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GENERALIZATION OF THE IRREDUCIBLE COEFFICIENT FUNCTIONS METHOD TO THE CASE OF FERMI PARTICLES: YUKAWA MODEL IN TWO-DIMENSIONAL SPACE-TIME

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GENERALIZATION OF THE IRREDUCIBLE COEFFICIENT FUNCTIONS METHOD TO THE CASE OF FERMI PARTICLES: YUKAWA MODEL IN TWO-DIMENSIONAL SPACE-TIME

S U M M A R Y

The method of works /1-11/ is generalized to the case of Fermi particles. The constructions of ground state functional (29),(30) and excited state functional (59)-(61) are given analogously with the Boson case /1/. Equations (37) for coefficient functions of ground state functional are derived; they are the Yukawa equivalent of $g\phi^4$ eqs. (5). Equations (37) are rewritten in dimensionless variables (§3); this step is useful for the investigation of strong coupling limit. We do not succeed, however, unlike $g\phi^4$ model /11/, in receiving strong coupling decomposition.

The construction of asymptotic states is given, § 4; so that the scattering problem is reduced to that of solving Ekstein equations /9,11/.

The transformation is given which turns the Tamm-Dankoff equations into equations of our method (item 1.1)

Introduction

We have proposed and are now developing a method in quantum theory of boson field /1-11/. Unlike the approach of general use /13/, we mainly consider not the scattering matrix but the vectors of physical states, that is the eigenfunctionals of the Hamiltonian. The investigation of the ground state functional Ω_0 and of the simplest excited state functionals provides the way for constructing the asymptotic state functionals; thereafter it is possible to determine the scattering matrix solving the Ekstein equation /9,12/*.

1. Our approach is based on the representation

$$\Omega_0 = e^{-\kappa} \tag{1}$$

of the ground state functional; here κ is the series in powers of field variables, e.g., for the model of neutral scalar field with the self-action $g\phi^4$

$$\begin{aligned} 2\kappa &= \int a(k)\,\phi(k)\,\phi(-k)\,dk \\ &+ \int C_4(k_1, k_2, k_3, k_4) \prod_1^4 (\phi(k_i)\,dk_i)\,\delta\Big(\sum_1^4 k_i\Big) \\ &+ \dots \qquad (2) \\ &= \sum_{n=1}^{\infty} p_{2n}(\phi). \end{aligned}$$

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* It is possible also if there exist the bound states.

Substituting (1) and (2) into the Schrödinger equation

$$(H - E_0)\Omega_0 = 0, \qquad (3)$$

where (for $g\phi^4$ model)

$$H = \frac{1}{2}\int dk\left[-\frac{\delta^2}{\delta\phi(k)\delta\phi(-k)} + (k^2+M^2)\phi(k)\phi(-k)\right]$$
$$+\, g\int \prod_1^4 (\phi(k_i)\,dk_i)\,\delta(\sum_1^4 k_i) \qquad (4)$$

gives the system of eqs.

$$E_0 = -\frac{1}{2}\delta(0)\int a(k)\,dk$$

$$a^2(k) = k^2+M^2+6\int C_4(k,-k,s,-s)\,ds$$

$$C_4(1,2,3,4)\,\Sigma_4 = 2g + 15\int C_6(1,2,3,4,s,-s)\,ds \qquad (5)$$

$$C_6(1,2,\ldots 6)\;\Sigma_6 + 4[C_4 C_4] = 28\int C_8(1,2,\ldots 6,s,-s)\,ds$$

. . . . .

which determines the coefficient functions of decomposition (2) and the ground state energy E_0.

1.1. Let us make some remarks here, concerning representation (1).

1) Equation (5) implies, that the coefficient functions $C_4, C_6 \ldots$ are irreducible in the following sense: None among homogeneous translationally-invariant polynomials p_{2n} of series (2) contains the part equal to the product of two homogeneous translationally invariant polynomials (such a product part necessary gives the singularity for

the coefficient function C_{2n} and according to (5) all these functions are regular).

2) Let us consider also another representation of the ground state functional

$$\Omega_0 = 1 + \int \gamma_2(k)\phi(k)\phi(-k)\,dk$$
$$+ \int \gamma_4(k_1,k_2,k_3,k_4)\prod_1^4(\phi(k_i)\,dk_i)\,\delta(\sum_1^4 k_i) \qquad (6)$$
$$+ \ldots\ldots = \sum_0^\infty q_{2n}(\phi).$$

One has, evidently

$$q_2 = -\frac{1}{2}p_2$$

$$q_4 = -\frac{1}{2}p_4 + \frac{1}{2}\left(\frac{1}{2!}p_2\right)^2. \qquad (6a)$$

. . . .

This formula and the irreducibility of the coefficient functions of expansion (2) imply that the coefficient functions of expansion (6) are not irreducible.

3) So representation (1), (2) is, in some sense, the simplest possible one.

4) The unphysical ground state energy is separated from essential part of eqs. (5): first of all, the second and subsequent equations of system (5) are to be solved with respect to functions a, C_4, $C_6 \ldots$ and then the first equation (5) gives the value of E_0.

5) The equations for the determination of coefficient functions of expansion (6) are similar to the Tamm-Dankov equations. The transformation of the coefficient functions according to formulae (6a) turns these equations into (5).

6) Let us substitute in (6) and (2) the meson creation operator $b^*(k)$ for the function $\phi(k)$; denote these new

expressions (6) and (2) by Ω_0' and κ', respectively. Let us search for the ground state (of the model $g\phi^4$) in the form

$$\Omega_0 = \Omega_0' \, |0>$$

(here $|0>$ is the state without mesons); then to determine the coefficient functions γ of the operator Ω_0' we get exactly the Tamm-Dankov equations. If, otherwise, we search for the ground state in the form

$$\Omega_0 = e^{-\kappa'}|0>,$$

then we get for the determination of the coefficient functions a, C_4, C_6... of the operator κ' by the method of 2. the system of equations similar to (5), though more complicated (the equations will contain combinations of the functions C up to the fourth order instead of combinations of the second order $[CC]$ in (5)); we mark this system of equations by (5a). So the connection

$$\Omega_0' = e^{-\kappa'}$$

defines (through the equations like (6a)) the transformation of unknown functions, which turns the Tamm-Dankov equations into our type equations (5a).

In practice, we use eqs. (5), and not (5a), for the sake of simplicity.

1.2. In works /1-11/ we were restricted to the case of boson quantum field theory. Now we are able to generalize our method to the case of the Fermi particles.

1.3. We shall consider the Yukawa model in two-dimensional space-time. This model was studied by Glimm and Jaffe /14/. Our investigation has little in common with that of /14/.

Our model is defined by the Hamiltonian

$$H = \int \psi^*(p)\,(\sigma_1 p + \sigma_3 \mu)\,\psi(p)\,dp$$
$$+\frac{1}{2}\int dk\,\Big[-\frac{\delta^2}{\delta\phi'(k)\,\delta\phi'(-k)} + (k^2+M^2)\,\phi'(k)\,\phi'(-k)\Big] \qquad (7)$$
$$+\,g\int \psi^*(p_1)\,\sigma_3\,\psi(p_2)\,\phi'(k)\,\delta(k+p_2-p_1)\,dp_1\,dp_2\,dk\,,$$

where σ_1 and σ_3 are Pauli matrices,

$$\psi^*(p)\,\psi(q) + \psi(q)\,\psi^*(p) = \delta(p-q)\,,$$
$$\psi(p)\,\psi(q) + \psi(q)\,\psi(p) = 0. \qquad (8)$$

1.4. We shall split the quantity μ according to

$$\mu = \nu + \lambda\,; \qquad (9)$$

The convenience of such splitting will be clarified later on.

1.5. It is useful to introduce two-component normalized eigenvectors $u(p)$ and $v(p)$ of the Dirac operator

$$d(p) = \sigma_1 p + \sigma_3 \nu\,, \qquad (10)$$

$$d(p)\,u(p) = E(p)\,u(p)\,,$$
$$d(p)\,v(p) = -E(p)\,v(p)\,, \qquad (11)$$

$$E(p) = \sqrt{p^2+\nu^2}\,, \qquad (12)$$

$$u^*(p)\,u(p) = v^*(p)\,v(p) = 1\,,$$
$$u^*(p)\,v(p) = 0\,. \qquad (13)$$

The formulae

$$\psi(p) = u(p)\,a_+(p) + v(p)\,a_-^*(-p)\,,$$
$$\psi^*(p) = u^*(p)\,a_+^*(p) + v(p)^*\,a_-(-p) \qquad (14)$$

introduce operators a_+^*, a_-^* and a_+^*, a_- of the annihilation and creation of the charge. From (8) and (14) it follows

$$[a^*_\alpha(p),\ a_\beta(q)]_+ = \delta(p-q)\delta_{\alpha\beta},$$
$$[a_\alpha(p),\ a_\beta(q)]_+ = 0, \tag{15}$$

here indices α, β take the values $\pm$. The operators of fermion charge and momentum take the form

$$Q \equiv \int \psi^*(p)\psi(p)\,dp$$
$$= \int dp\,[a^*_+(p)\,a_+(p) - a^*_-(p)\,a_-(p)], \tag{16}$$

$$P_F \equiv \int dp\,\psi^*(p)\,\psi(p)p$$
$$= \int dp\ p[a^*_+(p)\,a_+(p) + a^*_-(p)\,a_-(p)]. \tag{17}$$

So $a_\pm(p)$ $(a^*_\pm(p))$ are the operators of the annihilation (creation) of the particle with the charge ± 1 and momentum p. Note that in (16), (17) eqs. (13) are used.

1.6. Let us introduce new variable of meson field.

$$\phi'(k) = \gamma\,\delta(k) + \phi(k), \tag{18}$$

Sunstituting (14) and (18) into (7) transforms Hamiltonian into the form

$$H = h + h_I + V, \tag{19}$$

where

$$h = \int dp\ E(p)[a^*_+(p)\,a_+(p) + a^*_-(p)\,a_-(p)]$$
$$-\frac{1}{2}\int dk\ \frac{\delta}{\delta\phi(k)\,\delta\phi(-k)} + g\int dp_1\,dp_2\,dk\,\phi(k) \tag{20}$$

$$[A(p_1,p_2)(a^*_+(p_1)\,a_+(p_2) + a^*_-(p_1)\,a_-(p_2))\,\delta(p_1-p_2-k)$$
$$-B(p_1,p_2)\,a_+(p_1)\,a_-(p_2)\,\delta(p_1+p_2+k)],$$

$$h_I = \frac{1}{2}\int dk\,(k^2+M^2)\phi(k)\,\phi(-k) + \delta(0)\,(\frac{1}{2}M^2\gamma^2 - \int E(p)\,dp)$$
$$+(M^2\gamma - g\int A(p\ -p)\,dp)\,\phi(0) \tag{21}$$
$$+g\int dp_1\,dp_2\,dk\ B(p_1,p_2)\,a^*_+(p_1)\,a^*_-(p_2)\,\phi(k)\,\delta(p_1+p_2-k),$$

$$V = (\lambda + g\gamma)\int \psi^*(p)\,\sigma_3\psi(p)\,dp, \tag{22}$$

$$A(p_1,p_2) \equiv u^*(p_1)\,\sigma_3\,u(p_2)$$
$$= -v(-p_2)^*\,\sigma_3\,v(-p_1), \tag{23}$$

$$B(p_1,p_2) \equiv u^*(p_1)\,\sigma_3\,v(-p_2)$$
$$= v(-p_2)^*\,\sigma_3\,u(p_1). \tag{24}$$

Note that

$$A(p_2,p_1) = A(p_1,p_2) = A(-p_1,-p_2),$$
$$B(p_2,p_1) = -B(p_1,p_2) = B(-p_1,-p_2). \tag{25}$$

For proof of eqs. (23)-(25) the following formulae

$$v(p) = \sigma_1\,\sigma_3\,u(p),$$
$$u(-p) = \sigma_3\,u(p), \tag{26}$$

and the fact that the vector $u(p)$ is real, are useful.

1.7. Taking λ to be equal

$$\lambda = -g\gamma \tag{27}$$

we get rid of the third term in (19).

1.8. Define $|0>$ as the state of fermion vacuum:

$$a_{\pm}(p)\,|0> = 0 \tag{28}$$

for all p.

1.9. Now we are in a position to proceed to our generalization of the ground state functional construction (1), (2) to the case of Yukawa model.

2. Ground State

This generalization is given by formulae

$$\Omega_0 = e^{-\kappa}|0> \equiv \Sigma \frac{(-\kappa)^n}{n!}|0> \tag{29}$$

and

$$2\kappa = C_{01}\,\phi(0) + \int C_{02}(k_1,k_2)\,\phi(k_1)\phi(k_2)\,dk_1 dk_2\,\delta(k_1+k_2)$$

$$+\int C_{20}(p,q)\,a_+^*(p)\,a_-^*(q)\,dp\,dq\,\delta(p+q)$$

$$+\int C_{03}(k_1,k_2,k_3)\prod_1^3(\phi(k_i)dk_i)\,\delta(\Sigma k_i)$$

$$+\int C_{21}(p_1;q_1;k)\,a_+^*(p_1)\,a_-^*(q_1)\,\phi(k)\,dp_1 dq_1 dk\,\delta(p_1+q_1-k)$$

$$\cdots$$

$$= \sum_{n+r>0}\int C_{2n,r}(p_1,p_2,\ldots p_n;q_1,q_2,\ldots q_n;k_1,\ldots k_r),$$

$$\prod_1^n(a_+^*(p_i)\,a_-^*(q_i)\,dp_i dq_i)\prod_1^r(\phi(k_i)\,dk_i) \tag{30}$$

$$\delta\left(\sum_1^n(p_i+q_i) - \sum_1^r k_i\right).$$

Note, that all terms of expansion (30) commute with each other.

2.1. Let us substitute (29), (30) into Schrödinger equation (3) with Hamiltonian (19).
It is easily seen that

$$\frac{\delta^2}{\delta\phi(k)\,\delta\phi(-k)}e^{-\kappa} = e^{-\kappa}\left[-\frac{\delta^2\kappa}{\delta\phi(k)\,\delta\phi(-k)} + \frac{\delta\kappa}{\delta\phi(k)}\,\frac{\delta\kappa}{\delta\phi(-k)}\right] \tag{31}$$

$$a(p)\,\kappa^n\,|0> = n\kappa^{n-1}[a(p),\kappa]_-|0>; \tag{32}$$

$$a(p)\,e^{-\kappa}\,|0> = -e^{-\kappa}[a(p),\kappa]_-\,|0>, \tag{33}$$

$$a(p_1)\,a(p_2)\,e^{-\kappa}\,|0> =$$
$$= e^{-\kappa}[[a(p_1),\kappa]_-[a(p_2),\kappa]_- - a(p_1)a(p_2)\kappa]\,|0>. \tag{34}$$

So we get

$$[-E_0 + h_1 + \widehat{\kappa\kappa} - h\kappa]\,|0> = 0, \tag{35}$$

here, according to (19), (20), (31), (34)

$$\widehat{\kappa\kappa} = -\frac{1}{2}\int dk\,\frac{\delta\kappa}{\delta\phi(k)}\,\frac{\delta\kappa}{\delta\phi(-k)}$$

$$-g\int d\alpha\,d\beta\,dk\,\delta(\alpha+\beta+k)\,\phi(k)\,B(\alpha,\beta) \tag{36}$$

$$[a_+(\alpha),\kappa]_-[a_-(\beta),\kappa]_-.$$

The last term in (19) is, because of eq. (27), equal to zero and gives no contribution to (35).

2.2. Having got rid in (35) of the annihilation operators we represent left-hand side of (35) as a sum of homogeneous polynomials in variables ϕ, a^*_+ a^*_-. In order that such a sum would be equal to zero there must be equal to zero also each of the above-mentioned polynomials.

So from (35), (36) and (20), (21) the system of equations follows:

$$E_0 = \delta(0)\left[\frac{1}{2}\int C_{02}(k,-k)\,dk - \int E(p)\,dp + \frac{1}{2}M^2\gamma^2\right], \tag{37.0.0}$$

$$C_{01}\, C_{02}(0,0) = -2\left(g\int A(p,-p)\,dp - M^2\gamma\right) + 3\int C_{03}(0,s,-s)\,ds \tag{37.0.1}$$

$$-g\int B(\alpha,\beta)\, C\quad(\alpha;\beta)\,\delta(\alpha+\beta)\,d\alpha\, d\beta\,;$$

Let us take now the value γ such that right-hand side of (37.0.1) be equal to zero; then we have

$$C_{01} = 0, \tag{38}$$

this equality simplifies the subsequent equations;

$$C^2_{02}(k,-k) = k^2 + M^2 + 6\int C_{04}(k,-k,s,-s)\,ds$$

$$-g\int B(\alpha,\beta)\, C_{21}(\alpha;\beta;k)\,\delta(\alpha+\beta-k)\,d\alpha\, d\beta\,, \tag{37.0.2}$$

$$C_{03}(k_1,k_2,k_3)\,\Sigma_{03} = 10\int C_{05}(k_1,k_2,k_3,s,-s)\,ds$$

$$-g\,\mathrm{symm}(k_1,k_2,k_3)\int B(\alpha,\beta)\, C_{22}(\alpha;\beta;k_1,k_2)\,\delta(\alpha+\beta+k_3)\,d\alpha\, d\beta\,, \tag{37.0.3}$$

$$C_{04}(k_1,k_2,k_3,k_4) + \frac{3\cdot 3}{4}[C_{03}, C_{03}](k_1,k_2,k_3,k_4)$$

$$= 15\int C_{06}(k_1,k_2,k_3,k_4,s,-s)\,ds - g\,\mathrm{symm}(k_1,k_2,k_3,k_4)$$

$$\int B(\alpha,\beta)\, C_{23}(\alpha;\beta;k_1,k_2,k_3)\,\delta(\alpha+\beta+k_4)\,d\alpha\, d\beta\,, \tag{37.0.4}$$

. . . .

$$C_{20}(p;-p)\,\Sigma_{20} = \int C_{22}(p;-p;s,-s)\,ds\,, \tag{37.2.0}$$

$$C_{21}(p;q;k)\,\Sigma\quad + 2g\frac{1\cdot 1}{4}[C_{20}C_{20}B](p;q;k) + g[A\, C_{20}](p;q;k)$$

$$= 2gB(p,q) + 3\int C_{23}(p;q;k,s,-s)\,ds \tag{37.2.1}$$

$$-2\cdot 2\, g\int B(\alpha,\beta)\, C_{40}(\alpha,p;\beta,q)\,\delta(\alpha+\beta+k)\,d\alpha\, d\beta\,,$$

$$C_{22}(p;q;k_1,k_2)\,\Sigma_{22} + 2\frac{1\cdot 3}{4}[C_{21}\; C_{03}](p;q;k_1,k_2) + g[A\, C_{21}]$$

$$+2g\,\frac{1\cdot 1}{4}\left([C_{20}\, C_{21}B] + [C_{21}C_{20}\, B]\right)(p;q;k_1,k_2) \tag{37.2.2}$$

$$= 6\int C_{24}(p;q;k_1,k_2,s,-s)\,ds - 2\cdot 2\, g\,\mathrm{symm}(k_1,k_2)$$

$$\int B(\alpha,\beta)\, C_{41}(\alpha,p;\beta,q;k_1)\,\delta(\alpha+\beta+k_2)\,d\alpha\, d\beta\,,$$

$$C_{23}(p;q;k_1,k_2,k_3)\,\Sigma_{23} + 2\,\frac{1\cdot 4}{4}[C_{21}\; C_{04}] + 2\,\frac{2\cdot 3}{4}[C_{22}\; C_{03}] \tag{37.2.3}$$

$$+2g\,\frac{1\cdot 1}{4}\left([C_{20}\, C_{22}B] + [C_{22}\, C_{20}\, B] + [C_{21}C_{21}B]\right) + g[A\, C_{22}]$$

$$= 10\int C_{25}(p;q;k_1,k_2,k_3,s,-s)\,ds - 2\cdot 2\, g\,\mathrm{symm}(k_1,k_2,k_3)$$

$$\int B(\alpha,\beta)\, C_{42}(\alpha,p;\beta,q;k_1,k_2)\,\delta(\alpha+\beta+k_3)\, d\alpha\, d\beta \;,$$

. . . .

$$C_{40}(p_1,p_2;q_1,q_2)\,\Sigma_{40} + \frac{1\cdot 1}{4}[C_{21}C_{21}] = \int C_{42}(p_1,p_2;q_1,q_2;s,-s)\,ds \;, \tag{37.4.0}$$

$$C_{41}(p_1,p_2;q_1,q_2;k)\,\Sigma_{41} + 2\,\frac{1\cdot 2}{4}[C_{21}C_{22}] + g[A\,C_{40}] + 2g\,\frac{1\cdot 2}{4}([C_{20}C_{40}B] + [C_{40}C_{20}B])(p_1,p_2;q_1,q_2;k) \tag{37.4.1}$$

$$= 3\int C_{43}(p_1,p_2;q_1,q_2;k,s,-s)\,ds - 3\cdot 3\,g\int B(\alpha,\beta)\,C_{60}(\alpha,p_1,p_2;\beta,q_1,q_2)\,\delta(\alpha+\beta+k)\,d\alpha\, d\beta$$

and so on. In (37) we use notations like

$$\Sigma_{2n,r} \equiv \sum_1^n [E(p_i)+E(q_i)] + \sum_1^r C_{02}(k_i,-k_i)\;, \tag{39}$$

$$[C_{22}C_{03}](p;q;k_1,k_2,k_3) = \mathrm{symm}(k_1,k_2,k_3)\;, \tag{40}$$

$$C_{22}(p;q;k_1,p+q-k_1)\,C_{03}(k_2,k_3,-k_2-k_3)\;,$$

$$[C_{22}C_{41}B](p_i;p_2;q_1,q_2;k_1,k_2,k_3,k_4) =$$

$$= \mathrm{symm}(p_1,p_2;q_1,q_2;k_1,k_2,k_3,k_4)\;, \tag{41}$$

$$C_{22}(p_1;\beta;k_1,k_2)\,C_{41}(\alpha,p_2;q_1,q_2;k_3)\,B(\alpha,\beta)\;,$$

$$\alpha = k_3 - p_2 - q_1 - q_2, \qquad \beta = k_1+k_2-p_1\;,$$

$$[A C_{42}](p_1,p_2;q_1,q_2;k_1,k_2,k_3) = \mathrm{symm}(k_1,k_2,k_3)\,\{$$

$$A(p_1,p_1-k_3)\,C_{42}(p_1-k_3,p_2;q_1,q_2;k_1,k_2)$$

$$+A(p_2,p_2-k_3)\,C_{42}(p_1,p_2-k_3;q_1,q_2;k_1,k_2)$$

$$+A(q_1,q_1-k_3)\,C_{42}(p_1,p_2;q_1-k_3,q_2;k_1,k_2) \tag{42}$$

$$+A(q_1,q_2-k_3)\,C_{42}(p_1,p_2;q_1,q_2-k_3;k_1,k_2)\}\;,$$

symm denotes the operation of symmetrization over boson momenta (k) and antisymmetrization over the momenta of identical fermions (this is made over p and over q separately).

2.2.1. System (37) is Yukawa equivalent of eq.(5).

2.2.2. The splitting of mass parameter (9), item 1.4., has the only aim to remove the first term of expansion (30) and to simplify in this way eqs. (37).

2.3. It follows from (37.2.1) that:

$$C_{21}(p;q;k) = 2q\,B(p,q)/\Sigma_{21} + \dots \tag{43}$$

so that the last integral in eq. (37.0.2) diverges; introducing instead of M^2 a new parameter m^2 by the formula

$$M^2 = m^2 + g\int B(\alpha,-\alpha)\,C_{21}(\alpha;-\alpha;0)\,d\alpha \tag{44}$$

(compare with eq. (11) in /1/) we eliminate the divergence, eq. (37.0.2) taking the form

$$C^2_{02}(k,-k) = k^2 + m^2 + 6\int C_{04}(k,-k,s,-s)\,ds \tag{37.0.2a}$$

$$+ g\int d\alpha\,[\,B(\alpha,-\alpha)C_{21}(\alpha;-\alpha;0) - B(\alpha,k-\alpha)C_{21}(\alpha;k-\alpha;k)\,].$$

On substituting eq.(37.0.2a) for eq. (37.0.2), system (37) contains no divergences (with the only exception of eq. (37.0.1), see item 2.5).

2.4. Let us consider, for the proof of the latter statement, the asymptotics of some functions C_{nr} for large values of momenta. In order to get these asymptotics, it is sufficient to determine C_{nr} in the lowest order in coupling constant g; the higher order corrections are (relatively) small for large values of momenta. So (37.2.2) and (43) give

$$C_{22}(p;q;k_1,k_2) = -2g^2\,\mathrm{symm}(k_1,k_2)\,[\tag{45}$$

$$\frac{A(p,p-k_2)\,B(p-k_2,q)}{E(p-k_2)+E(q)+C_{02}(k_1)} + \frac{A(q,q-k_2)\,B(p,q-k_2)}{E(p)+E(q-k_2)+C_{02}(k_1,-k_1)}\,]$$

$$\cdot(E(p)+E(q)+C_{02}(k_1,-k_1)+C_{02}(k_2,-k_2))^{-1} + \ldots\,.$$

According to (23), (24) one has

$$A(p,p+s)B(p+s,-p) = u^*(p)\,\sigma_3\,u(p+s)\,u^*(p+s)\,\sigma_3\,v(p)$$

$$= u^*(p)\,\sigma_3\,\frac{\sigma_1(p+s)+\sigma_3\nu+E(p+s)}{2E(p+s)}\,\sigma_3\,v(p) \tag{46}$$

$$= -\frac{(2p+s)\,\nu}{E(p+s)\,E(p)}\,.$$

Here the columns u, v are handled in the same way as the solutions to Dirac equation /13/.

Substituting (45), (46) into (37.2.0) gives

$$C_{20}(p;-p) = O(|p|^{-3}),\quad |p|\to\infty. \tag{47}$$

Analogously, substituting (45) into (37.0.3) gives

$$C_{03}(k_1,k_2,k_3) = O([\max(|k_1|,|k_2|,|k_3|)]^{-3}). \tag{48}$$

Substituting into (37.0.4.) the following equation

$$C_{23} = -(g[\,A\,C_{22}] + \frac{2g}{4}[\,C_{21}C_{21}B])\,/\,\Sigma_{23} + \ldots \tag{49}$$

gives

$$C_{04}(k_1,k_2,k_3,k_4) = O([\max(|k_1|,|k_2|,|k_3|,|k_4|)]^{-3}). \tag{48a}$$

These examples show that all integrals in system (37), (37.0.2a) converge.

2.5. The only exception is eq. (37.0.1) where the integral

$$\int A(\alpha,-\alpha)\,d\alpha = \nu\int\frac{d\alpha}{E(\alpha)}$$

and quantity M^2 (44) logarithmically diverge. So, (38) and (43) give

$$\gamma = \nu/g + O((\ln\ell)^{-1})\,. \tag{50}$$

The divergence of M^2 makes system (37) supersensitive to the parameter γ: infinitesimal ($\approx(\ln\ell)^{-1}$) variation of this parameter implies finite (≈ 1) variation of functions C_{nr}. We meet such a phenomenon for the first time.

It follows from (50), (27), (9) that $\mu = O((\ln\ell)^{-1})$: system (37) is supersensitive also to the parameter μ.

Without performing transformation (18) all the functions would be infinite.

3. Transformation to Dimensionless Variables. Weak and Strong Coupling Approximations

Equation (7) implies the dimensions of $\phi(k)$, $\psi(p)$ and g to be those of inverse mass, square root of inverse mass and mass, respectively. (For determination of the dimension of variational derivative it is necessary to have in mind that $\delta\Phi/\delta\phi(k)$ is only the notation for the limit of expression

$[\Phi(\phi+\delta\phi)-\Phi(\phi)]/\int ds\,\delta\phi(s)$

for

$\delta\phi(s)=\lambda\delta(s-k)$, $\lambda\to 0$).

3.1. So, our quantum field theory model is completely determined by three constants of dimension of mass: ν (9), (10), (12); m (37.0.2a); g. Actually, the theory depends not on these three parameters, but on their two dimensionless combinations (compare /1/, item 2.1).

3.2. Any of parameters ν, m, g can be used to transform (37) to the dimensionless form.

Let us take, e.g.,

$$p=mp', \quad k=mk', \quad \phi(k)=m\phi'(k'), \quad a_{\pm}(p)=\frac{1}{\sqrt{m}}a'_{\pm}(p'), \quad a^{*}_{\pm}(p)=\frac{1}{\sqrt{m}}a'^{*}_{\pm}(p'), \tag{51}$$

(new operators a', a'^{*} satisfy canonical anticommutation relations (15) also).

Let us introduce new coefficient functions C'_{nr} via expansion (30), in powers of new function ϕ' and new operators a'^{*}.

In order to get the equations for the determination of functions C'_{nr} one must substitute into (37), (37.0.2a) the following quantities

$$\nu\to\nu/m, \tag{52}$$

$$m\to 1, \quad g\to g/m \tag{53}$$

(the parameter ν enters into eq. (37) through functions (12), (23), (24)). Let us make an additional transformation

$$C'_{nr}=C''_{nr}(g/m)^{n+r-2}. \tag{54}$$

The equation for C''_{nr} can be received from (37) by the following successive manipulations:

1) substituting (52),
2) putting g equal to unity,
3) adding the factor $(g/m)^2$ to the integral terms.

This latter form of ground state equation is suitable for investigating the weak coupling limit

$$|g/m|\ll 1 \tag{55}$$

and it is similar to eq. (14) of /1/.

3.3. For the investigation of strong coupling limit compare /7,11/ we substitute g for m in transformation (51) and define coefficient functions $\overline{C}_{nr}$ by expansion (30) in powers of new function ϕ' and new operators a'^{*}.

In order to get the equations for determining the functions $\overline{C}_{nr}$ one must substitute into (37), (37.0.2a) the following expressions

$$\gamma\to\gamma/g=\sigma, \quad m\to m/g=\delta, \quad g\to 1; \tag{56}$$

such a system will be denoted as (37-). It depends on two dimensionless parameters σ and δ (56). One may think, that to the strong coupling limit there correspond zero values of both these parameters. It seems to us, however, to be not correct.

3.4. We introduce a new parameter ϵ,

$$\epsilon^2 = \delta^2 + 6 \int \overline{C_{04}}(0,0,s,-s)\, ds . \tag{57}$$

Then eq. (37.0.2-) takes the form

$$\overline{C_{02}}(k,-k) = k^2 + \epsilon^2 + 6 \int ds [\overline{C_{04}}(k,-k,s,-s) - C_{04}(0,0,s,-s)]$$

$$+ \int d\alpha \; [\bar{B}(\alpha,-\alpha)\, \bar{C}_{21}(\alpha;-\alpha;0) - \bar{B}(\alpha,k-\alpha)\bar{C}_{21}(\alpha;k-\alpha;k)]; \tag{37.02-}$$

here $\bar{B}$ is the function (24) with the parameter ν replaced by σ (56).
According to (56) and /7/ we state that strong coupling limit corresponds to zero values of parameters σ and ϵ.

3.5. After work /11/ it is natural to expect, that strong coupling decomposition contains integer non-negative degrees of ϵ. We cannot, however, at present point out, how the parameter σ enters into this decomposition.

4. Excited States. Statement of the Scattering Problem.

The construction of excited state functionals given in /3,2/ can be almost straightforward generalized to our case. One must search for these functionals in the form

$$\Omega = U\Omega_0 , \tag{58}$$

where U is the expansion in powers of the function ϕ and operators $a^*_\pm$. In particular, the formulae

$$U_0(k) = \phi(-k) + \int \Gamma^k_{03}(k_1,k_2,k_3) \prod_1^3 (\phi(k_i)\, dk_i) \times$$
$$\times \delta(k + \sum_1^3 k_i) + \int \Gamma^k_{21}(p;q;k_1)\, a^*_+(p)\, a^*_-(q) \times$$
$$\times \phi(k_1)\, dp\, dq\, dk_1\, \delta(k + k_1 - p - q) \tag{59}$$
$$+ \dots$$

$$U_+(p) = a^*_+(p)$$
$$+ \int \Gamma^p_{12}(p_1;k_1,k_2)\, a^*_+(p_1)\, dp_1 \prod_1^2 (\phi(k_i)\, dk_i) \times$$
$$\times \delta(p - p_1 + k_1 + k_2) + \int \Gamma^p_{30}(p_1, p_2; q_1) \times \tag{60}$$
$$\times a^*_+(p_1)\, a^*_+(p_2)\, a^*_-(q)\, dp_1\, dp_2\, dq\, \delta(p - p_1 - p_2 - q)$$
$$+ \dots$$

$$U_-(q) = a^*_-(q) +$$
$$+ \int \Gamma^q_{12}(q_1;k_1,k_2)\, a^*_-(q_1)\, dq_1 \prod_1^2 (\phi(k_i)\, dk_i) \times$$
$$\times \delta(q - q_1 + k_1 + k_2) \tag{61}$$
$$+ \dots$$

define the one-particle excited states of the types:

"meson", "positively charged fermion", "negative charged fermion", respectively.

4.1. The products of commuting factors (59)-(60) define asymptotic states quite similarly to work [9]. So, the scattering problem is reduced to that of solving Ekstein equations [9,12].

4.2. For the computation of expressions

$$N = (\Omega_0, \Omega_0), \tag{62}$$

$$\bar{Z} = (\Omega_0, Z\Omega_0)/N \tag{63}$$

there exist the rules, analogous to that given in item 2.7 of [1].

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Распространение метода неприводимых коэффициентных функций на случай частиц Ферми: модель Юкавы в двумерном пространстве-времени

Метод предыдущих работ автора распространен на случай частиц Ферми. Указано преобразование неизвестных функций, переводящее уравнения типа Тамма-Данкова в уравнения нашего метода.

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Generalization of the Irreducible Coefficient Functions Method to the Case of Fermi Particles: Yukawa Model in Two-Dimentional Space-Time

See the Summary on the reverse side of the title-page.

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