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AUTOMODEL SOLUTIONS FOR MATRIX ELEMENTS OF CURRENTS IN CONFORMAL INVARIANT THEORIES

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1. Introduotion

The asymptotio dilatation invariance found in deep-inelastio lepton-hadron scattering has stimulated the investigation of an automodel behaviour in local Quantum Field Theory.

This problem has been studied in a rigorous manner on the basis of a Jost-Lehmann-Dyson representation for the one-particle matrix elements of the electromagnetic current commutator /1/. Sufficient oonditions for automodel behaviour have been formulated in terms of the spectral functions which furthermore constitutes a direct conneotion between the automodel behaviour and the light cone singularity of the commutator /2/.

We remark, that the automodel asymptotics of the structure funotions in deep inelastic scattering corresponds in some sense to an absenoe of dimensional quantities in the behaviour of the commutator near the light cone /3/. This leads to the concept of asymptotic dilatation invariance at small distances.

In the last time it oan be found a rising interest in the extension of asymptotio scale invarianoe to oonformal invariance at small distanoes. The conformal group includes the Poinoare group, dilatations and special conformal transformations, leaving the light oone $x^2=0$ invariant /4/.

In partioular some field theories with massless particles lead to the consideration of conformal invariance /5/.

In the following we study the consequenoes of conformal invariance for matrix elements of currents.

It is well known that conformal invariance alone allows the construction of the complete two- and three-point Green functions /6/ .

$$G_2(x_1,x_2) = \langle 0| \varphi_1(x_1)\varphi_2(x_2)|0\rangle = \frac{C}{(x_1-x_2)^{2d}} \qquad (1.1)$$

$$d_1 = d_2 = d$$

$$G_3(x_1,x_2,x_3) = \langle 0|\varphi_1(x_1)\varphi_2(x_2)\varphi_3(x_3)|0\rangle \qquad (1.2)$$

$$= C'(x_1-x_2)^{-2d_{12}}(x_1-x_3)^{-2d_{13}}(x_2-x_3)^{-2d_{23}}$$

$$2d_{12} = d_1+d_2-d_3$$

$$2d_{13} = d_1+d_3-d_2 \qquad 2d_{23} = d_2+d_3-d_1$$

where d_i are the dimensions of the fields $\varphi_i(x)$, C and C' are arbitrary constants.

In a series of papers /7,9/ a theoretical scheme for the evaluation of higher Green functions has been given. We remark however, that even if this constructive programm could be carried out, there remains a gap between the idealized conformal invariant theory and the description of real physical processes. The difficulty consists in the lack of reduction formalism connecting the n-point Green functions of conformal invariant theories with physical matrix elements. This comes out in the following way. Assume

1. conformal invariance including an invariant nondegenerate vacuum state ;
2. existence of a complete set of asymptotic states with positive definite norms.

Then the vacuum expectation value of the massless field $\varphi(x)$ and its Heisenberg current $j(x) = \Box\varphi(x)$ vanishes, i.e., $\langle 0|\varphi(x)j(y)|0\rangle = 0$ because of $d_j = d_\varphi + 2 \neq d_\varphi$. Therefore the two-point function of $j(x)$ also vanishes: $\langle 0|j(x)j(y)|0\rangle = 0$ and consequently $j(x)|0\rangle = 0$ .Applying well-known theorems /8/ we conclude $j(x) \equiv 0$, so that the usual reduction integrals become trivial. In other words: Either there are no particle states or we have a free field theory, if conformal invariance holds exactly.

Obviously conformal symmetry is strongly broken and should be considered as an asymptotic symmetry[1)] of interactions at short distances /9,11/.

One may try to circumvent the above mentioned difficulties by taking into account the possibility of spontaneous breaking of conformal symmetry connected with the appearence of degenerate vacuum states /10, 11/.

On the other hand, it is interesting to study the consequences if we impose conformal invariance for particular quantities only, which are of direct physical interest, e.g., one-particle matrix elements of electromagnetic currents. By neglecting the conditions of conformal symmetry for the remaining parts of the theory (leptons and photons) one introduces some sort of symmetry breaking from the beginning /14/.

In the following we consider conformal invariance of matrix elements of currents taken between spinless zero mass one-particle states. We apply the usual transformation laws for the currents under infinitesimal conformal transformations. Furthermore we assume that the massless one-particle states belong to an irreducible representation of the conformal group /15/ which remains irreducible when restricted to the Poincare group.

In this manner we arrive at a system of linear differential equations of second order which expresses the consequences of strict conformal invariance. The class of allowed solutions

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1) In this connection it is interesting to compare the approach /12/ to asymptotic dilatation invariance based on the renormalization group /13/.

is strongly restricted by the conditions of polynomial boundedness, causality and spectrality. We get regular as well as nonregular solutions (logarithmic behaviour for $x^2 \to 0$ and $t \to 0$).

Similar equations have been considered /17/ for scattering amplitudes on the basis of low energy theorems for the gravitational field and the conservation of conformal currents.

2. Transformation Properties of States and Fields

In this section we derive the differential equations, which express the conformal invariance of the one-particle matrix elements of the current product. We remember that the conformal group containing the following transformations

a. inhomogeneous Lorentz transformation

b. dilatations $x_\mu' = \lambda x_\mu$

c. special conformal transformations $x_\mu' = (1 - 2cx + c^2x^2)^{-1}(x_\mu - c_\mu x^2)$

may also be characterized as the maximal symmetry group leaving the light cone invariant (in some sense).

The generators of the infinitesimal transformations $P_\mu, M_{\mu\nu}$ (Poincare group), D (dilatations) and K_μ (special conformal transformations) fulfill the following commutation relations /7,15/

$$
\begin{aligned}
&[P_\mu, M_{\nu\lambda}] = i(g_{\mu\nu} P_\lambda - g_{\mu\lambda} P_\nu) \,, \\
&[M_{\mu\nu}, M_{\lambda\rho}] = i(g_{\nu\rho} M_{\mu\lambda} - g_{\nu\lambda} M_{\mu\rho} - g_{\mu\rho} M_{\nu\lambda} + g_{\mu\lambda} M_{\nu\rho}) \,, \\
&[D, P_\mu] = -i P_\mu \,, \qquad [D, K_\mu] = i K_\mu \,, \\
&[K_\mu, P_\nu] = 2i(g_{\mu\nu} D + M_{\mu\nu}), \quad [K_\mu, M_{\nu\lambda}] = i(g_{\mu\nu} K_\lambda - g_{\mu\lambda} K_\nu) \,, \\
&[D, M_{\mu\nu}] = 0 \,, \qquad [K_\mu, K_\nu] = 0 \,. \qquad (2.1)
\end{aligned}
$$

Because of

$$e^{i\alpha D} P^2 e^{-i\alpha D} = e^{2\alpha} P^2 \qquad (2.2)$$

the spectrum of the mass operator P^2 either consists of one isolated point $P^2 = 0$ or covers the real positive axis. Here we consider the irreducible representation of the conformal group with $P^2 = 0$ and spin $s = 0$ obtained by an extension of the corresponding representation of the Poincare group. Following Mack and Todorov /15/ this representation is constructed with the help of a set of creation and destruction operators acting on a conformal invariant vacuum $|0\rangle$:

$$
\begin{aligned}
&|\vec{p}\rangle = a^+(\vec{p})|0\rangle \,, \qquad p_0 = |\vec{p}| \,, \\
&[a(\vec{p}), a^+(\vec{p}\,')] = 2|\vec{p}|\, \delta(\vec{p} - \vec{p}\,') \qquad (2.3)
\end{aligned}
$$

and satisfying specific commutation relations with the generators G of the group

$$
\begin{aligned}
&[a^+(\vec{p}), G] = O_G(\vec{p}, \nabla_p)\, a^+(\vec{p}) \,, \\
&[a(\vec{p}), G] = \overline{O}_G(\vec{p}, \nabla_p)\, a(\vec{p}) \,. \qquad (2.4)
\end{aligned}
$$

Here the differential operators O_G; $\overline{O}_G$ corresponding to $G = P_\mu, M_{\mu\nu}, D, K_\mu$ are defined by

$$
\begin{aligned}
\pm P_\mu &: \quad p_\mu = (|\vec{p}|, \vec{p}) \\
M_{ik} &: \quad i(p_i \nabla_k - p_k \nabla_i) \\
M_{0i} &: \quad i|\vec{p}|\nabla_i \\
D &: \quad i(1 + \vec{p}\vec{\nabla}) \\
\pm K_i &: \quad -p_i \Delta_p - 2(1 + \vec{p}\vec{\nabla})\nabla_i \\
\pm K_0 &: \quad -|\vec{p}|\Delta_p \,. \qquad (2.5)
\end{aligned}
$$

The upper or lower sign corresponds to O_G or $\overline{O}_G$, respectively ($\nabla_K \equiv \frac{\partial}{\partial p^K}$).

In this way we obtain the action of the generators on the state vectors

$$G|\vec{p}\rangle = O_G(\vec{p},\nabla)|p\rangle \quad . \tag{2.6}$$

For the investigation of matrix elements of currents or fields we need their commutation relations with group generators. Let be $\chi_G(x)$ a field transforming according to a representation of the Poincare group, characterized by some $D(j_1, j_2)$ of SL(2,C):

$$[\chi_\sigma(x), P_\mu] = i\partial_\mu \chi_\sigma(x), \tag{2.7}$$

$$[\chi_\sigma(x), M_{\mu\nu}] = \{i\delta_{\sigma\sigma'}(x_\mu\partial_\nu - x_\nu\partial_\mu) + (\Sigma_{\mu\nu})_{\sigma\sigma'}\}\chi_{\sigma'}(x),$$

where $\Sigma_{\mu\nu}$ are the generators in the $D(j_1, j_2)$ representation, e.g., $\Sigma_{\mu\nu} = 0$ for the scalar case, $(\Sigma_{\mu\nu})_{\alpha\varrho} = i(g_{\mu\alpha}g_{\nu\varrho} - g_{\mu\varrho}g_{\nu\alpha})$ in the vector oase. As usual /7/ we take the oommutation relations between field and the generators as

$$[\chi_\sigma(x), D] = -i(d + x^\nu\partial_\nu)\chi_\sigma(x), \tag{2.8}$$

$$[\chi_\sigma, K_\mu] = -i\{(2dx_\mu + 2x_\mu x^\nu\partial_\nu - x^2\partial_\mu)\delta_{\sigma\sigma'} + 2ix^\nu(\Sigma_{\mu\nu})_{\sigma\sigma'}\}\chi_{\sigma'}(x)$$

corresponding to the choice $K_\mu = 0$ /15/. Here d is the dimension of the field. For the above introduced massless fields which allow a particle interpretation in the sense of the second quantization d is oonneoted with the spin of the field by the relation $d = 1 + j$, e.g., $d = 1$ for soalar bosons and $d = 3/2$ for fermions. If we write the commutation relations eq. (2.8) in the form

$$[\chi(x), G] = O_G(x, \partial_x)\chi(x), \tag{2.9}$$

the consequences of conformal invariance of the matrix elements $\langle p|\chi(x)\chi(y)|q\rangle$ are expressed by a set of partial differential equations

$$[\overline{O}_G(\vec{p},\nabla_p) + O_G(\vec{q},\nabla_q) + O_G(x,\partial_x) + O_G(y,\partial_y)] \times$$

$$\langle p|\chi(x)\chi(y)|q\rangle = 0, \tag{2.10}$$

where $G = P_\mu, M_{\mu\nu}, D, K_\mu$.

3. Solutions of the Differential Equations for the Matrix Elements of Currents

Here we will study the consequences of dilatation and oonformal invariance for matrix elements of products of two local soalar or vector ourrents.

a. Scalar Currents

Let us begin with matrix element of scalar ourrents between massless one-particle states

$$f(x,p,q) = \langle p|j(x)j(0)|q\rangle \tag{3.1}$$

which satisfies

$$f^*(x,p,q) = e^{-ix(p-q)}f(-x,q,p) \tag{3.2}$$

for hermitean currents.

Though we are interested particularly in the forward case, the equations must be solved in the general oase $p \neq q$ to inoorporate all available information.

From eq. (2.10) we get

$$\left[2d + x^\nu \partial_\nu - (1+\vec{p}\vec{\nabla}_p) - (1+\vec{q}\vec{\nabla}_q)\right] f(x,p,q) = 0 \qquad (3.3)$$

for $G = D$,

$$\left[2dx_i + 2x_i x^\nu \partial_\nu - x^2 \partial_i + i\left(p_i \Delta_p + 2\vec{p}\vec{\nabla}_p \nabla_{p_i} + 2\nabla_{p_i}\right) - i\left(q_i \Delta_q + 2\vec{q}\vec{\nabla}_q \nabla_{q_i} + 2\nabla_{q_i}\right)\right] f(x,p,q) = 0 \qquad (3.4)$$

for $G = K_i$,

and

$$\left[i|\vec{p}|\Delta_p - i|\vec{q}|\Delta_q + 2dx_0 + 2x_0 x^\nu \partial_\nu - x^2 \partial_0\right] f = 0 \qquad (3.5)$$

for $G = K_0$.

Remark that $p_0 = |\vec{p}|$ and $q_0 = |\vec{q}|$ which must be taken into account.

The dimension d of the current remains arbitrary in this connection.

Because of Lorentz invariance and the normalization (2.3) the function $f(x,p,q)$ depends on invariant variables only which we choose as x^2 and the dimensionless combinations

$$z_1 = x(p+q)\,, \quad z_2 = x(p-q)\,, \quad \omega = x^2/pq)\,.$$

It is convenient to use the ansatz

$$f(x,p,q) = (-x^2 - ix_0\varepsilon)^{1-d}\, e^{\frac{iz_2}{2}}\, \phi(z_1,z_2,\omega) \qquad (3.6)$$

which solves the equation of dilatation invariance (3.3).
The function ϕ is not determined by dilation invariance.
From general principles we know the following properties

$\phi^*(z_1,z_2,\omega) = \phi(-z_1,z_2,\omega)$ (hermiticity of current)

and

(1) $\phi(z_1,z_2,\omega)$ is an entire function of z_1, z_2,

(2) $\phi(z_1,z_2,\omega) = \phi(-z_1,-z_2,\omega)$.

The last properties reflecting the causality of the current will be derived in the next section from a Dyson representation of f.

Let us now insert the ansatz (3.6) into equations (3.4,5). As result we obtain a set of independent differential equations for ϕ:

$$\begin{aligned}
&z_2\left[\phi_{11}+\phi_{22}+\tfrac{1}{4}\phi\right] + 2z_1\phi_{12} + 4\phi_2 + 4\omega\phi_{23} - 2z_2\phi_3 - 2z_2\omega\phi_{33} = 0 \\
&z_1 z_2\left[\phi_{11}+\phi_{22}+\tfrac{1}{4}\phi\right] + 2z_1^2\phi_{12} + 2z_1\phi_2 + 2z_1\omega\phi_{23} + 2z_2\omega\phi_{13} - 2\omega\phi_{12} = 0 \qquad (3.7) \\
&z_2^2\left[\phi_{11}+\phi_{22}+\tfrac{1}{4}\phi\right] + 2z_1z_2\phi_{12} + 2z_2\phi_2 + 2z_2\omega\phi_{23} + 2z_1\omega\phi_{13} \\
&\quad + 2\omega\phi_3 + 2\omega^2\phi_{33} + \omega\left[\phi_{11}+\phi_{22}+\tfrac{1}{4}\phi\right] = 0\,,
\end{aligned}$$

where

$$\phi_1 = \frac{\partial\phi}{\partial z_1}\,, \quad \phi_2 = \frac{\partial\phi}{\partial z_2}\,, \quad \phi_3 = \frac{\partial\phi}{\partial\omega}\,\ldots$$

It is useful to introduce a new set of variables

$$z_1\,, \quad z_2\,, \quad z_3 = z_1^2 - z_2^2 - 2\omega$$

and to write $\phi(z_1,z_2,\omega) = H(z_1,z_2,z_3)$. After some algebra the system (3.7) transforms into

$$H_{11} + H_{22} + \tfrac{1}{4}H - 4z_3 H_{33} - 4H_3 = -\frac{4z_2}{z_3 + z_2^2 - z_1^2}\, H_2 \qquad (3.8)$$

$$H_{12} = \frac{2z_1}{z_3 + z_2^2 - z_1^2} H_2 \quad ,$$

$$H_{23} = -\frac{1}{z_3 + z_2^2 - z_1^2} H_2 \quad . \qquad (3.8)$$

The general solution of the last two equations

$$H_2(z_1, z_2, z_3) = C(z_2)(z_1^2 - z_2^2 - z_3)^{-1}$$

will be inserted into the first equation in order to determine

$$C(z_2) = \alpha \sin\frac{z_2}{2} + \beta \cos\frac{z_2}{2} \quad .$$

From the symmetry property 2. follows $\beta = 0$ so that

$$H_2(z_1, z_2, z_3) = \alpha \frac{\sin\frac{z_2}{2}}{z_1^2 - z_2^2 - z_3} \quad .$$

Therefore we have

$$H(z_1, z_2, z_3) = K(z_1, z_3) + \frac{\alpha}{2}\int_{-\infty}^{+\infty} dt\, \varepsilon(z_2 - t)\frac{\sin\frac{t}{2}}{z_1^2 - z_3 - t^2} \quad . \qquad (3.9)$$

The first eq. (3.8) puts restrictions on the function K :

$$K_{11} + \frac{1}{4}K - 4z_3 K_{33} - 4K_3 = 0 \quad . \qquad (3.10)$$

The integral in expr. (3.9) represents a function with a logarithmic singularity at $2\omega = z_1^2 - z_2^2 - z_3 = 0$, i.e.

$$\int_{-\infty}^{+\infty} dt\, \varepsilon(z_2 - t)\frac{\sin t/2}{z_1^2 - z_3 - t^2}\bigg/_{\omega \sim 0} \sim \frac{\sin\frac{z_2}{2}}{z_2}\log\omega \quad . \qquad (3.9')$$

That means, all solutions with $\alpha \neq 0$ are singular in the limit of forward direction $p = q, \omega = 0$. Let us turn to the solution of eq. (3.10) for K. Applying the method of separation of variables $K(z_1, z_3) = \chi(z_1)\eta(z_3)$ we obtain ordinary second order differential equations for $\chi(z_1)$ and $\eta(z_3)$:

$$\chi'' + \left(\frac{1}{4} + \gamma\right)\chi = 0$$

$$z_3\eta'' + \eta' + \frac{\gamma}{4}\eta = 0 \quad , \qquad (3.11)$$

γ is the constant of separation.

The solutions with the properties (1) and (2) are

$$\chi(z_1) = \cos z_1\sqrt{\gamma + \frac{1}{4}} \quad , \qquad (3.12)$$

$$\eta(z_3) = J_0\left(\sqrt{\gamma z_3}\right) \quad .$$

As it will be shown later, the conditions of polynomial boundedness, spectrality and causality restrict γ to the value $\gamma = 0$. In terms of generalized functions the regular solutions for the matrix element of current products can be written $(\alpha = 0)$

$$f(x, p, q) = (-x^2 - ix_0\varepsilon)^{1-d}\, e^{\frac{i}{2}x(p+q)} \times \qquad (3.13)$$

$$\times \int d\gamma\, \sigma(\gamma)\cos\left[x(p+q)\sqrt{\gamma + \tfrac{1}{4}}\right] J_0\left[\sqrt{\gamma\left(4(p+q)x\right) - 2(p+q)x^2}\right] ,$$

where $\sigma(\gamma)$ is localized at the point $\gamma = 0$

$$\sigma(\gamma) = \sum_{n=0}^{\infty} a_n \, \delta^{(n)}(\gamma) \quad . \qquad (3.14)$$

As examples we give two special solutions :

1. $\sigma = \delta(\gamma)$

$$\langle p | j(x)\, j(0) | q \rangle = (-x^2 - i x_0 \varepsilon)^{1-d}\, e^{\frac{i}{2} x(p-q)} \cos \frac{x(p+q)}{2} \quad . \qquad (3.15)$$

2. $\sigma = \delta'(\gamma)$

$$\langle p | j(x)\, j(0) | q \rangle = (-x^2 - i x_0 \varepsilon)^{1-d}\, e^{\frac{i}{2} x(p-q)} \times \qquad (3.16)$$

$$\times \left[\cos \frac{x(p+q)}{2} \left((px)(qx) - \tfrac{1}{2}(pq)x^2 \right) + x(p+q) \sin \frac{x(p+q)}{2} \right] .$$

The first example may be understood for $d = 2$ as a Born term corresponding to the ansatz (*see* Fig. 1)

$$j(x) = \,:\, \varphi^2(x) \,:\, \quad , \quad d_\varphi = 1 \, .$$

For $d = 3$, however, the matrix element corresponds to the diagram of Fig.2, or equivalently

$$j(x) = \,:\, \varphi^3(x) \,:\, \quad , \quad d_\varphi = 1 \, .$$

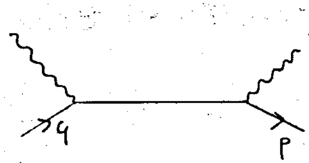

Fig.1

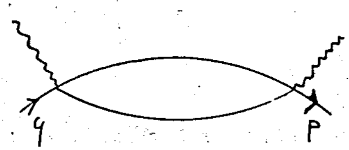

Fig.2

It should be remarked that the spectrality condition restricts the solutions very strongly. For instance, the regular solution

$$K(z_1, z_3) = \frac{\sin \frac{1}{2}\sqrt{v}}{\sqrt{v}} \quad , \quad v = z_1^2 - z_3$$

satisfying the conditions (1) and (2) violates the spectrality condition. This follows from a comparison with the Dyson representation (see the next section).

Let us look for another form of the singular solution, which clearly exhibits its spectral properties. We know that there exists such a solution depending on z_2 and ω only. Therefore with $\phi = \bar{\phi}(z_2, \omega)$ the system (3.7) becomes

(3.17)

$$\bar{\phi}_{22} + \tfrac{1}{4}\bar{\phi} + 2\bar{\phi}_3 + 2\omega\bar{\phi}_{33} = 0 \; ,$$

$$z_2\left(\bar{\phi}_{22} + \tfrac{1}{4}\bar{\phi}\right) + 2\bar{\phi}_2 + 2\omega\bar{\phi}_{23} = 0 \quad . \qquad (3.17)$$

Using the Fourier transform

$$\bar{\phi}(z_2, \omega) = \int d\mu \; e^{\frac{1}{2}\mu z_2} g(\omega, \mu)$$

the first equation (3.17) is equivalent to

$$\frac{\partial^2 g}{\partial \omega^2} + \frac{1}{\omega}\frac{\partial g}{\partial \omega} + \frac{1-\mu^2}{8\omega} g = 0$$

with the solution

$$g(\omega, \mu) = K_0\left(\sqrt{-\tfrac{1}{2}\omega(1-\mu^2)}\right) \psi(\mu) \; .$$

This is compatible with the second equation if $\psi(\mu) = \Theta(1-\mu^2)$.

Therefore we have

$$\bar{\phi}(z_2, \omega) = \int_{-1}^{+1} d\mu \; e^{\frac{1}{2}\mu z_2} K_0\left(\sqrt{-\tfrac{1}{2}\omega(1-\mu^2)}\right) \quad . \qquad (3.18)$$

This represents an analytic function in the variable ω with a cut along the real axis. At $\omega=0$ it has a logarithmic singularity

$$\bar{\phi}(z_2,\omega)/_{\omega\sim 0} \sim \frac{1}{z_2}\sin\frac{z_2}{2}\,\log(-\omega) .$$

In section 4 it will be shown, that this solution fulfils the conditions from causality and spectrality, independent of the dimension d.

b. Vector Currents

More important is the investigation of matrix elements of conserved vector currents, e.g. the electromagnetic current

$$M^{\mu\mu'}(x,x',p,q)=\langle p|j^{\mu}(x)\,j^{\mu'}(x')|q\rangle . \qquad (3.19)$$

Let us begin with a kinematical decomposition of $M^{\mu\mu'}$ which takes into account current conservation as well as conformal transformation properties:

$$M^{\mu\mu'}(x,x',p,q)=\partial_\nu\partial'_{\nu'}\{[L^{\mu\mu'}(x,x')L^{\nu\nu'}(x,x')-L^{\nu\mu'}(x,x')L^{\mu\nu'}(x,x')]M_1(x,x',p,q)\}$$
$$+(\delta_\nu{}^\mu\Box-\partial^\mu\partial_\nu)(\delta^{\mu'}_{\nu'}\Box'-\partial'^{\mu'}\partial'_{\nu'})\{L^{\nu\nu'}(x,x')M_2(x,x',p,q)\} \qquad (3.20)$$

with $\Box=\partial^\mu\partial^\nu g_{\mu\nu}$.

This decomposition has been derived in /18/ by looking for a bilocal tensor

$$L^{\alpha\alpha'}(x,x')=\partial^\alpha\partial'^{\alpha'}\log(x-x')^2 \qquad (3.21)$$

which is form invariant with regard to conformal transformations and allows contractions between tensors at different space-time points.

We are especially interested in the transformation properties of $L^{\alpha\alpha'}(x,x')$ under infinitesimal conformal transformations. Direct calculation gives

$$\{\delta^\alpha_\rho[k_\mu(x,d,s=1)]^{\alpha'}_{\rho'}+\delta^{\alpha'}_{\rho'}[k_\mu(x',d,s=1)]^{\alpha}_{\rho}\}L^{\rho\rho'}(x,x')=$$
$$=L^{\alpha\alpha'}(x,x')\{k_\mu(x,d-1,s=0)+k_\mu(x',d-1,s=0)\} \qquad (3.22)$$

with (compare (2.8))

$$[k_\mu(x,d,s=1)]^\alpha_\rho=-i\{(2dx_\mu+2x_\mu x^\nu\partial_\nu-x^2\partial_\mu)\delta^\alpha_\rho$$
$$+2x^\nu(\delta^\alpha_\mu g_{\nu\rho}-\delta^\alpha_\nu g_{\mu\rho})\}, \qquad (3.23)$$

$$k_\mu(x,d,s=0)=-i\{2dx_\mu+2x_\mu x^\nu\partial_\nu-x^2\partial_\mu\} .$$

Let us shortly write the decomposition (3.20)

$$M^{\mu\mu'}(x,x',p,q)=\sum_{i=1,2}C_i^{\mu\mu'}\cdot M_i$$

It turns out that for d=3 the action of the differential operator $O_{k_\mu}(x,\partial_x)$ on the bilocal operator is especially simple

$$\{\delta^{\alpha'}_{\rho'}[k_\mu(x,d=3,s=1)]^\alpha_\rho+\delta^\alpha_\rho[k_\mu(x',d=3,s=1)]^{\alpha'}_{\rho'}\}C_i^{\rho\rho'}(x,x')=$$
$$=C_i^{\alpha\alpha'}(x,x')\{k_\mu(x,d=0,s=0)+k_\mu(x',d=0,s=0)\} . \qquad (3.24)$$

The choice d=3 is quite natural because this is just the dimension of a conserved vector current. This allows the evaluation of the equations (2.10) for the matrix elements of conserved currents in the form

$$[\bar{O}_{K\mu}(\vec{p},\nabla_p)+O_{K\mu}(\vec{q},\nabla_q)+O_{K\mu}(x,\partial_x)+O_{K\mu}(x',\partial_{x'})]M^{\rho\rho'}(x,x',p,q)=$$

$$=\sum_i\{\delta^{\alpha}_{\rho}\,\delta^{\alpha'}_{\rho'}\,[\bar{O}_{K\mu}(\vec{p},\nabla_p)+O_{K\mu}(\vec{q},\nabla_q)]+\delta^{\alpha'}_{\rho'}[k_\mu(x,d=3,s=1)]^{\alpha}_{\rho}+$$

$$+\;\delta^{\alpha}_{\rho}[k_\mu(x',d=3,s=1)]^{\alpha'}_{\rho'}\}\,C_i^{\rho\rho'}M_i(x,x',p,q)$$

$$=\sum_i C_i^{\alpha\alpha'}(x,x')\{\bar{O}_{K\mu}(\vec{p},\nabla)+O_{K\mu}(\vec{q},\nabla_q)+ \qquad (3.25)$$

$$+k_\mu(x,d=0,s=0)+k_\mu(x',d=0,s=0)\}M_i(x,x',p,q)=0.$$

The linear independence of the kinematical tensors $C_i^{\alpha\alpha'}(x,x')$ leads to our previous equations for the scalar functions M_i where the parameter d is now fixed to the value zero. In a similar way we write the equation of dilatation invariance as

$$[\bar{O}_D(\vec{p},\nabla_p)+O_D(\vec{q},\nabla_q)+O_D(x,\partial_x)+O_D(x',\partial_{x'})]\,M^{\rho\rho'}(x,x',p,q)=$$

$$=\sum_i\{\bar{O}_D(\vec{p},\nabla_p)+O(\vec{q},\nabla_q)+D(x,d=3)+D(x',d=3)\}C_i^{\rho\rho'}M_i \qquad (3.26)$$

$$=\sum_i C_i^{\rho\rho'}(x,x')\{\bar{O}_D(\vec{p},\nabla_p)+O(\vec{q},\nabla_q)+D(x,d=0)+D(x',d=0)\}M_i=0,$$

where $D(x,d)=-i(d+x^\nu\partial_\nu)$.

Again the scalar functions M_i have to fulfill the familiar eq. (3.3) with $d=0$.

Therefore the conformal invariant matrix elements of conserved vector currents can be reconstructed from the already known expressions (3.13, 18) for the matrix elements of scalar currents.

4. Restrictions from Causality and Spectrality

In order to study the restrictions from causality and spectrality we shall at first establish a Dyson representation for matrix elements of the product of two currents and afterwards investigate its general consequences.

For an illustration we consider at the beginning the diagonal case. Taking into account causality and spectrality the matrix element of the current commutator between massless states can be represented by a DJL representation /16/

$$\tilde{F}(x,p)=\langle p|[j(x),j(0)]|p\rangle \qquad (4.1)$$

$$=\frac{1}{2\pi}\int_{-1}^{+1}d\alpha\, e^{i\alpha(px)}\int_0^\infty d\lambda^2\,\psi(\alpha,\lambda^2)\,D(x,\lambda^2)$$

with

$$D(x,\lambda^2)=\frac{1}{i(2\pi)^3}\int dq\,\varepsilon(q_0)\,\delta(q^2-\lambda^2)\,e^{iqx}$$

If the matrix element is dilatation invariant

$$\tilde{F}(x,p)=t^{2l}\,\tilde{F}(tx,t^{-1}p) \qquad (4.2)$$

the spectral function is a homogeneous generalized function of order $l-2$ with respect to the variable λ^2

$$\psi(\alpha,t\lambda^2)=t^{l-2}\,\psi(\alpha,\lambda^2),\quad t>0. \qquad (4.3)$$

This leads immediately to

$$\Psi(\alpha,\lambda^2) = \Psi_0(\alpha)\, f_\beta(\lambda^2) \quad ,$$

$$\tilde{F}(x,P) = \frac{1}{2\pi}\, \varepsilon(px)\, G(px)\, f_{-\beta}(x^2) \qquad (4.4)$$

with /20/

$$f_\beta(t) = \begin{cases} \dfrac{t_+^{\beta-1}}{\Gamma(\beta)} & , \ \beta > 0 \\ f'_{\beta+1}(t) & , \ \beta < 0 \ . \end{cases} \qquad (4.5)$$

From the DJL representation follows, that the function

$$G(px) = \frac{1}{2\pi} \int_{-1}^{+1} d\alpha\, e^{i\alpha(px)}\, \Psi_0(\alpha) \qquad (4.6)$$

is an entire even function of px, if $\Psi_0(\alpha)$ has been chosen even.

In general however, we must consider the non-diagonal matrix element of the commutator

$$\tilde{F}(x,P,\Delta) = \langle p | [\, j(\tfrac{x}{2}),\, j(-\tfrac{x}{2})\,] | q \rangle$$

$$= \tilde{M}(x,P,\Delta) - \tilde{M}(-x,P,\Delta) \ , \qquad (4.7)$$

where

$$\tilde{M}(x,P,\Delta) = \langle p |\, j(\tfrac{x}{2})\, j(-\tfrac{x}{2})\, | q \rangle \ ,$$

$$P = \tfrac{1}{2}(p+q) \ , \quad \Delta = p-q \ , \quad p^2 = q^2 = 0 \qquad (4.8)$$

Because of $P^2>0$ a general DJL-representation can be written down

$$F(Q,P,\Delta) = \int dx\, e^{iQx}\, \tilde{F}(x,P,\Delta) \qquad (4.9)$$

$$= \int du\, d\lambda^2\, \varepsilon(Q_0-u_0)\, \delta((Q-u)^2-\lambda^2)\, \Psi(u,P,\Delta,\lambda^2)$$

The spectral function can be chosen as an even function with respect to u_σ in any case. Its support is restricted by

$$(u,\lambda^2) : \begin{cases} 0 \le \lambda^2 < \infty & , \\ P^2+u^2 \ge 2|Pu| & , \ |u_0| \le P_0 \ . \end{cases} \qquad (4.10)$$

Hermiticity of the currents implies the symmetry relation

$$\Psi^*(u,P,\Delta,\lambda^2) = \Psi(u,P,-\Delta,\lambda^2) .$$

Let us introduce the positive and negative frequency parts of representation (4.9)

$$F_\pm(Q,P,\Delta) = \int du\, d\lambda^2\, \Theta(\pm(Q_0-u_0))\, \delta((Q-u)^2-\lambda^2)\, \Psi(u,P,\Delta,\lambda^2) \qquad (4.11)$$

and their Fourier transform

$$\tilde{F}_\pm(x,P,\Delta) = \frac{1}{2\pi} \int du \int d\lambda^2\, e^{iux}\, D_\pm(x,\lambda^2)\, \Psi(u,P,\Delta,\lambda^2) \qquad (4.12)$$

with

$$D_\pm(x,\lambda^2) = \frac{1}{i(2\pi)^3} \int dq\, e^{iqx}\, \Theta(\pm q_0)\, \delta(q^2-\lambda^2)$$

$$= \frac{i\lambda^2}{4\pi^2}\, \frac{K_1\left(\sqrt{(-x^2 \pm i x_0 \varepsilon)\lambda^2}\right)}{\sqrt{(-x^2 \pm i x_0 \varepsilon)\lambda^2}} \ .$$

Our task consists in the confirmation of a DJL representation for the matrix elements $\tilde{M}(x,P,\Delta)$. For this reason we compare the following equation

$$F(Q,P,\Delta) = F_+(Q,P,\Delta) - F_-(Q,P,\Delta)$$
$$= F_+(Q,P,\Delta) - F_+(-Q,P,\Delta) \quad (4.13)$$

with the Fourier transform of eq. (4.7)

$$F(Q,P,\Delta) = M(Q,P,\Delta) - M(-Q,P,\Delta) . \quad (4.14)$$

Obviously the difference

$$Z(Q,P,\Delta) = M(Q,P,\Delta) - F_+(Q,P,\Delta) \quad (4.15)$$

is an even function with respeot to Q_μ . The support of Z is contained in the region

$$(Q+P)^2 \geq 0 \quad , \quad Q_0 + P_0 \geq 0$$

being the common support of M and F_+ . Because of the just mentioned symmetry it is finally restricted to

$$Q^2 + P^2 \geq 2|QP| \quad , \quad |Q_0| \leq P_0 \quad .$$

Therefore the Fourier transform $\widetilde{Z}(x,P,\Delta)$ is an even entire function of x_μ . Consequently the matrix element of the product of two soalar currents oan be represented as

$$f(x,p,q) = \langle p|j(x)j(0)|q\rangle = e^{\frac{i}{2}x(p+q)}\left\{\widetilde{F}_+(x,P,\Delta) + \widetilde{Z}(x,P,\Delta)\right\} \quad (4.16)$$

The function $\widetilde{Z}$ does not effeot any one of our conclusions, so we omit it. Moreover it turns out that our solutions oan be represented by the term $\widetilde{F}_+$ alone.

Because we are interested in dilatation invariant matrix

elements only, we have to deal with a restricted class of spectral functions. The simplest type of such spectral functions is

$$\mathcal{Y}(u,P,\Delta,\lambda^2) = f_\rho(\lambda^2)\,\mathcal{Y}_0(u,P,\Delta) , \quad (4.17)$$

where the homogeneous distribution f_ρ has already been defined in eq. (4.5) and $\mathcal{Y}_0$ is a homogeneous function

$$\mathcal{Y}_0(tu, tP, t\Delta) = t^{-4}\,\mathcal{Y}_0(u,P,\Delta) .$$

Insertion of this spectral function into the DJL representation leads to

$$f(x,p,q) = C_d\,(-x^2 - ix_0\epsilon)^{1-d}\, e^{\frac{i}{2}x(p+q)}\,\phi(x,P,\Delta) \quad (4.18)$$
$$d = \rho+2 \, , \quad c_d = (d-2)\,4^{d-1}$$

with

$$\phi(x,P,\Delta) = \frac{1}{(2\pi)^3}\int du\, e^{iux}\,\mathcal{Y}_0(u,P,\Delta) \quad (4.19)$$
$$(\; P^2 + u^2 \geq 2|Pu| \;,\; |u_0| \leq P_0 \;) \quad .$$

From this representation it follows at onoe that $\phi(x,P,\Delta)$ is an even entire function in the variable x_μ which finally leads to the earlier used properties (1),(2) . A further investigation of ϕ can be done most appropriately in the Breit system

$$\vec{P} = 0 \, , \quad P_0 = \Delta_3 = p \quad , \quad \Delta_0 = 0 \; .$$

The expressions for the earlier used invariant variables are

$$z_1 = 2px_0 \, , \quad z_2 = 2px_3 \, , \quad z_3 = 4p^2 x_\perp^2 \; .$$

The speotral function can now be written with the dimensionless variable $\mu_6 = p^{-1}u_6$ as

$$\mathcal{Y}_0 = \mathcal{Y}_0(u_0, u_3, \vec{u}_\perp, p) = p^{-4}\,\mathcal{Y}_0(\mu_0, \mu_3, \mu_\perp) \quad (4.20)$$

In this manner the integral (4.19) takes the form

$$\Phi(x,P,\Delta) = H(z_1,z_2,z_3) \tag{4.21}$$

$$= \frac{1}{8\pi^2}\int\limits_{|\mu_0|+\sqrt{\mu_3^2+\mu_\perp^2}\le 1} d\mu_0\, d\mu_3\, d\mu_\perp^2\, e^{\frac{1}{2}(\mu_0 z_1-\mu_3 z_2)} J_0\!\left(\frac{\mu_\perp\sqrt{z_3}}{2}\right)\Psi_0(\mu).$$

The range of integration is shown in Fig.3

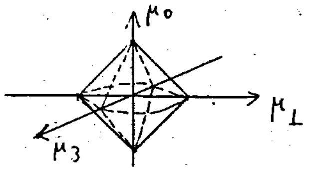

Fig.3

It is now easy to rederive our regular solutions (3.13) in the framework of the Dyson representation. Remember $\frac{\partial H}{\partial z_2}=0$ and therefore

$$\Psi_0(\mu) = \delta(\mu_3)\, \chi(\mu_0,\mu_\perp)$$

and

$$H = K(z_1,z_3) = \frac{1}{8\pi^2}\int d\mu_0\, d\mu_\perp^2\, e^{\frac{1}{2}\mu_0 z_1} J_0\!\left(\frac{\mu_\perp\sqrt{z_3}}{2}\right)\chi(\mu_0,\mu_\perp) \tag{4.22}$$

Applying eq. (3.10) for K we get an equation for the spectral function:

$$(\mu_0^2-\mu_\perp^2-1)\,\chi(\mu_0,\mu_\perp)=0$$

with the solution

$$\chi(\mu_0,\mu_\perp) = \delta(\mu_0^2-\mu_\perp^2-1)\, G(\mu_\perp^2).$$

The support condition (i.e. ,spectrality properties) localizes the function $G(\mu_\perp^2)$ to the point $\mu_\perp^2=0$: $G(\mu_\perp^2)=\sum b_n\,\delta^{(n)}(\mu_\perp^2)$.

As a result we obtain

$$K(z_1,z_3) = \int \frac{d\mu_\perp^2}{\sqrt{1+\mu_\perp^2}}\cos\!\left[\frac{z_1}{2}\sqrt{1+\mu_\perp^2}\right] J_0\!\left(\frac{\mu_\perp\sqrt{z_3}}{2}\right) G(\mu_\perp^2) \tag{4.23}$$

which coincides with the already known solution if we substitute $\mu_\perp^2/4 \to \gamma$. Let us furthermore discuss the automodel behaviour of the conformal invariant matrix elements in the Bjorken region

$$\nu = 2PQ \to +\infty \quad , \quad Q^2\to\infty$$

$$\xi = -\frac{Q^2}{\nu} \quad , \quad \vec{n} = \frac{\vec{Q}}{|\vec{Q}|} \quad , \quad \rho = P_0\cdot\tfrac{1}{2}|\vec{\Delta}| \qquad fix .$$

For spectral functions of the type (4.17) an application of the well known technics /1/ leads to the result

$$F(Q,P,\Delta) \sim \frac{\nu^K}{\Gamma(1+K)}\int\limits_{|\mu_0|+|\vec{\mu}|\le 1} d\mu_0\, d\vec{\mu}\,(\vec{n}\vec{\mu}+\mu_0-\xi)_+^K\, \Psi_0(\mu_0,\mu_3,\mu_\perp) \tag{4.24}$$

Especially for the regular solutions described by the spectral function

$$\Psi = \rho^{-4} f_\beta(d^2)\,\delta(\mu_3)\,\delta(\mu_0^2-\mu_\perp^2-1)\sum c_n\,\delta^{(n)}(\mu_\perp^2)$$
$$\beta = d-2$$

we get

$$F(\nu,\xi,\vec{n}) \sim \frac{\nu^{\beta-1}}{\Gamma(\beta)}\int\frac{d\vec{\mu}_\perp}{2\sqrt{1+\mu_\perp^2}}\Big[\big(\vec{n}_\perp\vec{\mu}_\perp+\sqrt{1+\mu_\perp^2}-\xi\big)_+^{\beta-1} -$$
$$-\big(\vec{n}_\perp\vec{\mu}_\perp-\sqrt{1+\mu_\perp^2}-\xi\big)_+^{\beta-1}\Big]\sum c_n\,\delta^{(n)}(\mu_\perp^2) \tag{4.25}$$
$$\sim \nu^{\beta-1}\,\phi_{\beta-1}(\xi),$$

where $\phi_{\beta-1}$ appears as a series of terms $\left[(1-\xi)_+^{\beta-1-m}-(-1-\xi)_+^{\beta-1-m}\right]$, $m\ge 0$.

The nonregular solution (3.18) for conformal invariant matrix elements demands the consideration of the general form of dilatation invariant spectral functions

$$\Psi(u,P,\Delta,\lambda^2) = p^{2K-4}\,\Psi_0\left(\mu,\tau=\frac{\lambda^2}{p^2}\right) \qquad (4.26)$$

$$k = d-3\,,\quad p^2 = P^2\,.$$

In this case the matrix element of the product of two scalar currents has the representation

$$f(x,p,q) = e^{\frac{i}{2}x(p-q)}\,\tilde{F}_T(x,P,\Delta) \qquad (4.27)$$

$$= (-x^2 - ix_0\varepsilon)^{1-d}\, e^{\frac{i}{2}x(p-q)} \int\limits_{|\mu_0|+|\vec{\mu}|\le 1} d\mu\, e^{\frac{i}{2}(\mu_0 \xi_1 - \mu_3\xi_2 - \mu_\perp\sqrt{\xi_3}\cos\varphi)}\,\overline{\Psi}_0(\mu,\omega)$$

where

$$\overline{\Psi}_0(\mu,\omega) = \frac{i}{2\pi}\, p^{2(d-3)}(-x^2)^{d-1}\int_0^\infty d\lambda^2\, D_+(x,\lambda^2)\,\Psi_0(\mu,\tau) \qquad (4.28)$$

$$= -\frac{2^{1-d}}{(2\pi)^3}\,\omega^{d-1}\int_0^\infty d\tau\,\tau\,\frac{K_1(\sqrt{1-\frac{1}{2}\omega\tau})}{\sqrt{1-\frac{1}{2}\omega\tau}}\,\Psi_0(\mu,\tau)$$

is an analytical function of ω with singularities confined to the positive real axis. This carries over to the matrix element itself as a nontrivial consequence of the Dyson representation. A generalized power behaviour (4.5) with respect to λ^2 for $k = d-3 > -1$ leads to a function $\overline{\Psi}_0(\mu,\omega)$ which does not depend on ω.

Turning to the solutions found in the foregoing section, we remark that the nonregular solution (3.18) can be described in terms of the spectral function

$$\overline{\Psi}_0(\mu,\omega) = \delta(\mu_0)\,\delta(\vec{\mu}_\perp)\,K_0\left(\sqrt{1-\tfrac{1}{2}\omega(1-\mu_3^2)}\right). \qquad (4.29)$$

Its support is the section

$$|\mu_3| \le 1\,,\quad \mu_0 = \mu_\perp = 0$$

lying inside the region allowed by the conditions of causality and spectrality (see Fig.3). Some interpretation of this solution can be obtained by an inspection of its behaviour in the Bjorken region. The simple form of the spectral function leads to a special form of the Dyson representation (4.9)

$$F(Q,P,\Delta) = \int\limits_{|\alpha|\le 1,\,\lambda^2\ge 0} d\alpha \int d\lambda^2\, \varepsilon(Q_0)\,\delta((Q-\alpha\Delta)^2-\lambda^2)\,\Psi(\alpha,\lambda^2) \qquad (4.30)$$

As a consequence of this expression $F(Q,P,\Delta)$ is different from zero for

$$K_+^2(1-\alpha) + K_-^2(1-\alpha) = \Delta^2\left(\tfrac{1}{4}-\alpha^2\right) + \lambda^2 \ge \tfrac{1}{4}\Delta^2\,,$$

only, where $K_\pm^2 = (Q \pm \frac{1}{2}\Delta)^2$ are the mass variables of the currents. This means that there are asymptotic contributions to the absorbtive part, i.e. $F(Q,P,\Delta)$ provided one current mass at least tends to $+\infty$. In distinction from eq.(4.25) the asymptotic behaviour now reads $\nu^{d-3}\log\nu/t$.

5. Conclusions

Starting from the equations of dilatation and conformal invariance we have found an infinite set of solutions for the matrix elements of local current products with arbitrary dimensions (real), fulfilling the conditions of causality and spectrality.

The regular solutions eqs. (3.13) have the character of generalized Born terms for $d = 2,3 \ldots$. That means, in the Bjorken region the scaling function are localized at $\xi = \pm 1$. We remark however, that only for particular cases an interpretation with the help of a power type ansatz /19/ for the current

$$j(x) = \, :[\varphi(x)]^{d}: \, , \quad d_\varphi = 1 \, ,$$

is possible. No clear physical interpretation for the nonregular solution has been found. It should be added, that the most important problem, consisting in a reasonable symmetry breaking remains open. In particular, the possibility of a spontaneous symmetry breaking with degenerate vacuum states has to be taken into account.

Therefore the search for nontrivial consequences of conformal symmetry which do not contradict the general principles of Quantum Field Theory represents a complicated problem which deserves further investigations.

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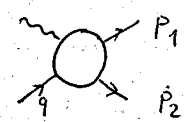
Appendix

Conformal Invariance for an Amplitude with One Off-Shell Particle

Let us apply conformal invariance to the matrix element (see Fig.4)

$$f(z_1, z_2, z_3) = \langle p_1 p_2 | j(0) | q \rangle \tag{A.1}$$

Fig.4



where $z_1 = p_1 q$, $z_2 = p_2 q$, $z_3 = p_1 p_2$, $p_1^2 = p_2^2 = q^2 = 0$.
In general the mass of the current $(p_1 + p_2 - q)^2 = m^2$ is different from zero. Analogously to eq. (2.10) we get now the conditions

$$\left[\bar{O}_G(\vec{p}_1, \nabla_{p_1}) + \bar{O}_G(\vec{p}_2, \nabla_{p_2}) + O_G(\vec{q}, \nabla_q) + O_G(x, \partial_x) \right] f = 0 \tag{A.2}$$

For $G = D$ this reads

$$(d-3) f - (z_1 f_1 + z_2 f_2 + z_3 f_3) = 0 \tag{A.3}$$

whereas the equations (A.2) for $G = K_\mu$ take the form

$$\begin{aligned} z_1^2 f_{11} + z_3^2 f_{33} + 2 z_1 z_3 f_{13} + z_2 (z_1 - z_3) f_{22} & \\ + z_1 (f_1 + f_2) + z_3 (f_3 - f_2) &= 0 \\ z_2^2 f_{22} + z_3^2 f_{33} + z_1 (z_2 - z_3) f_{11} + 2 z_2 z_3 f_{23} & \\ + z_2 (f_1 - f_2) + z_3 (f_3 - f_1) &= 0 \end{aligned} \tag{A.4}$$

$$(A.4) \quad z_1^2 f_{11} + z_2^2 f_{22} + 2 z_1 z_2 f_{12} + z_3(z_1+z_2) f_{33} + z_2(f_3 - f_2) + z_1(f_3 - f_1) = 0. \tag{A.4}$$

Let us introduce the variables

$$\omega_1 = \frac{z_1}{z_3} \quad , \quad \omega_2 = \frac{z_2}{z_3}$$

and solve eq. (A.3) by the ansatz

$$f(z_1, z_2, z_3) = z_3^{d-3}\, g(\omega_1, \omega_2) \ . \tag{A.5}$$

The remaining differential equations for g are

$$(1-\omega_1-\omega_2)[\omega_2 g_{22} + g_2] + (d-3)[2\omega_2 g_2 - (d-3) g] = 0 \tag{A.6}$$

$$(1-\omega_1-\omega_2)[\omega_1 g_{11} + g_1] + (d-3)[2\omega_1 g_1 - (d-3) g] = 0$$

$$(1-\omega_1-\omega_2)[\omega_1^2 g_{11} + \omega_2^2 g_{22} + 2\omega_1\omega_2 g_{12} + \omega_1 g_1 + \omega_2 g_2]$$

$$+(d-3)(\omega_1+\omega_2)[2\omega_1 g_1 + 2\omega_2 g_2 + (d-3) g] = 0$$

In the following we will look for solutions with smooth extrapolation off the mass shell. More correctly we demand that $g(\omega_1, \omega_2)$ together with its partial derivatives g_i, g_{ij} behave continuously at

$$1 - \omega_1 - \omega_2 = \frac{m^2}{s} \rightarrow 0$$

Then the equations are consistent for d=3 only. In this case the system (A.7) takes the simple form

$$\begin{aligned} \omega_1 g_{11} + g_1 &= 0 \\ \omega_2 g_{22} + g_2 &= 0 \\ \omega_1 \omega_2 g_{12} &= 0 \ . \end{aligned} \tag{A.7}$$

The solution is

$$g(\omega_1, \omega_2) = a \log \omega_1 + b \log \omega_2 + c \ . \tag{A.8}$$

In terms of the Mandelstam variables s,t,u we can write

$$\langle p_1 p_2 | j(0) | q \rangle = a \log\left(-\frac{u}{s}\right) + b \log\left(-\frac{t}{s}\right) + c \ . \tag{A.9}$$

Crossing symmetry may give further restrictions for the choice of a and b. Especially complete s,t,u crossing symmetry demands $a = b = 0$ and the matrix element reduces to a constant, which can be interpreted as the lowest order term of a φ^4 theory. This is in accordance with the conclusions reached by Gross and Wess /17/. In general, however, we get in opposite to /17/ strong restriction beyond that coming from dilatation invariance alone.

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