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M.A.Smondyrev, A.N.Tavkhelidze

TWISTED EIKONAL GRAPHS AND QUASIPOTENTIAL STRUCTURE

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TWISTED EIKONAL GRAPHS AND QUASIPOTENTIAL STRUCTURE

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Перекрученные эйкональные графы и структура квазипотенциала

На основе метода прямолинейных путей исследуется асимптотика перекрученных эйкональных графов, которая затем используется для восстановления асимптотического квазипотенциала.

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Twisted Eikonal Graphs and Quasipotential Structure

On the basis of the straight-line path method the asymptotics is studied for the twisted eikonal graphs, which violates the eikonal representation with effective Yukawa potential for the sum of ladder diagrams even in the lowest orders of perturbation theory. The results obtained are employed for reconstructing the asymptotical quasipotential.

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Dubna, 1974

Investigations of the eikonal representation by straight-line path method /1,2/ have shown that the eikonal formula means only the account of the virtual processes in which type of particles with large momenta does not change.

Noneikonal contributions to the amplitude appear when changing the sort of leading particle /3/, i.e., when allowing for the processes with large momenta transferred from nucleons to mesons and vice versa. In this connection it was mentioned in ref. /3/ that one should also consider twisted graphs obtained from ladder diagrams by transposing two nucleons, when investigating the high-energy asymptotics of the two-nucleon scattering amplitude within the scalar model. The possibility of transferring large momenta by mesons results in that the contribution from such graphs can dominate over the eikonal ones in the same order of perturbation theory. This circumstance leads to the fact that the local quasipotential in the region of high energies will be represented by a power-series in the coupling constant, each term of which gives a correction to the Yukawa interaction, corresponding to the traditional eikonal approximation. In what follows we shall consider in detail these diagrams and also will study how to reconstruct the asymptotic quasipotential from them. To the second order of perturbation theory the only twisted graph does exist (Fig. 1):

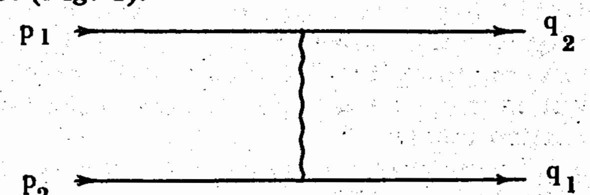

Fig. 1

with the known asymptotics $\frac{1}{s}$ (recall that $s=(p_1+p_2)^2$ and $t = (p_1 - q_1)^2$).

To the fourth order we have already two such diagrams. One of them (see Fig. 2)

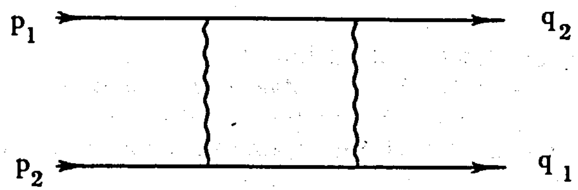

Fig. 2

possesses a more weak asymptotics $1/s^2$ than the corresponding nontwisted graph. The other (see Fig. 3)

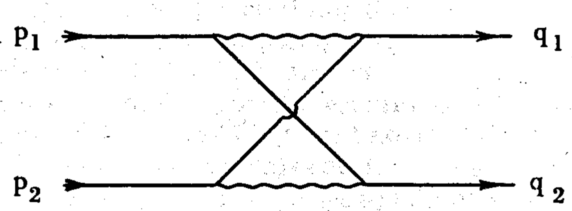

Fig. 3

has the asymptotics $\ln s/s$ that results in the breaking of the eikonal representation for the sum of generalized ladder graphs in the fourth order /3/ already. Recall that noneikonal contributions (of which the possibility was pointed out in ref./4/) do appear only in the eighth order of perturbation theory.

In the subsequent order we have six twisted diagrams. The diagram drawn in Fig. 4

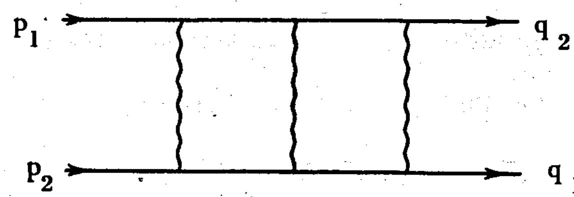

Fig. 4

possesses the most weak asymptotics $\frac{1}{s^3}$.

The next pair of graphs (fig. 5)

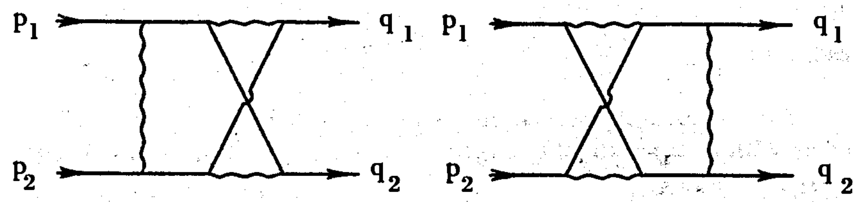

Fig. 5

in the limit $s \to \infty$ with t fixed behaves as $\frac{\ln s}{s^2}$,

i.e., has the same asymptotics as nontwisted graphs in this order. The diagrams in Fig. 6

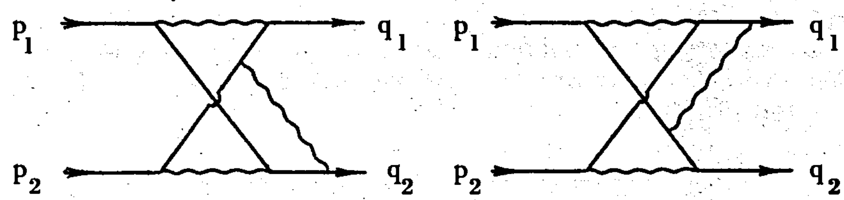

Fig. 6

have the asymptotics $\frac{1}{s}$ what is stronger than the eikonal formula gives.

Finally, the last graph in the sixth order

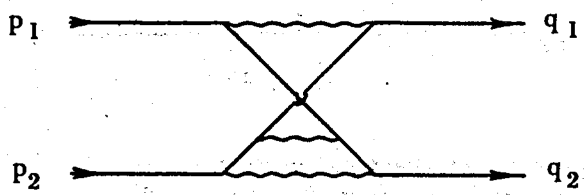

Fig. 7

behaves like $\frac{\ln^2 s}{s}$.

The consideration of the first six orders allows one to conjecture that in the higher orders the diagrams of the shape in Fig. 8

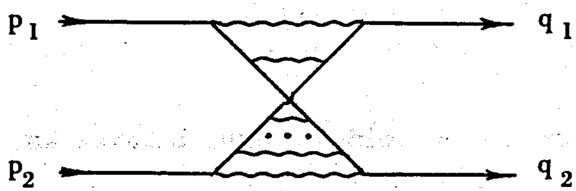

Fig. 8

with asymptotics $g^{2n}\frac{\ln^{n-1}s}{s}$ will dominate.

If only the leading asymptotic terms in each order of perturbation theory are summed up, as it is usually made when deriving the eikonal representation, then one gets the following asymptotic expression for the sum F of twisted graphs:

$$F\Big|_{\substack{s\to\infty \\ t-\text{fixed}}} \sim -i\,\frac{g^2}{(2\pi)^4}\,s^{\alpha(t)} , \qquad (1)$$

$$\alpha(t) = -1 + \frac{g^2}{8\pi^2}\,\frac{1}{\sqrt{-t\,(4m^2-t)}}\,\ln\frac{\sqrt{1-\frac{4m^2}{t}}+1}{\sqrt{1-\frac{4m^2}{t}}-1} . \qquad (2)$$

With such a summation, the coefficient for $s^{\alpha(t)}$ and the expression for $\alpha(t)$ are computed up to an accuracy of g^2 only. However, already from (1) and (2) it follows that within the framework of scalar model the sum of ladder graphs leads by no means to the eikonal representation proper to the Yukawa potential scattering. Indeed as has been already mentioned, the twisted graphs are due to the identity of scattering particles. Within the framework of quasipotential scattering theory the particle identity implies necessarily the exchange forces in two-particle interactions as it holds in quantum mechanics.

The standard method of constructing the local quasipotential by perturbation theory developed in works /5,6/ can be generalized in different ways when the exchange forces are present. Here we will briefly describe a method based on introducing the normal and exchange interaction parts through the expression

$$V(s;p,k) = g(s;p-k) + h(s;p+k) . \qquad (3)$$

The quasipotential scattering amplitude is here represented by the sum of two terms /7/

$$T(s;p,k) = \mathcal{G}(s;p,k) = \mathcal{H}(s;p,k) , \qquad (4)$$

satisfying the system of linear equations

$$\begin{pmatrix}\mathcal{G}\\ \mathcal{H}\end{pmatrix} = \begin{pmatrix} g\\ h\end{pmatrix} + \begin{pmatrix} g & h\\ h & g\end{pmatrix} \times \begin{pmatrix}\mathcal{G}\\ \mathcal{H}\end{pmatrix} , \qquad (5)$$

where the symbol "×" means integration

$$\int \frac{dq}{\sqrt{m^2+q^2}}\;\frac{1}{m^2+q^2-E^2} .$$

For a scattering of two identical particles we have

$$\mathcal{h}(s;p+k) = \hat{P}\, g(s;p-k) = g(s;p+k), \tag{6}$$
$$\mathcal{H}(s;p,k) = \hat{P}\,\mathcal{G}(s;p,k) = \mathcal{G}(s;p,-k) = \mathcal{G}(s;-p,k),$$

where $\hat{P}$ is the transposition operator for coordinates of the two particles. With this, the fucntion $\mathcal{G}$ obeys the conventional equation by Logunov-Tavkhelidze *

$$\mathcal{G} = g + g \times \mathcal{G}. \tag{7}$$

The equation (7) can be used to construct the local quasipotential g over the given perturbation series defining the amplitude G:

$$g_2 = [\mathcal{G}_2],$$
$$g_4 = [\mathcal{G}_4] - [g_2 \times g_2], \tag{8}$$
$$g_6 = [\mathcal{G}_6] - [g_2 \times g_4] - [g_4 \times g_2] - [g_2 \times g_2 \times g_2]$$

and so on.

The symbol [...] means the "local" continuation off the mass shell $E^2 = p^2 + m^2 = k^2 + m^2$ of an arbitrary function $A(E;p,k) = A(s,t,u,\delta)$, where

$$s = 4E^2, \quad t = -(p-k)^2, \quad u = -(p+k)^2,$$
$$\delta = p^2 - k^2.$$

In this notation we have

$$[A(s,t,u,\delta)] = A(s,t,4m^2 - s - t, 0). \tag{9}$$

The quasipotential constructed in this way makes it possible, in its turn, to reconstruct the initial scattering amplitude on the mass shell.

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*Eq. (7) follows from (6) if one takes into account that in the case of identical particles the integration over intermediate two-particle states contains the statistical factor $\frac{1}{2!}$, and $g \times \mathcal{G} = \mathcal{h} \times \mathcal{H}$, $g \times \mathcal{H} = \mathcal{h} \times \mathcal{G} = \hat{P}(g \times \mathcal{G})$.

We should stress, however, that perturbation theory defines the amplitude T as a whole but not $\mathcal{G}$ and $\mathcal{H}$ parts separately, i.e.

$$T_{2n}(s,t) = [\mathcal{G}_{2n}(E;p,k) + \mathcal{H}_{2n}(E;p,k)]. \tag{10}$$

Defining

$$F_{2n}(s,t) = [\mathcal{G}_{2n}(E;p,k)],$$
$$B_{2n}(s,u) = [\mathcal{H}_{2n}(E;p,k)], \tag{11}$$

which are connected in the case of scattering of identical particles by the symmetry relation

$$F_{2n}(s,t) \leftrightarrow B_{2n}(s,u) \quad \text{at} \quad t \leftrightarrow u, \tag{12}$$

we have

$$T_{2n}(s,t) = F_{2n}(s,t) + B_{2n}(s,u). \tag{13}$$

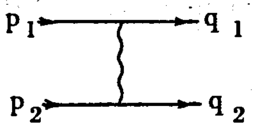
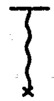
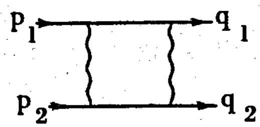
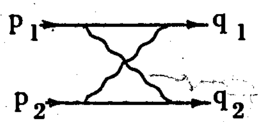
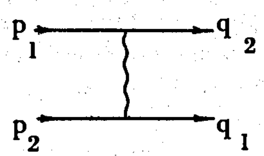
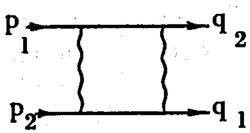
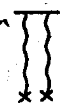
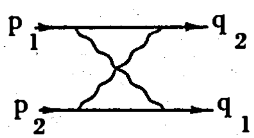
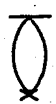
In general, the splitting (13) is not unique. As additional condition fixing this splitting one may employ the analyticity properties. In particular, one may assume that the quantities $F_{2n}(s,t)$ and $B_{2n}(s,u)$ are analytic functions of momentum transfer with singularities at $t > 0$ and $u > 0$, respectively, and obey the nonsubtracted dispersion relation.

In this paper, with the main task on reconstructing the local quasipotential by perturbation theory in the region of asymptotically high energies, we will formulate the following condition:

$F_{2n}(s,t)$ is defined by the leading asymptotic term of the amplitude T_{2n} in the region $s \to \infty$, t - fixed (the forward scattering).

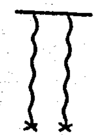
$B_{2n}(s,u)$ is defined by the leading asymptotic term of the amplitude T_{2n} in the region $s \to \infty$, u - fixed (the backward scattering).

The following Table exemplifies the method of constructing the local quasipotential proceeding from the set of twisted and usual eikonal graphs on the basis of the condition stated above.

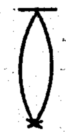
Normal graphs	F -contributions	B -contributions
	g^2 	$0(\frac{1}{s})$
	$-g^4\frac{\ln(-s)}{s}$ 	$0(\frac{1}{s^2})$
	$g^4\frac{\ln s}{s}$ 	$g^4\frac{\ln s}{s}$ 
Twisted graphs	**F -contributions**	**B -contributions**
	$0(\frac{1}{s})$	$\hat{P}$ 
	$0(\frac{1}{s^2})$	$-g^4\frac{\ln(-s)}{s}\hat{P}$ 
	$g^4\frac{\ln s}{s}$ 	$g^4\frac{\ln s}{s}\ \hat{P}$ 

Here the following notations are used:

 $\sim \frac{1}{\Delta_\perp^2 + \mu^2}$ corresponding, in the language of quasipotential graphs, to the single scattering on Yukawa potential at high energies and fixed momenta transfer

 $\sim \int \frac{d^2k_\perp}{(k_\perp^2+\mu^2)\,[(\Delta_\perp+k_\perp)^2+\mu^2]}$

is the two-dimensional contraction corresponding to the double scattering on Yukawa potential

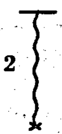
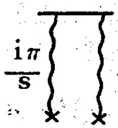
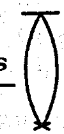
 $\sim \int \frac{d^2k_\perp}{(k_\perp^2+m^2)\,[(\Delta_\perp+k_\perp)^2+m^2]}$,

that corresponds to the contribution to scattering from the exchange by nucleon-antinucleon pair.

The action of operator $\hat{P}$ turns, obviously, into the substitution

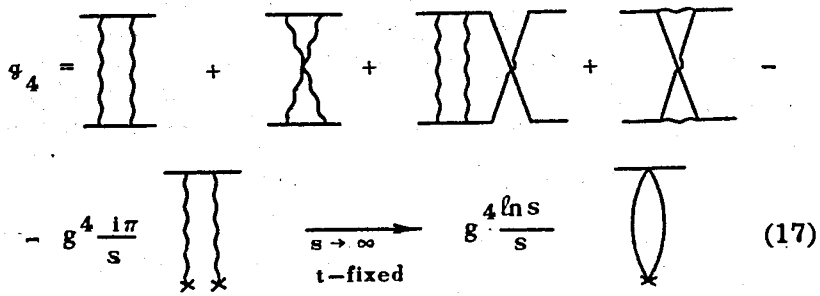
$$\hat{P}\,\Delta_\perp = \hat{P}\,(p-k)_\perp \to (p+k)_\perp \qquad (14)$$

Summing the usual eikonal and twisted graphs we get for the scattering amplitude:

$$T = (1+\hat{P})\,[\,g^2$$  $$+\ g^4\frac{i\pi}{s}$$  $$+\ g^4\frac{\ln s}{s}$$  $$+\dots\,]. \qquad (15)$$

Making use of the above procedure the local quasipotential can now be reconstructed over perturbation theory

$\mathcal{g}_2$ =  $\xrightarrow[t-\text{fixed}]{s\to\infty}$  , (16)



and so on.

As has been indicated above, g_2 represents the conventional Yukawa potential in the phase of eikonal representation. The relation (17) defines the correction of non-Yukawa type which originates from the graph in Fig. 3. In the momentum space this correction to quasipotential is given by the formula

$$g_4(q^2) = \frac{\ln s}{s} \frac{g^4}{2(2\pi)^7} \int \frac{d^2 k_\perp}{(k_\perp^2 + m^2)[(\Delta_\perp + k_\perp)^2 + m^2]} \tag{18}$$

where the replacement $\Delta_\perp^2 = -t \to q^2$ should be performed after integrating.

Introducing α -representation we obtain from (18):

$$g_4(q^2) = \frac{g^4}{4(2\pi)^6} \frac{\ln s}{s} \int_0^1 \frac{d\alpha}{\alpha(1-\alpha)\, q^2 + m^2}. \tag{19}$$

The representation (19) allows one to calculate the quasipotential in the coordinate representation

$$g_4(r) = \frac{g^4}{4(2\pi)^6} \frac{\ln s}{s} \cdot \int_0^1 d\alpha \int dq \frac{e^{iqr}}{\alpha(1-\alpha)\, q^2 + m^2} =$$

$$= \frac{g^4}{2(2\pi)^4} \frac{\ln s}{s} \frac{K_0(2mr)}{r}. \tag{20}$$

We see that g_4 is asymptotically smaller than the leading term (Yukawa potential) of quasipotential independent of s. However, even in the fourth order g_4 gives larger contribution to the scattering amplitude than the second iteration of Yukawa potential that results in breaking of the eikonal formula. At short distances this potential behaves as $\frac{\ln r}{r}$, i.e., it is more singular than the Yukawa potential. The connection of noneikonal terms with increasing of singularity of the quasipotential corrections was pointed out in refs. /8,9/.

To conclude we note that the method described above can be applied to calculations of the asymptotical quasipotential in higher orders of perturbation theory.

The authors express their deep gratitude to N.N.Bogolubov for interest in the work and valuable remarks and also to B.M.Barbashov, A.V.Efremov, A.A.Logunov, D.V.Shirkov and V.S.Vladimirov for fruitful discussions.

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