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DETERMINATION OF NUCLEAR SPECTROSCOPIC FACTORS FROM DATA ON DIFFERENTIAL CROSS SECTIONS FOR ELASTIC NEUTRON SCATTERING ON LIGHT NUCLEI

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DETERMINATION OF NUCLEAR SPECTROSCOPIC FACTORS FROM DATA ON DIFFERENTIAL CROSS SECTIONS FOR ELASTIC NEUTRON SCATTERING ON LIGHT NUCLEI

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Определение ядерных спектроскопических факторов на основе данных по дифференциальному сечению упругого рассеяния нейтронов на легких ядрах

Найдены значения констант связей (спектроскопических факторов) dpn, $^{12}C\,^{11}C\,n$ и $^{14}N\,^{13}N\,n$ путем экстраполяции данных по дифференциальным сечениям к соответствующим полюсам. Рассматриваются некоторые общие вопросы применения этого метода к легким ядрам.

Препринт Объединенного института ядерных исследований.
Дубна, 1974

Dubnička S., Dumbrajs O.V. E2 - 7731

Determination of Nuclear Spectroscopic Factors from Data on Differential Cross Sections for Elastic Neutron Scattering on Light Nuclei

The values of the coupling constants (spectroscopic factors) dpn, $^{12}C\,^{11}C\,n$ and $^{14}N\,^{13}N\,n$ have been derived by means of the extrapolation of differential cross section data to the corresponding poles. Some general questions concerning the application of this method to light nuclei are considered.

Preprint. Joint Institute for Nuclear Research.
Dubna, 1974

1. Introduction

A spectroscopic factor is a measure of the probability of the system being composed of the constituents in a particular channel. It is the magnitude of the asymptotic part of the wave function of a composite system in that channel. Therefore spectroscopic factors must be given the same status as other nuclear parameters, such as the binding energy, charge radius and form factors.

In our previous work [1] (see also other related works [2–4]), hereafter referred as I, we described in detail a new method of determination of spectroscopic factors based on the extrapolation of data on differential cross section for elastic particle-nucleus scattering to the poles. The method was tested practically in the cases of $n\,^3H$ and $n\,^3He$ elastic scattering. It produced good results * and we urged to apply this method to other elastic processes of both the neutron and the proton on other nuclei.

In this work we extend the analysis to neutron scattering. There exists a complete compilation [5] of differential cross section data for neutron elastic scattering on light nuclei ($Z \leq 20$). However, after examination of the data

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* Unfortunately in I we used the erroneous relation between the residue of a spin averaged forward scattering amplitude and the coupling constant. This resulted in the ≈20% reduction of the value of the $^3H\,dn$ coupling constant. We elaborate this point at the end of Appendix of this paper.

presented in this compilation, we singled out three nuclei only: deuteron, ^{12}C and ^{14}N as being appropriate for our purpose. Deuteron has been chosen to compare our analysis with the previous one /3/ , ^{12}C and ^{14}N turned out to be the only nuclei for which good data (tabulated with given errors) exist: ref. /6–8/ for deuteron, ref. /9/ for ^{12}C and ref. /10/ for ^{14}N ** . For other nuclei either there are no data at all, or data are presented in form of graphs only. Besides in many cases the references given in compilation /5/ turned out to be inaccessible to us. Heavier nuclei than ^{14}N were abandoned because form factor effects might have distorted the analysis. Of course, this bound $Z \leq 14$ is rather arbitrary and on the whole we admit that the question about the size effects is not very clear for us.

We refer the reader for the description of the method to I and proceed directly with the concrete analysis.

2. Singularities of the Elastic nd , n ^{12}C and n ^{14}N Scattering in the $\cos\theta$ Plane for Fixed Energy

The singularities of the elastic nd , n ^{12}C and n ^{14}N scattering amplitudes are assumed to be found by application of the Landau's method /11,12/ to the Feynman diagrams of the corresponding processes. They are shown explicitly for some lower order diagrams in figs.1,2,3. The positions of the singularities are determined according to formulas analogous to those given in I. As already was mentioned in I, the excited states of the "pole" nucleus should be considered as the extra singularities unless they decay through strong interactions only. This worsens the situation because in practice it means that the position of the lowest of such excited states should be considered as the nearest left-hand

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** We used, in fact, the data for natural carbon and natural nitrogen, the overwhelming constituents of which are just ^{12}C and ^{14}N .

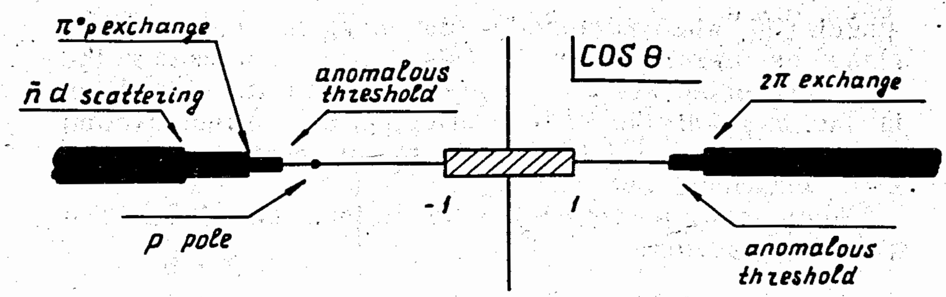


Fig. 1. The analytic structure of amplitude for nd elastic scattering in the $\cos\theta$ plane. The figure is not drawn in scale.

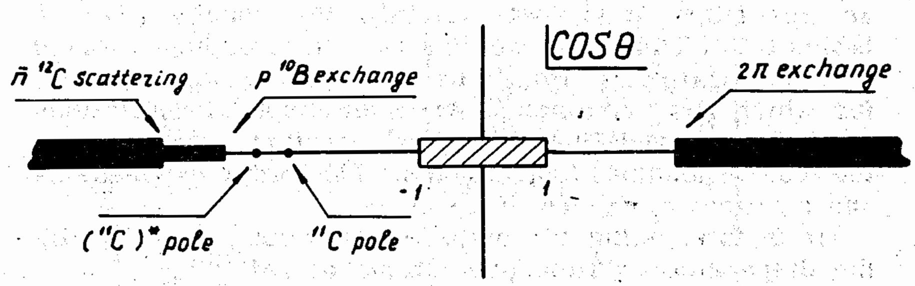


Fig. 2. The analytic structure of amplitude for n ^{12}C elastic scattering in the $\cos\theta$ plane. Only the first excited state of ^{11}C is shown. The figure is not drawn in scale.

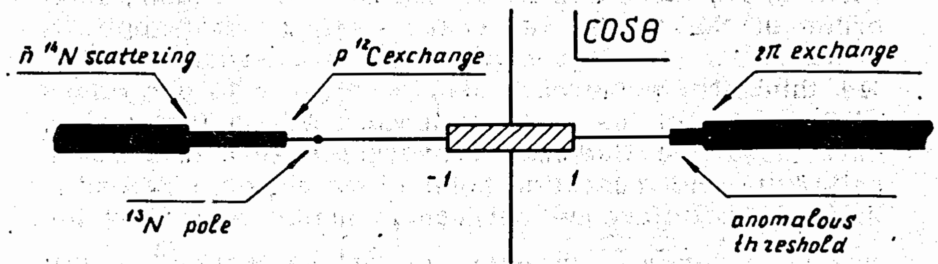


Fig. 3. The analytic structure of amplitude for n ^{14}N elastic scattering in the $\cos\theta$ plane. The figure is not drawn in scale.

singularity when performing the mapping of the $\cos\theta$ plane as described in I. This is just the case in the $n^{12}C$ scattering as it is seen in fig. 2. On the other hand in the case of the $n^{14}N$ scattering the lowest excited state of ^{13}N is high enough to be buried under the $p^{12}C$ exchange cut.

Some details about the anomalous thresholds are given in Appendix.

3. Results and Conclusions

After mapping the entire $\cos\theta$ plane onto an unifocal ellipse in the z-plane and carrying out the fits in a way as described in I, we obtained the results given in tables 1,2,3. There r denotes a residue of a spin-averaged forward scattering amplitude in the laboratory system, for which it is common to write the dispersion relations and g is a redefined coupling constant, which enters the corresponding Lagrangians. The exact expressions and relations are given in Appendix.

It is interesting to compare our values of r_{dpn} with the dispersion relation predictions of ref.[13] r_{dpn} = =0.081 ± 0.002. On the other hand, the analysis of the nd scattering along the lines similar to ours has been carried out already earlier in ref. [3] We find some discrepancies between our results and those of ref. [3] . First of all, there is a difference in choice of the optimal order of the fit. In two cases we find that the optimal order is for one degree less than that used in ref. [3] . We think that preference should be given to our values of the order of the fit because we, contrary to ref. [3] , have used the Cutkosky convergence test function in estimating the truncation point of the series. Secondly, there is a difference between formulas connecting the residue r (or g_N in notations of the work [3]) with the differential cross section. We do not understand the origin of the simple formula (2) of the ref. [3] . However, it is important to note that our complicated but precise expressions (A.9), (A.17) lead to ~ 1.5 times smaller

Table 1

The results of the fit and the values of the coupling constant g_{dpn} . Here M denotes the optimal order of the Tschebyscheff polynomial and ndf means the number of degrees of freedom

Energy (MeV)	M	χ^2/ndf	$r_{dpn} \pm \Delta r_{dpn}$	$g^2_{dpn} \pm \Delta g^2_{dpn}$
4.50	3	3.15	0.070±0.004	0.278±0.014
5.50	3	3.19	0.060±0.003	0.241±0.013
5.64	3	0.55	0.072±0.003	0.287±0.012
7.01	3	0.56	0.083±0.002	0.331±0.009
9.04	4	0.49	0.068±0.006	0.271±0.023
14.30	5	1.50	0.081±0.004	0.323±0.015

Table 2
The results of the fit and the values of the coupling constant $^{12}C^{11}Cn$. Conventions are the same as in Table 1

Energy (MeV)	M	χ^2/ndf	$r_{^{12}C^{11}Cn} \pm \Delta r_{^{12}C^{11}Cn}$	$g^2_{^{12}C^{11}Cn} \pm \Delta g^2_{^{12}C^{11}Cn}$
1.45	3	0.55	13.7±1.9	0.99±0.14
2.02	3	1.99	9.8±1.2	0.71±0.09
2.15	3	0.51	13.8±1.9	1.00±0.13
2.28	3	1.86	13.3±0.8	0.97±0.06
2.51	3	2.29	15.7±0.5	1.14±0.04
2.76	3	2.49	19.5±0.6	1.41±0.05
2.95	3	1.61	28.9±0.7	2.09±0.05
3.05	3	0.99	11.5±0.5	0.84±0.04
3.25	3	1.20	22.1±0.5	1.60±0.04
3.51	3	0.88	24.5±0.4	1.78±0.03
3.76	3	3.38	21.0±0.4	1.52±0.03

Table 3
The results of the fit and the values of the coupling constant $^{14}N^{13}Nn$. Conventions are the same as in Table 1

Energy (MeV)	M	χ^2/ndf	$r_{^{14}N^{13}Nn} \pm \Delta r_{^{14}N^{13}Nn}$	$g^2_{^{14}N^{13}Nn} \pm \Delta g^2_{^{14}N^{13}Nn}$
6.78	5	1.43	−0.162±0.032	438±86
7.41	5	0.89	−0.255±0.020	620±54
7.93	5	1.17	−0.303±0.014	821±39
8.35	5	0.75	−0.228±0.013	618±34
8.57	5	2.05	−0.223±0.012	605±32
9.37	5	0.88	−0.223±0.012	605±32
10.10	5	1.71	−0.251±0.009	679±25
10.93	5	2.08	−0.220±0.008	597±21

values of the residue than corresponding values of ref. [3].

Concerning the values of the coupling constants given in tables 2 and 3 we do not know what to compare them with. The evaluations of the forward dispersion relations exist neither for $n\,^{12}C$ nor for $n\,^{14}N$ scattering. They would provide the values of $r_{^{12}C\,^{11}Cn}$ and $r_{^{14}N\,^{13}Nn}$.

Perhaps one could also determine the corresponding coupling constants (spectroscopic factors) from some specific nuclear models (see discussion in I). In any case it would be very interesting to evaluate these quantities in some other way, because, in principle, there are two indications that the method used in this work might have not produced reliable results for such "heavy" nuclei as ^{12}C or ^{14}N .First of all, poles are far away from the physical region ($z_{^{11}C} \approx -24$, $z_{^{13}N} \approx -7$ should be compared with $z_d \approx -1.5$) even at the highest energies used in the fit. Secondary, the form factor dependence could be already very important. If it will turn out that the values for the coupling constants $^{12}C\,^{11}Cn$ and $^{14}N\,^{13}Nn$ found in this work are incorrect that will indicate the limitation of the method in nuclear physics in its present form, at least.

We express our deep gratitude to M.Plate and J.Tambergs for valuable discussions and help in numerical calculations.

Appendix

I. The anomalous thresholds marked in figs. 1 and 3 are found by means of investigation of analytic properties of the triangle diagrams shown in figs. 4a, 5a and 6a, respectively. The positions of the right-hand side anomalous thresholds (see figs. 1 and 3) in t -variable are determined according to the following formulas:

$$t_{01} = 4m_p^2 - \frac{[m_d^2 - m_n^2 - m_p^2]^2}{m_n^2} \approx 1.4\,m_\pi^2 \qquad (A.1)$$

$$t_{03} = 4m_p^2 - \frac{[m_{^{14}N}^2 - m_{^{13}C}^2 - m_p^2]^2}{m_{^{13}C}^2} \approx 3.1\,m_\pi^2 \qquad (A.2)$$

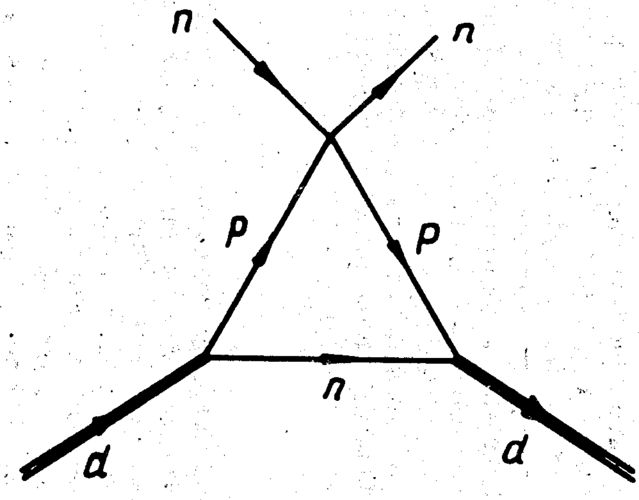
The position of the left-hand side anomalous threshold (see fig. 1) of the nd elastic scattering in u -variable is given by the expression:

$$u_{01} = m_p^2 + m_\pi^2 + \frac{m_\pi}{2m_n^2}\{\sqrt{[4m_p^2 m_n^2 - (m_p^2 + m_n^2 - m_d^2)^2](4m_n^2 - m_\pi^2)} - m_\pi(m_p^2 + m_n^2 - m_d^2)\} \approx 48.1\,m_\pi^2 \quad . \qquad (A.3)$$

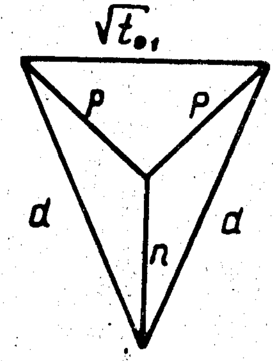
The relations (A.1-3) are found by means of the corresponding dual diagrams ref. [14,15] (see figs. 4b, 5b and 6b) obtained from the triangle diagrams by mapping closed loops into vertices and vertices into closed polygons. The right-hand side anomalous threshold is not marked in fig. 2 because the lowest anomalous threshold on physical sheet in t -variable ($t_{02} \approx 6.6\,m_\pi^2$) is higher than the lowest normal threshold. There are no anomalous thresholds on the physical sheet in u -variable for the $n\,^{12}C$ and $n\,^{14}N$ elastic scattering at all.

II. In our method coupling constants are defined as a measure of the strength of interaction of particles in vertices of Feynman diagrams. They can be explicitly introduced as numerical factors in effective Lagrangians for corresponding interactions. For instance, the following couplings fulfilling all requirements can be constructed for the description of vertices of pole diagrams 7a,b,c, respectively

$$\mathcal{L}_{Ia} = G_{dpn}\,\bar{\psi}_n \gamma_5 \gamma_\mu \psi_p D^\mu + h.c. \qquad (A.4)$$

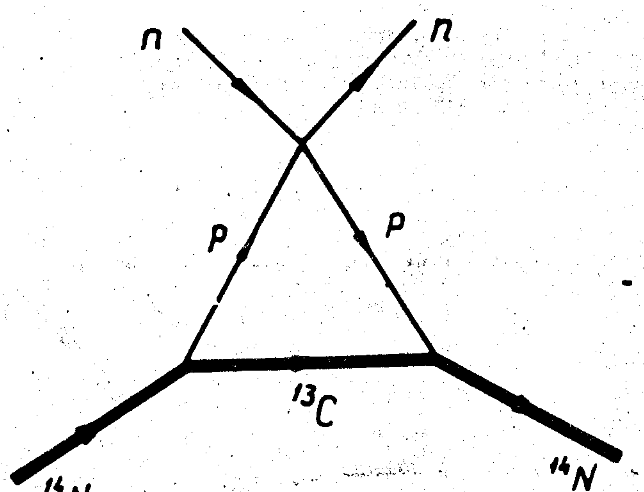


a)

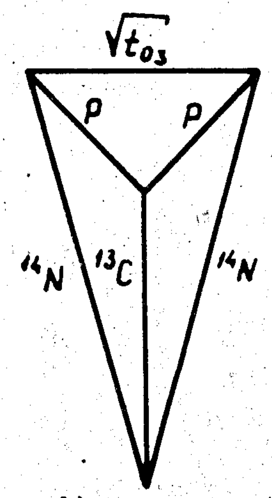


b)

Fig. 4a, b. Triangle and its dual diagrams giving the right-hand anomalous threshold for nd elastic scattering in the $\cos\theta$ plane (see fig. 1).



a)



b)

Fig. 5a, b. Triangle and its dual diagrams giving the right-hand anomalous threshold for $n^{14}N$ elastic scattering in the $\cos\theta$ plane (see fig. 3).

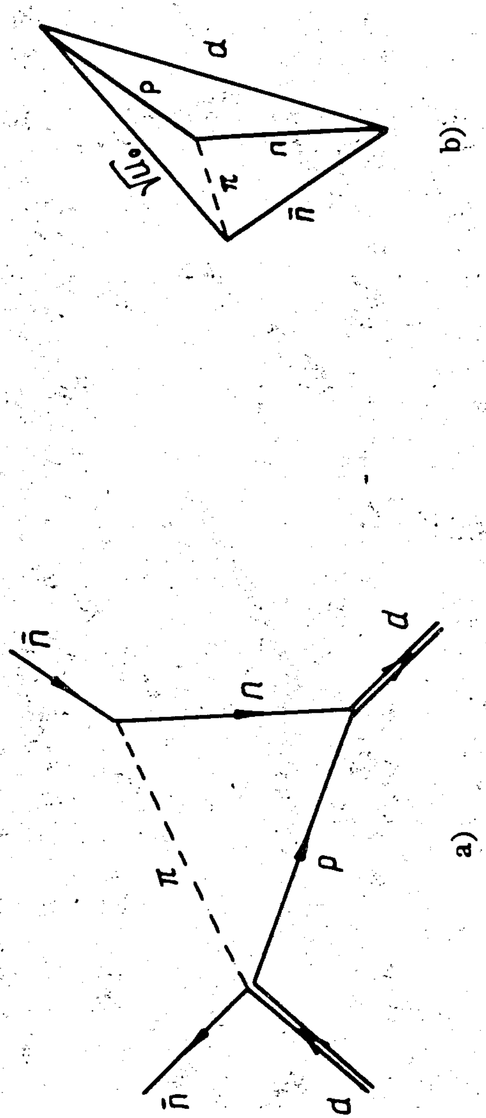


Fig. 6a, b. Triangle and its dual diagrams giving the left-hand anomalous threshold for nd elastic scattering in the $\cos\theta$ plane (see fig. 1).

$$\mathcal{L}_{Ib} = G_{^{12}C\,^{11}Cn}\,\bar{\psi}^{\mu}_{^{11}C}\,\psi_n\,\partial_\mu\phi_{^{12}C} + h.c. \qquad (A.5)$$

$$\mathcal{L}_{Ic} = G_{^{14}N\,^{13}Nn}\,\bar{\psi}_{^{13}N}\,\gamma_\mu\,\psi_n\,U^{\mu}_{^{14}N} + h.c. \qquad (A.6)$$

Taking into account the fact that the Lagrangian has the dimension $[L^{-4}]$, $\psi^{\mu}_{^{11}C}$, $\psi_{^{13}N}$, ψ_p , ψ_n fields have dimension $[L^{-3/2}]$ and D^{μ} , $U^{\mu}_{^{14}N}$, $\phi_{^{12}C}$ fields have dimension $[L^{-1}]$ (L means the length in $\hbar = c = 1$ units) one can see immediately that the coupling constants G_{dpn} , $G_{^{14}N\,^{13}Nn}$ are dimensionless and the coupling constant $G_{^{12}C\,^{11}Cn}$ has dimension $[L]$.

The coupling constants g in tables 1-3 are connected with the coupling constants introduced in Lagrangians (A.4-6) through relation

$$g^2 = \frac{G^2}{4\pi}. \qquad (A.7)$$

The squared residues of the scattering amplitudes at the corresponding poles defined through the expression

$$\lim_{z\to z_{pole}} [\,z(x) - z(x_{pole})]^2\,\frac{d\sigma(z)}{d\Omega} \equiv (Res_{pole})^2 = \sum_{n=1}^{M} A_n B_n T_n(z_{pole}) \qquad (A.8)$$

are related to the coupling constants g^2_{dpn} , $g^2_{^{12}C\,^{11}Cn}$ and $g^2_{^{14}N\,^{13}Nn}$ in the following way:

$$(Res_p)^2 = g^4_{dpn}\,\frac{10(\hbar c)^2}{24sk^4}\,\{[9m_n^2 m_d^2 + 3m_d^2(m_n^2 + C) +$$

$$+\frac{8}{m_d^2}(m_d^2 + C)(A_1 + B)^2 + 2(A_1 + B)^2\,] -$$

$$-4[-m_n^2(A_1+k^2)+(A_1+B)(m_n^2+C)-2\frac{m_n^2}{m_d^2}(A_1+B)(m_d^2+C)+$$

$$+\frac{2}{m_d^2}(A_1+k^2)(A_1+B)^2]+2(m_n-m_p)[-9m_n(A_1+B)-$$

$$-3m_n(A_1+k^2)-6\frac{m_n}{m_d^2}(m_d^2+C)(A_1+B)]+$$

$$+4[-m_n^2(m_n^2+C)+3m_n^4+\frac{1}{m_d^2}(A_1+B)^2(m_n^2+C)-$$

$$-3\frac{m_n^2}{m_d^2}(A_1+B)^2-2\frac{m_n^2}{m_d^2}(A_1+k^2)(A_1+B)+\frac{2}{m_d^4}(A_1+B)^3(A_1+k^2)]-$$

$$-4(m_n-m_p)[-m_n(m_n^2+C)+3m_n^3-3\frac{m_n}{m_d^2}(A_1+B)^2-$$

$$-\frac{m_n}{m_d^2}(A_1+k^2)(A_1+B)+2\frac{m_n}{m_d^4}(A_1+B)^2(m_d^2+C)]+$$

$$+(m_n-m_p)^2[(m_n^2+C)+9m_n^2+\frac{4}{m_d^2}(A_1+B)(A_1+k^2)+$$

$$+\frac{4}{m_d^4}(A_1+B)^2(m_d^2+C)]\}\left(\frac{dz}{dx}\right)^2_{x=x_p} \qquad \text{(A.9)}$$

$$(Res_{11_C})^2=g^4_{12_C\,11_{Cn}}\frac{10(\hbar c)^2}{8sk^4}\{[2(A_2+B)(A_2+k^2)-m^2_{12_C}C]D^2+$$

$$+2m_n(2A_2+C)DE+(2m_n^2+C)E^2\}\left(\frac{dz}{dx}\right)^2_{x=x_{11_C}} \qquad \text{(A.10)}$$

where

$$D=m^2_{12_C}+C-\frac{1}{3}[2m_n(m_{11_C}-m_n)+m^2_{12_C}]+$$

$$+\frac{m_{11_C}+m_n}{3m_{11_C}}(A_2+B-m^2_{12_C})-\frac{2}{3m^2_{11_C}}(A_2+B-m^2_{12_C})^2-$$

$$-\frac{m_{11_C}-m_n}{3m_{11_C}}(m^2_{11_C}+C+A_2+k^2)$$

$$E=-\frac{2m^2_{12_C}}{3m_{11_C}}(m^2_{12_C}+C)-\frac{2}{3m_{11_C}}(A_2+B-m^2_{12_C})(A_2+B)+$$

$$+(m_{11_C}-m_n)(m^2_{12_C}+C)+\frac{m_{11_C}-m_n}{3}(2A_2+2B-m^2_{12_C})-$$

$$-\frac{2(m_{11_C}-m_n)}{3m^2_{11_C}}(A_2+B-m^2_{12_C})^2$$

$$(Res_{13_N})^2=g^4_{14_N\,13_{Nn}}\frac{10(\hbar c)^2}{24sk^4}\{[9m_n^2m^2_{14_N}+3m^2_{14_N}(m_n^2+C)+$$

$$+\frac{8}{m^2_{14_N}}(m^2_{14_N}+C)(A_3+B)^2+2(A_3+B)^2]-$$

$$-4[-m_n^2(A_3+k^2)+(A_3+B)(m_n^2+C)-2\frac{m_n^2}{m^2_{14_N}}(A_3+B)(m^2_{14_N}+C)+$$

$$+\frac{2}{m^2_{14_N}}(A_3+k^2)(A_3+B)^2]+2(m_n+m_{13_N})[-9m_n(A_3+B)-$$

$$-3m_n(A_3+k^2)-6\frac{m_n}{m^2_{14_N}}(m^2_{14_N}+C)(A_3+B)]+$$

$$+4[-m_n^2(m_n^2+C)+3m_n^4+\frac{1}{m_{14_N}^2}(A_3+B)^2(m_n^2+C)-$$

$$-3\frac{m_n^2}{m_{14_N}^2}(A_3+B)^2-2\frac{m_n^2}{m_{14_N}^2}(A_3+k^2)(A_3+B)+$$

$$+\frac{2}{m_{14_N}^4}(A_3+B)^3(A_3+k^2)]-$$

$$-4(m_n+m_{13_N})[-m_n(m_n^2+C)+3m_n^3-3\frac{m_n}{m_{14_N}^2}(A_3+B)^2-$$

$$-\frac{m_n}{m_{14_N}^2}(A_3+k^2)(A_3+B)+2\frac{m_n}{m_{14_N}^4}(A_3+B)^2(m_{14_N}^2+C)]+$$

$$+(m_n+m_{13_N})^2[(m_n^2+C)+9m_n^2+\frac{4}{m_{14_N}^2}(A_3+B)(A_3+k^2)+$$

$$+\frac{4}{m_{14_N}^4}(A_3+B)^2(m_{14_N}^2+C)]\}\left(\frac{dz}{dx}\right)^2_{x=x_{13_N}} \qquad (A.11)$$

where

$$A_1=\sqrt{(k^2+m_n^2)(k^2+m_d^2)} \qquad B=k^2x_{pole}$$

$$A_2=\sqrt{(k^2+m_n^2)(k^2+m_{12_C}^2)} \qquad C=k^2(1-x_{pole})$$

$$A_3=\sqrt{(k^2+m_n^2)(k^2+m_{14_N}^2)} \qquad x=\cos\theta$$

The formulas (A.9-11) are derived in the same way as in I from the differential cross section

$$\frac{d\sigma}{d\Omega}=\frac{1}{4\pi^2 s}\,\frac{1}{(2s_1+1)(2s_2+1)}\sum_{s_i,s_f}|M(s,t)|^2 \qquad (A.12)$$

(here s_1, s_2 are spins of the particles in initial state) substituting for $M(s,t)$ the corresponding pole contributions to the invariant nd, n ^{12}C and n ^{14}N amplitudes

$$M_p(s,t)=-\frac{m_n}{2}\frac{G^2_{dpn}}{u-m_p^2}\{\bar{u}(p_2)(\gamma\epsilon_1)(\gamma t_2)(\gamma\epsilon_2)u(p_1)- \qquad (A.13)$$

$$-2(p_1\epsilon_2)\bar{u}(p_2)(\gamma\epsilon_1)u(p_1)+(m_n-m_p)\bar{u}(p_2)(\gamma\epsilon_1)(\gamma\epsilon_2)u(p_1)\}$$

$$M_{^{11}C}(s,t)=+\frac{m_n}{2}\frac{G^2_{^{12}C\,^{11}Cn}}{u-m^2_{^{11}C}}\{[(t_1t_2)-\frac{m^2_{^{12}C}}{3}+$$

$$+\frac{2m_n}{3m_{^{11}C}}\{(p_1t_2)-m^2_{^{12}C}\}-\frac{2m_n(m_{^{11}C}-m_n)}{3}+$$

$$+\frac{2}{3m^2_{^{11}C}}\{(p_1t_2)-m^2_{^{12}C}\}\{(t_1t_2)-(p_1t_1)\}-$$

$$-\frac{m_{^{11}C}-m_n}{3m_{^{11}C}}\{(t_1t_2)+(p_1t_1)-(p_1t_2)+m^2_{^{12}C}\}]\bar{u}(p_2)(\gamma t_2)u(p_1)+$$

$$+[(m_{^{11}C}-m_n)(t_1t_2)-\frac{m^2_{^{12}C}}{3m_{^{11}C}}\{(t_1t_2)+(p_1t_1)\}+$$

$$+\{\frac{m^2_{^{12}C}}{3m_{^{11}C}}-\frac{2}{3m_{^{11}C}}(p_1t_2)\}\{(p_1t_2)-m^2_{^{12}C}\}+$$

$$+\frac{m_{^{11}C}-m_n}{3}\{2(p_1t_2)-m^2_{^{12}C}\}+\frac{2(m_{^{11}C}-m_n)}{3m^2_{^{11}C}}\{(p_1t_2)-m^2_{^{12}C}\}\times$$

$$\times\{(t_1t_2)-(p_1t_1)\}]\bar{u}(p_2)u(p_1)\} \qquad (A.14)$$

$$M_{^{13}N}(s,t)=-\frac{m_n}{2}\frac{G^2_{^{14}N^{13}Nn}}{u-m^2_{^{13}N}}\{\bar{u}(p_2)(\gamma\epsilon_1)(\gamma t_2)(\gamma\epsilon_2)u(p_1)-$$

$$-2(p_1\epsilon_2)\bar{u}(p_2)(\gamma\epsilon_1)u(p_1)+(m_n+m_{^{13}N})\bar{u}(p_2)(\gamma\epsilon_1)\times$$

$$\times(\gamma\epsilon_2)u(p_1)\} \qquad (A.15)$$

calculated by means of the Feynman rules from the diagrams shown in figs. 7a,b,c, respectively (the propagator of ^{11}C with spin value 3/2 was taken from the paper/16/) and taking the limit

$$\lim_{x\to x_{pole}}\left[\frac{z(x)-z(x_{pole})}{x-x_{pole}}\right]^2(x-x_{pole})^2\frac{d\sigma(x)}{d\Omega}\Big|_{pole}. \qquad (A.16)$$

The residues r (also given in tables 1,2,3) of spin averaged forward scattering amplitudes in the laboratory system are related to g^2 by means of the expressions:

$$r_{dpn}=\frac{g^2_{dpn}}{6m^2_d}\left\{\frac{m^2_n}{2}+\frac{3}{2}(m^2_d-m^2_p)+3m_n m_p-\frac{(m^2_n+m^2_d-m^2_p)^2}{2m^2_d}\right\} \qquad (A.17)$$

$$r_{^{12}C\,^{11}Cn}=\frac{g^2_{^{12}C\,^{11}Cn}}{m_{^{12}C}}\left\{\omega_0\left[\frac{(m_{^{12}C}\,\omega_0-m^2_{^{12}C})^2}{m^2_{^{11}C}}+\frac{m_n}{3}(m_{^{11}C}-m_n)-\right.\right.$$

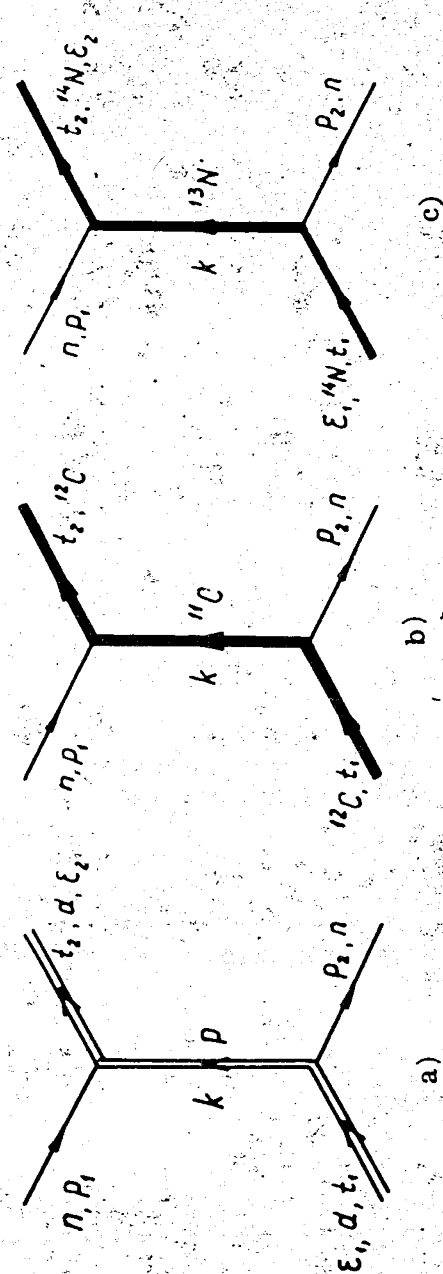


Fig. 7a,b,c. The proton, ^{11}C and ^{13}N pole contributions to the nd, n^{12}C and n^{14}N elastic scattering amplitudes, respectively.

$$- \frac{m_{12_C}\, m_n}{3 m_{11_C}}\, \omega_0 \Big] + \frac{m_n}{3} \Big[\frac{m_{12_C}}{m_{11_C}} \left(2\omega_0^2 - 3 m_{12_C}\, \omega_0 + 2 m_{12_C}^2 \right) -$$

$$-(m_{11_C} - m_n)(\omega_0 + m_{12_C}) - \frac{m_{12_C}\, m_n}{m_{11_C}^2} \left(\omega_0^2 - 2 m_{12_C}\, \omega_0 + m_{12_C}^2 \right) \Big] \Big] \Big\},$$

where

$$\omega_0 = \frac{m_{12_C}^2 + m_n^2 - m_{11_C}^2}{2 m_{12_C}} \qquad \text{(A.18)}$$

$$r_{^{14}N\, ^{13}N\, n} = \frac{g^2_{^{14}N\, ^{13}N n}}{6 m^2_{^{14}N}} \Big\{ \frac{m_n^2}{2} + \frac{3}{2} \left(m^2_{^{14}N} - m^2_{^{13}N} \right) - 3 m_n\, m_{^{13}N} -$$

$$- \frac{\left(m_n^2 + m^2_{^{14}N} - m^2_{^{13}N} \right)^2}{2 m_{^{14}N}} \Big\}. \qquad \text{(A.19)}$$

They are obtained by taking the limit

$$r = \lim_{\omega \to \omega_0} (\omega - \omega_0)\, f_{pole}(\omega, 0), \qquad \text{(A.20)}$$

where ω is the incident neutron laboratory total energy, ω_0 is the position of the corresponding pole in the ω-plane and $f_{pole}(\omega,0)$ is defined by the following expression

$$f_{pole}(\omega, 0) = \frac{1}{2\pi m_{target}} \frac{1}{(2s_1 + 1)(2s_2 + 1)} \sum_{s_1, s_f} M_{pole}(s, 0), \qquad \text{(A.21)}$$

where $M_{pole}(s,0)$ are pole contributions (A.13-15) to the invariant amplitudes of corresponding processes taken for $t=0$.

At the end we would like to indicate the discrepancy

between the definition of $f_{pole}(\omega, 0)$ by (A.21) and by (A.14) in our previous paper I for the deuteron pole. The expression (A.14) in I must be multiplied by the factor $\frac{\sqrt{s}}{m_{^3H}}$ which comes from the well known relation between the forward scattering amplitude in the laboratory system and the forward scattering amplitude in the centre of mass system

$$\frac{f_{lab}(s,0)}{\mathcal{Q}_{lab}} = \frac{f_{c.m.}(s,0)}{\mathcal{Q}_{c.m.}}, \qquad \text{(A.22)}$$

where

$$\mathcal{Q}_{c.m.} = \mathcal{Q}_{lab} \frac{m_{target}}{\sqrt{s}} \qquad \text{(A.23)}$$

After this correction our averaged value of the residue r ($r_{^3Hdn} \approx 0.38$) over the first three energies is exactly the same as that obtained in ref. /13/ ($r_{^3Hdn} = 0.382 \pm 0.040$) from the forward dispersion relations for the nd scattering. Then the corresponding spectroscopic factor takes the value $C^2_{^3Hdn} \approx 3.6$.

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