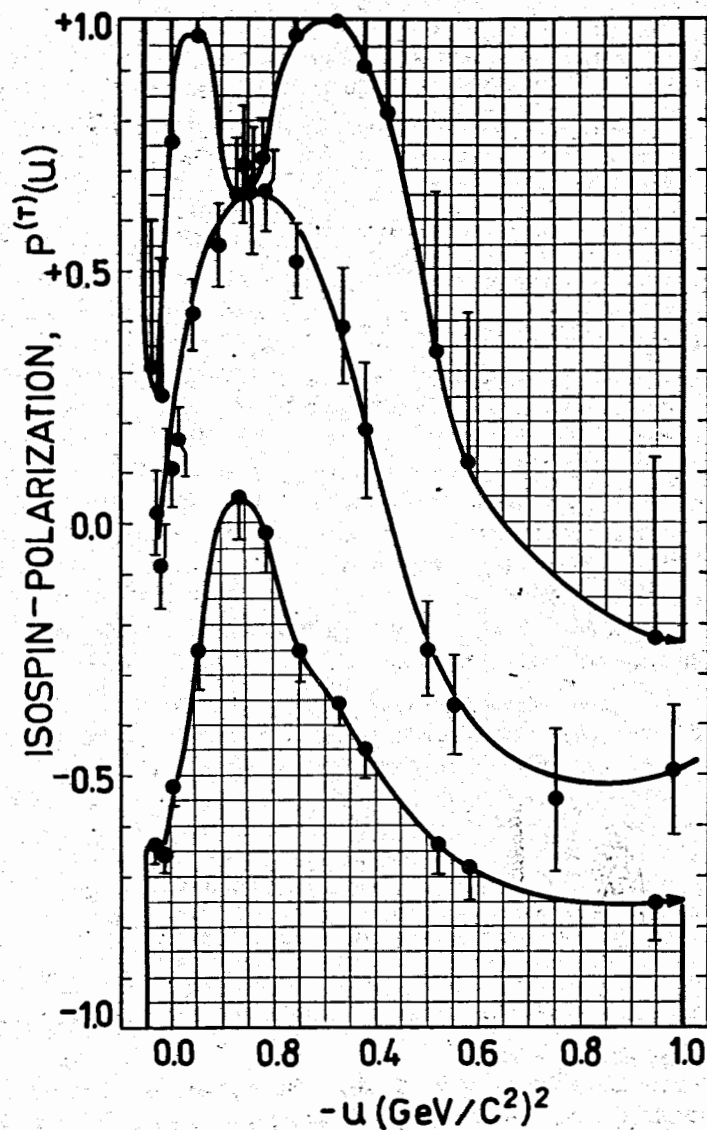
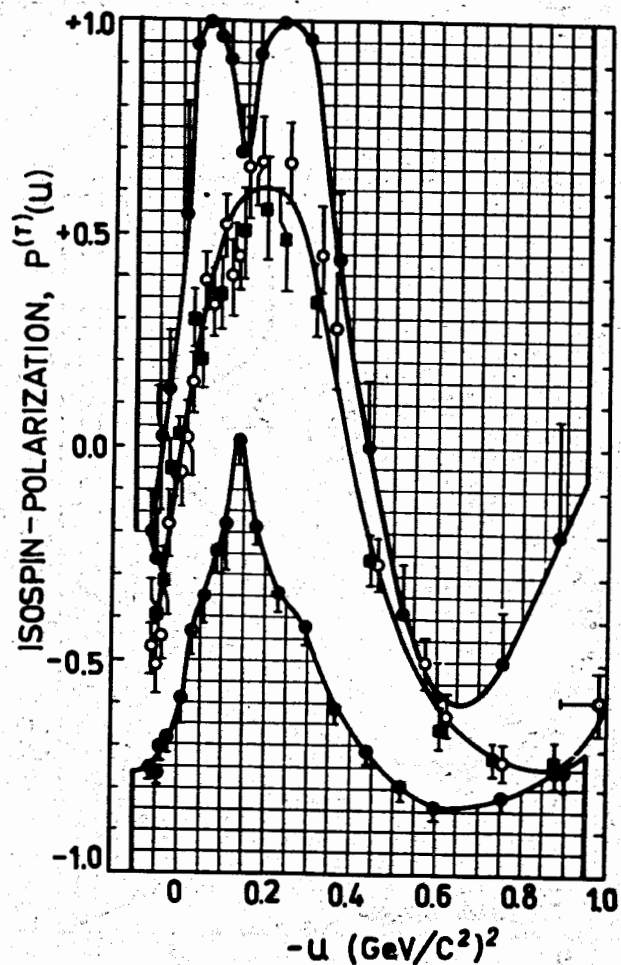


Fig. 4a. The backward isospin polarization parameter  $P^{(T)}$  (○ ■) and the isospin bounds  $P_{\min}^{(T)}$  ( ),  $P_{\max}^{(T)}$  (●) at (○) 5.9 GeV/c and 6.0 GeV/c determined from the data of Owen et al.<sup>/8/</sup> and Boright et al.<sup>/9/</sup>; b) The backward isospin polarization parameter  $P^{(T)}$  (●) and the isospin bounds  $P_{\min}^{(T)}$  ( ),  $P_{\max}^{(T)}$  (●) at  $P_{LAB} = 10$  GeV/c determined from the data of Owen et al.<sup>/8/</sup> and Boright et al.<sup>/9/</sup>.

Fig. 4b.

the saturation of the upper bound  $P^{(T)} \approx P_{\max}^{(T)} \approx -1/3$  observed at  $p_{\text{LAB}} = 0.296 \text{ GeV/c}$  in entire  $\cos \theta_{\pi}$  range is obviously an effect of  $\Delta(1236)$  resonance. The saturations of the upper bounds (see Figs. (2-4)) at  $0.614 \text{ GeV/c}$  ( $\cos \theta_{\pi} = -1$ );  $0.658 \text{ GeV/c}$  ( $-1 \leq \cos \theta_{\pi} \leq -0.5$ );  $p_{\text{LAB}} = 0.710 \text{ GeV/c}$  ( $-1 \leq \cos \theta_{\pi} \leq -0.9$ );  $p_{\text{LAB}} = 0.777 \text{ GeV/c}$  ( $\cos \theta_{\pi} = -1$ );  $p_{\text{LAB}} = 1.0 \text{ GeV/c}$  ( $\cos \theta_{\pi} = -0.25$  and  $\cos \theta_{\pi} = +1$ );  $p_{\text{LAB}} = 6 \text{ GeV/c}$  and  $10 \text{ GeV/c}$  ( $u = -0.15 (\text{GeV/c}^2)^2$ ) are all due to  $\sigma_{1u} \approx 0$ . The zero of  $\sigma_{1u}$  at  $p_{\text{LAB}} = 6$  and  $10 \text{ GeV/c}$  was observed by Barger et al. /2/, who found that this is a quadratic zero consistent with the wrong-signature-nonsense zero of  $N_{\alpha}$ -Regge pole at  $\alpha = -1/2$ . Also, the indications for  $\sigma_{1u} \approx 0$  at  $p_{\text{LAB}} \approx 0.65 \text{ GeV/c}$  and  $p_{\text{LAB}} = 0.80 \text{ GeV/c}$  for  $\cos \theta_{\pi} = -1$  can be observed from Fig. 5. Next, the saturation of the lower isospin bound corresponding to  $\sigma_0 \approx 0$  is observed (Fig. 2) at  $p_{\text{LAB}} = 0.710 \text{ GeV/c}$  for  $\cos \theta_{\pi} = -0.25$ . An interesting effect seems to be at  $p_{\text{LAB}} = 1.12$  and  $1.43 \text{ GeV/c}$  (see Fig. 3) for  $\cos \theta_{\pi} \approx 0.4$ , where the values of the nucleon isospin parameters  $P_{\max}^{(T)}$  and  $P_{\min}^{(T)}$  are near  $-1/3$  due to a possible zero of  $s$ -channel  $1/2$ -isospin exchange amplitude near the physical region. The zero of  $\sigma_{1s}$  is also indicated by Hohler et al. /6/ at  $p_{\text{LAB}} \approx 1.070 \text{ GeV/c}$  for  $\cos \theta_{\pi} = 0.5$ . Also, from Fig. 5 one can see that  $P^{(T)} \approx P_{\max}^{(T)} \approx -1/3$  at  $p_{\text{LAB}} = 0.3 \text{ GeV/c}$ ;  $1.4 \text{ GeV/c}$ ;  $2.6 \text{ GeV/c}$ ;  $3.0 \text{ GeV/c}$ ;  $4.3 \text{ GeV/c}$ ;  $5.0 \text{ GeV/c}$  and  $5.3 \text{ GeV/c}$  for  $\theta_{\pi} = 180^\circ$ .

These are indications in favour of  $\sigma_{3s} \gg \sigma_{1s}$  due to  $\Delta(1236)$ ,  $\Delta(1890)$ ,  $\Delta(2420)$ ,  $\Delta(2560)$ ?,  $\Delta(3000)$ ?,  $\Delta(3230)$  and  $\Delta(3300)$ ?-resonances in  $\pi N$ -backward scattering at  $\theta_{\pi} = 180^\circ$ .

We see also from Fig. 5, that the upper and lower isospin bounds tend to be degenerated at  $p_{\text{LAB}} \approx 0.740$ ,  $1.1$  and  $2.2 \text{ GeV/c}$  due to the zeros of  $\sigma_{3s}$  and  $\sigma_{3u}$  from the neighbourhood of the physical region.

On the other hand, we have calculated  $\sigma_{1s}$ ,  $\sigma_0$ ,  $\sigma_{1u}$  from the experimental data and have verified that the

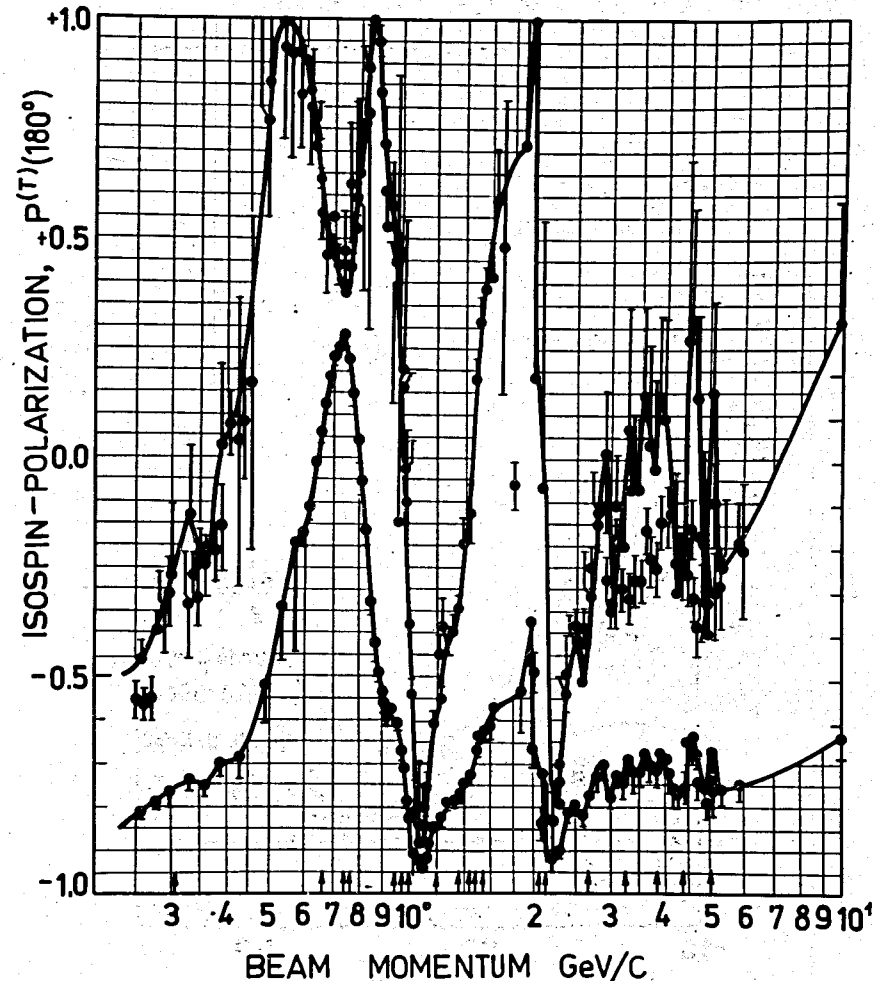


Fig. 5. The nucleon isospin polarization parameters  $P^{(T)}$  ( $\bullet$ )  $P_{\min}^{(T)}$  ( $\circ$ ) and  $P_{\max}^{(T)}$  ( $\bullet$ ) at  $\theta = 180^\circ$  for  $0.25 \text{ GeV/c} \leq p_{\text{LAB}} \leq 10 \text{ GeV/c}$  determined from the experimental data /8,9/.

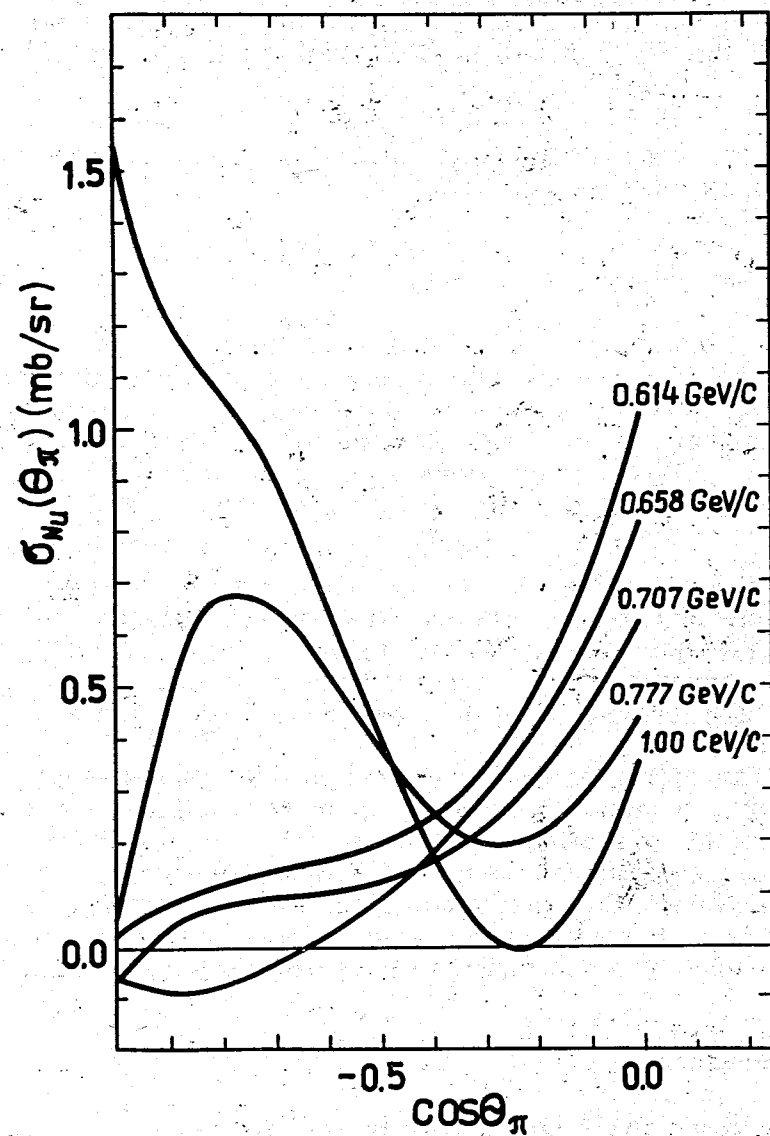
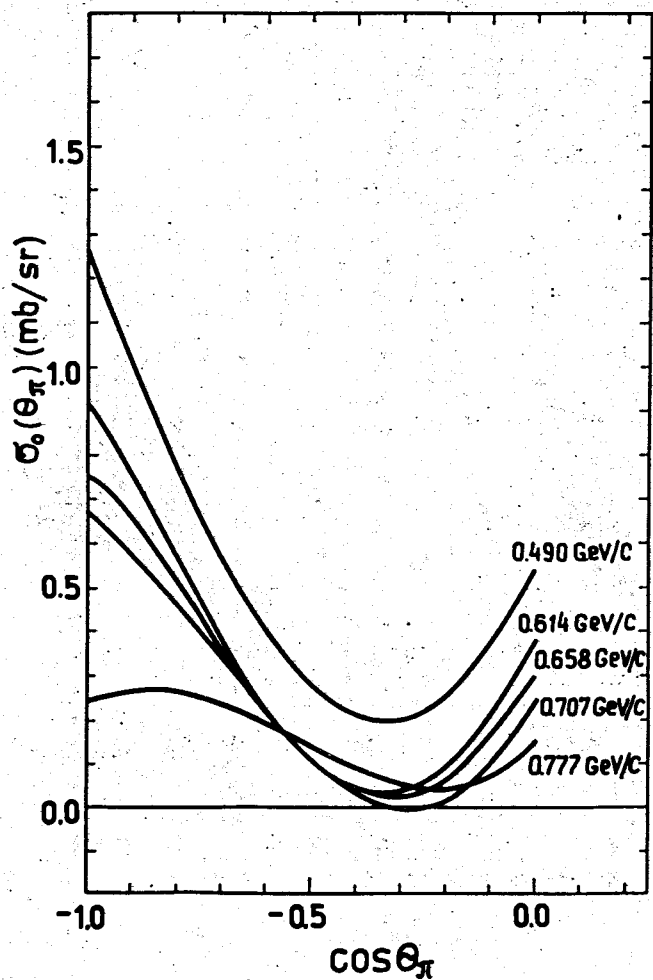


Fig. 6. a. The  $\sigma_0$  -cross sections at  $P_{LAB} = 0.490$ ; 0.614; 0.658; 0.707 and 0.777 GeV/c obtained using the coefficients from the original polynomial fits /8,9/ ; b. The  $\sigma_{1u}$  -cross sections at  $P_{LAB} = 0.614$ ; 0.658; 0.707; and 1.0 GeV/c obtained using the coefficients from the original polynomial fits /8,9/.

Fig. 6b

positions of the zeros are in accord with that obtained from the study of isospin polarization parameters  $P^{(T)}$ ,  $P_{\min}^{(T)}$ ,  $P_{\max}^{(T)}$ .

The results for  $\sigma_0$  and  $\sigma_{1u}$  are presented in Figs. 6a,b.

#### 4. Conclusions

We summarize the results as follows.

The values of the nucleon isospin polarization parameter satisfy the isospin bounds, obtained from the isospin invariance, at any angle for the all energies considered above. All the predictions (see Table I) relative to the values of the isospin polarization parameters  $P^{(T)}$ ,  $P_{\min}^{(T)}$ ,  $P_{\max}^{(T)}$  when one of the isospin exchange amplitudes in the  $s$ ,  $t$ ,  $u$ -channels have a zero near the physical region, are in good agreement with the experimental data. So that the branching ratio of different  $\pi N$ -cross sections for the zero position of resonance poles are consistent with that obtained from the isospin invariance.

Therefore, from the presented data we see that the isospin invariance is valid up to conventional electromagnetic corrections in the whole  $\cos\theta_\pi$  region excepting some points at  $\cos\theta_\pi = -1$  discussed in ref. /4/. The saturation of the isospin polarization bounds are systematically connected with the zeros or resonance poles of the pion-nucleon scattering amplitude near the physical region.

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Received by Publishing Department  
on February 18, 1974.