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THEORY OF DYNAMICAL AFFINE AND CONFORMAL SYMMETRIES AS GRAVITY THEORY

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S u m m a r y

The invariance with respect to the infinite-dimensional general covariance group is equivalent to the invariance with respect to the affine and conformal groups simultaneously. The nonlinear realizations of affine group (the Poincare group being the stability subgroup) lead to the symmetry tensor field as the Goldstone field. The requirement that the theory also corresponds at the same time to the realizations of conformal group, results uniquely in the tensor field theory which equations coincide with the Einstein equations. Thus, it becomes clear that the theory of gravitation is that of spontaneous breaking of the affine and conformal symmetries, just as the chiral dynamics is the theory of the spontaneous breaking of the chiral symmetry. The analogy established suggests some new aspects of possible gravity effects in elementary particle physics.

1. Introduction

The gravitational field is known to be a gauge field providing invariance of the Einstein gravity theory under the group of general coordinate transformations. This exhibits a deep analogy between the gravitational field and the Yang-Mills fields - which are gauge fields for the internal symmetries.

Another deep analogy does exist, that one between gravitons in the theory of gravitation and pions in the $SU(2) \times SU(2)$ chiral dynamics based on nonlinear realizations of the chiral symmetry. The chiral invariance is achieved via appropriate interactions with (massless) pion field and the general covariance is ensured by the appropriate interactions with massless gravitational field. These interactions are introduced replacing usual derivatives by covariant derivatives nonlinearly dependent on the pion field (in chiral symmetry) and on the gravitational field (in gravity theory). The chiral symmetry is spontaneously broken and (massless) pions are the Goldstone bosons.

In the present paper it will be shown that the general relativity is that of simultaneous nonlinear realizations of affine and conformal symmetries. Gravitons prove to be the Goldstone bosons for these symmetries*. We shall

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* Attempts to consider gravitons as the Goldstone particles were undertaken also earlier /1,2/. However they were connected with the hypothetical spontaneous breaking of the Lorentz invariance along a specific direction. In our case the Lorentz symmetry is unbroken and vacuum is invariant under Lorentz transformations.

derive the Einstein equations from the requirement of invariance under the affine and conformal groups realized nonlinearly which become linear on the Poincare group. The derivation is analogous with that of the chiral dynamics equations in the theory of nonlinear realizations of chiral symmetry [3,4].

The general covariance group

$$\delta x_\mu = f_\mu(x), \qquad (1)$$

where $f_\mu(x)$ are arbitrary functions of coordinates x_1, x_2, x_3, $x_4 = ict$ contain an infinite number of parameters -expansion coefficients of $f_\mu(x)$ as series in powers of coordinates. Its algebra includes an infinite number of generators:

$$L_\mu^{n_1 n_2 n_3 n_4} = -i x_1^{n_1} x_2^{n_2} x_3^{n_3} x_4^{n_4} \partial_\mu \quad \left(\partial_\mu = \frac{\partial}{\partial x_\mu}\right). \qquad (2)$$

The nonlinear realization theory has been developed for finite-parameter groups [4]. How can it be applied to the interesting for us infinite parameter group (1)? The key to solving this problem gives theorem [5] found by one of us: This theorem states that the infinite-dimensional algebra (2) is a closure of the finite-dimensional algebras of SL(4,R) and conformal groups. (For us here it will be convenient to consider the affine group of all linear transformations $x'_\mu = a_{\mu\nu} x_\nu + c_\mu$ instead of the special linear group SL(4,R)). The generator of special conformal transformations in the coordinate space $K_\mu = -i(x^2 \partial_\mu - 2x_\mu x \cdot \partial)$ is quadratic in coordinates. Its commutator with the generator $-i x_\mu \partial_\nu$ is also quadratic in x. Commuting these commutators with each other we obtain operators cubic in x, and so on. In [5] we have shown that any generator of the general covariance group $L_\mu^{n_1 n_2 n_3 n_4}$ (2) can be represented as a linear combination of repeated commutators of generators of the special linear and conformal groups. Hence it follows that any theory invariant simultaneously with respect to the special linear and conformal groups will be invariant also with respect to the general covariance group. In this way, we arrive naturally at a new

approach to the gravity theory which rests on the invariance of this theory under the finite-parameter conformal and special linear groups having a structure essentially simpler than that of the infinite-parameter general covariance group.

In Nature no conservation laws do exist corresponding to the special linear and conformal transformations. Therefore SL(4,R) and conformal symmetries should be dynamical, spontaneously broken. Accordingly, we will consider their nonlinear realizations so that only their good algebraic subgroup - the Lorentz group (as well as translations) will be represented by linear homogeneous transformations of fields.

Nonlinear realizations of finite-parameter symmetry groups including that of space-time symmetries were studied in a number of papers [6-9] in several papers the conformal symmetry was considered, the linear group was discussed by C.Isham, A.Salam and Strathdee [7].

This paper is planned as follows. The second section describes nonlinear realizations of the affine group. Here the symmetric tensor field $h_{\mu\nu}(x)$ is required to serve as the Goldstone field. In the definition of covariant derivatives some nonminimal terms are allowed, and because of this the corresponding theory is not fixed quite strictly. In the third section the necessary information is given concerning dynamic realization of the conformal group. The general realization theory prescribes two Goldstone fields, vector and scalar fields $\phi_\mu(x)$ and $\phi(x)$. However, specific properties of the conformal group make it possible to represent the vector field $\phi_\mu(x)$ as the gradient of scalar field $\phi(x)$, so only the scalar field remains to be dealt with. When investigating the nonlinear realizations we use essentially the Cartan differential forms following D.V.Volkov [8]. Then in the fourth, principal, section we show that the consistency requirement for nonlinear realizations of the affine and conformal groups fixes uniquely a form of covariant derivatives, and we formulate the rules for writing down the invariant action.

In the next section we identify the theory obtained with the Einstein general relativity. The analogy established between gravitation theory and theories of nonlinear realization of groups of the internal symmetries (chiral and other ones) leads to statement of a series of problems. Solving of them would facilitate more deep understanding of the role of gravitation for elementary particle theory, for instance, for formation of particle mass spectra. Some of these problems are discussed briefly in conclusion.

2. Nonlinear Realizations of the Affine Group

Let us proceed to describe the nonlinear realization of the affine group, A(4), that of all linear transformations in the four-dimensional space-time $x'_\mu = a_{\mu\nu} x_\nu + c_\mu$. The affine group is the semidirect product of the group L(4,R) and that of translations, $A(4) = P_4 \subset\!\!\!\times\, L(4,R)$ and contains the Poincare group as a subgroup. Its algebra consists of generators of the Lorentz group $L_{\mu\nu}$, those of the special linear transformations including dilatations, $R_{\mu\nu}$, and generators of translations P_μ

$$\frac{1}{i}[L_{\mu\nu}, L_{\rho\tau}] = \delta_{\mu\rho} L_{\nu\tau} - \delta_{\mu\tau} L_{\nu\rho} - (\mu \to \nu)$$

$$\frac{1}{i}[L_{\mu\nu}, R_{\rho\tau}] = \delta_{\mu\rho} R_{\nu\rho} + \delta_{\mu\tau} R_{\nu\rho} - (\mu \to \nu) \qquad (3)$$

$$\frac{1}{i}[R_{\mu\nu}, R_{\rho\tau}] = \delta_{\mu\rho} L_{\tau\nu} + \delta_{\mu\tau} L_{\rho\nu} + (\mu \to \nu)$$

$$\frac{1}{i}[L_{\mu\nu}, P_\rho] = \delta_{\mu\rho} P_\nu - (\mu \to \nu)$$

$$\frac{1}{i}[R_{\mu\nu}, P_\rho] = \delta_{\mu\rho} P_\nu + (\mu \to \nu). \qquad (4)$$

In a vector representation the generators $L_{\mu\nu}$ and $R_{\mu\nu}$ may be defined in the matrix form

$$(L_{\mu\nu})_{\alpha\beta} = -i(\delta_{\mu\alpha}\delta_{\nu\beta} - \delta_{\mu\beta}\delta_{\nu\alpha})$$

$$(R_{\mu\nu})_{\alpha\beta} = -i(\delta_{\mu\alpha}\delta_{\nu\beta} + \delta_{\mu\beta}\delta_{\nu\alpha}). \qquad (5)$$

Only the Poincare subgroup of the affine group is algebraic and associated with the true conservation laws, conservation of total momentum and that of total angular momentum, hence the affine symmetry can be only dynamical. Therefore we shall consider the nonlinear realizations of A(4) which become linear only on its subgroup - that of Poincare. The translation group is an invariant subgroup, so we need to consider the realizations of A(4) in a quotient space, A(4)/L, where L is the Lorentz group. Following the general theory /4-9/ we introduce a symmetric tensor field $h_{\mu\nu}(x)$ and define the action of a group element as follows:

$$g: g e^{ix_\mu P_\mu} e^{\frac{1}{2} i h_{\mu\nu}(x) R_{\mu\nu}} = e^{ix'_\mu P_\mu} e^{\frac{i}{2} h'_{\mu\nu}(x') R_{\mu\nu}} e^{\frac{i}{2} u_{\mu\nu}(h,g) L_{\mu\nu}}, \qquad (6)$$

where x'_μ are the transformed coordinates, $h'_{\mu\nu}(x')$ is the transformed field $h_{\mu\nu}(x)$ and $u_{\mu\nu}(h,g)$ depends upon a group element g and field $h_{\mu\nu}(x)$. The action of A(4) on an arbitrary field $\Psi(x)$ is defined as follows:

$$g: \Psi'(x') = e^{\frac{i}{2} u_{\mu\nu}(h(x),g) L^{\Psi}_{\mu\nu}} \Psi(x), \qquad (7)$$

where $L^{\Psi}_{\mu\nu}$ are matrices representing the generators of the Lorentz group for the field $\Psi(x)$. For instance, $L^{\Psi}_{\mu\nu} = 0$ for scalar, $L^{\Psi}_{\mu\nu} = \frac{1}{2}\sigma_{\mu\nu}$ for a spinor, $(L^{\Psi}_{\mu\nu})_{\alpha\beta} = -i(\delta_{\mu\alpha}\delta_{\nu\beta} - \delta_{\nu\alpha}\delta_{\mu\beta})$ for a vector, etc., and the corresponding infinitesimal transformations $\delta\Psi(x) = \Psi'(x') - \Psi(x)$ are written as

$$\delta\phi(x) = 0 \quad (a) \qquad \delta\Psi(x) = \frac{i}{4} u_{\mu\nu}(h(x),g)\,\sigma_{\mu\nu}\,\Psi(x) \quad (b)$$

$$\delta a_\mu(x) = u_{\mu\nu}(h(x),g)\, a_\nu(x) \quad (c) \tag{8}$$

The group property of the transformations (6), (7), (8) is checked immediately. One can easily be convinced that the Poincare group is represented by the standard linear transformations. For example, for translations we have $g = e^{ic_\mu P_\mu}$ $u_{\mu\nu} = 0$ $x'_\mu = x_\mu + c_\mu$, $h'_{\mu\nu}(x') = h_{\mu\nu}(x)$, $\Psi(x') = \Psi(x)$ for the Lorentz transformations

$g = e^{i\frac{1}{2}\beta_{\mu\nu}L_{\mu\nu}}$ we see that $u_{\mu\nu} = \beta_{\mu\nu}$

and does not dependent on $h_{\mu\nu}(x)$, i.e., in accordance with (6) all fields undergo usual Lorentz transformations. For special linear transformations g contains the factor $e^{\frac{i}{2}a_{\mu\nu}R_{\mu\nu}}$ and $u_{\mu\nu}$ depends essentially on $h_{\mu\nu}$, so all fields but $h_{\mu\nu}$ transform through the Lorentz group with parameters $u_{\mu\nu}(h,g)$ nonlinearly dependent on $h_{\mu\nu}(x)$. In the lowest order in $a_{\mu\nu}$ infinitesimal transformations of $h_{\mu\nu}(x)$ and $u_{\mu\nu}(h,g)$ have the form

$$\delta h_{\mu\nu} = h'_{\mu\nu}(x') - h_{\mu\nu}(x) = \sum_{mn} b_{mn} (h^m(x)\, a\, h^n(x))_{\mu\nu}, \tag{9}$$

$$u_{\mu\nu}(h(x),g) = \sum_{mn} c_{mn} (h^m(x)\, a\, h^n(x))_{\mu\nu}, \tag{10}$$

where

$$(h^m a h^n)_{\mu\nu} = h_{\mu\sigma_1} \dots h_{\sigma_{m-1}\sigma_m}\, a_{\sigma_m\rho_1}\, h_{\rho_1\rho_2} \dots h_{\rho_n\nu}$$

and coefficients b_{mn} and c_{mn} are given by the generating functions

$$\sigma_1(x,y) = \sum_{mn} b_{mn} x^m y^n = (x-y)\,\mathrm{cth}(x-y), \tag{11}$$

$$\sigma_2(x,y) = \sum_{mn} c_{mn} x^m y^n = -\,\mathrm{th}\left(\frac{x-y}{2}\right). \tag{12}$$

Let us now introduce (in the vector representation (5)) the important quantity:

$$r_{\mu\nu}(x) = e^{\frac{i}{2}h_{\alpha\beta}R_{\alpha\beta}} = (e^h)_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu} + \frac{1}{2}h_{\mu\sigma}h_{\sigma\nu} + \dots \tag{13}$$

and its inverse $\quad r^{-1}_{\mu\nu}(x) = (e^{-h})_{\mu\nu}$. $\quad$ (14)

According to (6) the infinitesimal transformations of these Lorentz tensors corresponding to (9) have the form

$$\delta r^{\pm 1}_{\mu\nu}(x) = \pm a_{\mu\sigma} r^{\pm 1}_{\sigma\nu} - r^{\pm 1}_{\mu\sigma} u_{\sigma\nu}(h,a). \tag{15}$$

The identities equivalent to (10) (in the matrix form) are:

$$\{u(h,g), r^{\pm 1}(x)\}_{\mu\nu} = \pm[a, r^{\pm 1}]_{\mu\nu} \tag{16}$$

and they ensure that $\delta r^{\pm 1}_{\mu\nu}$ are symmetrical in indices. It is possible to construct the functions of the field transforming linearly. Those quantities are represented by squares of $r_{\mu\nu}$ and $r^{-1}_{\mu\nu}$.

$$g_{\mu\nu} = r_{\mu\sigma}(x) r_{\sigma\nu}(x) = (e^{2h})_{\mu\nu} \qquad g^{\mu\nu} = r^{-1}_{\mu\sigma} r^{-1}_{\sigma\nu} = (e^{-2h})_{\mu\nu} \tag{17}$$

$$\delta g_{\mu\nu}(x) = a_{\mu\sigma} g_{\sigma\nu}(x) + g_{\mu\sigma}(x) a_{\sigma\nu}$$
$$\delta g^{\mu\nu}(x) = -a_{\mu\sigma} g^{\sigma\nu} - g^{\mu\sigma} a_{\sigma\nu}. \tag{18}$$

These quantities correspond to the covariant and contravariant metric tensors in the general relativity. This similarity is very profound. By changing the field variables it is possible to introduce the linearly transformed contravariant and covariant quantities for any fields with integer spins. Thus, for a vector field $a_\mu(x)$

$$A_\mu(x) = r_{\mu\nu}(x) a_\nu(x); \quad A^\nu(x) = r^{-1}_{\mu\nu}(x) a_\nu(x) \quad A^\mu = g^{\mu\nu} A_\nu \tag{19}$$

are covariant and contravariant vectors, respectively. With (15) taken into account from (8c) it follows

$$\delta A^\mu(x) = -a_{\mu\nu} A^\nu(x); \quad \delta A_\nu(x) = a_{\mu\nu} A_\nu(x). \tag{20}$$

Analogously, for any number of vector indices they may be transformed to contra or covariant ones by multiplying them by $r^{-1}_{\mu\nu}$ or $r_{\mu\nu}$, respectively.

For fields of the half-integer spin it is not possible to pass to the linearly transformed quantities what is due to the absence of finite-dimensional spinor representations of affine group. Note that the nonlinear spinor law (10b) in gravity theory has been found in 1965 /10/.

Because of the nonlinear dependence of transformation laws on the Goldstone field $h_{\mu\nu}(x)$ one should define the covariant field derivatives, which transform under the affine transformations through the representations of Lorentz group with parameters dependent on $h_{\mu\nu}$ in the same way as the fields themselves do. Usual derivatives are not suitable. The covariant derivatives are defined much easy with the help of the Cartan differential forms /2,8,9,11/. We introduce the notation $G = e^{ix_\mu P_\mu} e^{\frac{i}{2} h_{\alpha\beta}(x) R_{\alpha\beta}}$ and consider the transformation properties of the expression $G^{-1} dG$ where differential d acts on x_μ and $h_{\alpha\beta}(x)$. From (6) it follows that

$$G'^{-1} dG' = e^{\frac{i}{2} u_{\mu\nu} L_{\mu\nu}} G^{-1} g d(g^{-1} G e^{-\frac{i}{2} u_{\mu\nu} L_{\mu\nu}}) = \qquad (21)$$
$$= e^{\frac{i}{2} u_{\mu\nu} L_{\mu\nu}} (G^{-1} dG) e^{-\frac{i}{2} u_{\mu\nu} L_{\mu\nu}} + e^{\frac{i}{2} u_{\mu\nu} L_{\mu\nu}} d e^{-\frac{i}{2} u_{\mu\nu} L_{\mu\nu}}$$

as g does not depend on x. Expanding $G^{-1} dG$ over the generators of affine group

$$G^{-1} dG = i\omega^P_\mu(d) P_\mu + \frac{i}{2}\omega^R_{\mu\nu}(d) R_{\mu\nu} + \frac{i}{2}\omega^L_{\mu\nu}(d) L_{\mu\nu} \qquad (22)$$

we obtain the Cartan differential forms

$$\omega^P_\mu(d) = r_{\mu\nu}(x) dx_\nu \quad (a) \qquad \omega^R_{\mu\nu}(d) = \frac{1}{2}\{ r^{-1}(x), dr(x)\}_{\mu\nu} \quad (b)$$
$$\omega^L_{\mu\nu}(d) = \frac{1}{2}[r^{-1}(x), dr(x)]_{\mu\nu} \quad (c), \qquad (23)$$

where the matrix notation is used, for instance, $[r^{-1}, dr]_{\mu\nu} \equiv \equiv r^{-1}_{\mu\sigma} dr_{\sigma\nu} - dr_{\mu\sigma} r^{-1}_{\sigma\nu}$. The generators $R_{\mu\nu}$, P_μ and $L_{\mu\nu}$ form representations of Lorentz group. Therefore from (21) it follows that under action of affine group the <u>Cartan forms undergo the Lorentz transformations with parameters $u_{\mu\nu}(h(x), g)$, the form $\omega^L_{\mu\nu}(d)$ getting also the additive contribution</u>

$$(L_{\mu\nu}\omega^L_{\mu\nu}(d))' = e^{\frac{i}{2} u_{\alpha\beta} L_{\alpha\beta}} \omega^L_{\mu\nu}(d) L_{\mu\nu} e^{-\frac{i}{2} u_{\alpha\beta} L_{\alpha\beta}} - \qquad (24)$$
$$-2i e^{\frac{i}{2} u_{\alpha\beta} L_{\alpha\beta}} d e^{-\frac{i}{2} u_{\alpha\beta} L_{\alpha\beta}}.$$

This additive term allows one to obtain the covariant differential for an arbitrary field $\Psi(x)$. From (24) and (9) it follows that unlike the usual differential the covariant one

$$D\Psi(x) = (d + \frac{i}{2}\omega^L_{\mu\nu} L^\Psi_{\mu\nu}) \Psi(x) \qquad (25)$$

transforms by the same representation as the field $\Psi(x)$ itself, $(D\Psi(x))' = e^{\frac{i}{2} u_{\mu\nu} L_{\mu\nu}} (D\Psi(x))$. The form $\omega^R_{\mu\nu}(d)$ (23b) is identified with the covariant differential of $h_{\mu\nu}(x)$. To construct covariant derivatives the use should be made of the Cartan form $\omega_\mu(d)$ (23a) as just this one (and not dx_μ) has appropriate transformation properties. The covariant derivative of the preferred field $h_{\mu\nu}(x)$ is defined as ($\partial_\mu = \frac{\partial}{\partial x_\mu}$).

$$\nabla_\lambda h_{\mu\nu} = \frac{\omega^R_{\mu\nu}(d)}{\omega^P_\lambda(d)} = \frac{1}{2} r^{-1}_{\lambda\tau}(x) \{ r^{-1}(x), \partial_\tau r(x)\}_{\mu\nu}. \qquad (26)$$

The minimal covariant derivative of an arbitrary field is written in the form

$$\nabla_\lambda \Psi(x) = \frac{D\Psi(x)}{\omega^P_\lambda(d)} = r^{-1}_{\lambda\tau}\partial_\tau \Psi(x) + \frac{i}{2} v^{min}_{\mu\nu,\lambda} L^\Psi_{\mu\nu} \Psi(x), \qquad (27)$$

where

$$v^{min}_{\mu\nu,\lambda}(x) = \frac{1}{2} r^{-1}_{\lambda\tau}(x) [r^{-1}(x), \partial_\tau r(x)]_{\mu\nu}. \qquad (27')$$

Transformation properties of a covariant derivative, $\nabla_\lambda \Psi(x)$ do not change if one will replace $v^{min}_{\mu\nu,\lambda}$ by

$$V_{\mu\nu,\lambda} = v_{\mu\nu,\lambda} + c_1 (\nabla_\mu h_{\nu\lambda} - \nabla_\nu h_{\mu\lambda}) +$$

$$+c_2(\delta_{\mu\lambda}\nabla_\nu h_{\sigma\sigma}-\delta_{\nu\lambda}\nabla_\mu h_{\sigma\sigma})+c_3(\delta_{\mu\lambda}\nabla_\tau h_{\nu\tau}-\delta_{\nu\lambda}\nabla_\tau h_{\mu\tau}). \quad (28)$$

Nonminimal terms added contain the first derivatives of $h_{\mu\nu}$ as well as $v^{min}_{\mu\nu,\lambda}$ do. Therefore the general form of a covariant derivative

$$\nabla_\lambda\Psi(x)=r^{-1}_{\lambda\tau}\partial_\tau\Psi(x)+\frac{i}{2}V_{\mu\nu,\lambda}L^{\Psi}_{\mu\nu}\Psi(x) \quad (29)$$

is not well fixed and contains arbitrary constants c_1, c_2, c_3. It will be shown below that values of these constants are defined by the conformal invariance requirement.

The invariant volume element is given by the outer product:

$$dV=-i\omega^P_1(d)\wedge\omega^P_2(d)\wedge\omega^P_3(d)\wedge\omega^P_4(d)=$$

$$=\det\|r_{\mu\nu}(x)\|d^4x=e^{sp\|h_{\mu\nu}\|}d^4x. \quad (30)$$

The action $\int\mathfrak{L}(x)\det\|r_{\mu\nu}\|d^4x$ will be invariant relative to the affine group if the Lagrangian density $\mathfrak{L}(\Psi(x)$, $\nabla_\lambda\Psi(x)$, $\nabla_\lambda h_{\mu\nu}(x)$ is a usual scalar of the Lorentz group. Indeed, all fields $\Psi(x)$, their covariant derivatives and that of $h_{\mu\nu}(x)$ transform by representations of the Lorentz group (though with parameters $u_{\mu\nu}(h,g)$ dependent on $h_{\mu\nu}$). A further specification of the theory is achieved through imposing the conformal invariance requirement.

3. Dynamical Conformal Symmetry

Realizations of the conformal group in the quotient space over the Lorentz group were considered in [6–8]. For us the following facts are needed: The conformal group algebra includes generators of the Poincare group, $L_{\mu\nu}$, P_ν (their commutators are listed above, see (4)), and generators of the scale and special conformal transformations, D and K_μ

$$[L_{\mu\nu},D]=0,\ [L_{\mu\nu},K_\rho]=i\delta_{\mu\rho}K_\nu-(\mu\to\nu),\ [K_\mu,K_\nu]=0 \quad (31)$$

$$[P_\mu,D]=-iP_\mu,\ [K_\mu,D]=iK_\mu,\ [P_\mu,K_\nu]=2i(\delta_{\mu\nu}D-L_{\mu\nu}).$$

A conformal group element $\bar{g}$ can be represented as follows:

$$\bar{g}=e^{ic_\mu P_\mu}e^{i\beta_\mu K_\mu}e^{i\beta D}e^{\frac{i}{2}\beta_{\mu\nu}L_{\mu\nu}}, \quad (32)$$

where c_μ, β_μ, β and $\beta_{\mu\nu}$ are transformation parameters. Now we introduce the Goldstone fields $\phi_\mu(x)$ and $\sigma(x)$, and denote

$$\bar{G}(x)=e^{ix_\mu P_\mu}e^{i\phi_\mu(x)K_\mu}e^{i\sigma(x)D}. \quad (33)$$

The action of conformal group is defined as (quite analogously to (6))

$$\tilde{g}:\ \tilde{g}\bar{G}(x)=\bar{G}'(x')e^{\frac{i}{2}\bar{u}_{\mu\nu}(x,\phi_\mu,\sigma,\tilde{g})L_{\mu\nu}}. \quad (34)$$

The infinitesimal special conformal and scale transformations of x, $\phi_\mu(x)$ and $\sigma(x)$ are the following:

$$\delta x_\mu=x^2\beta_\mu-2(\beta\cdot x)x_\mu-\lambda x_\mu$$

$$\delta\sigma(x)=2(\beta\cdot x)+\lambda \quad (35)$$

$$\delta\phi_\mu(x)=[1+2(x\phi(x))]\beta_\mu+2(x\cdot\beta)\phi_\mu(x)-2(\beta\cdot\phi(x))x_\mu+\lambda\phi_\mu(x).$$

Note that $\phi_\mu(x)$ transforms by the same law as $\frac{1}{2}\partial_\mu\sigma(x)$.

All other fields transform according to their representations of the Lorentz group (see eq. (7)) but now with parameters $\bar{u}_{\mu\nu}$. The infinitesimal transformation (with $\bar{g}$ given by eq. (32)) results in:

$$\bar{u}_{\mu\nu}=\beta_{\mu\nu}+2(\beta_\mu x_\nu-\beta_\nu x_\mu). \quad (36)$$

For instance, for a vector field

$$\delta a_\mu(x) = a'_\mu(x') - a_\mu(x) = \bar{u}_{\mu\nu} a_\nu(x) =$$
$$= \beta_{\mu\nu} a_\nu(x) + 2\beta_\mu (x a(x)) - 2x_\mu (\beta a(x)). \tag{37}$$

Now we decompose $\bar{G}^{-1}(x) d\bar{G}(x)$ into the generators P_μ, $L_{\mu\nu}$, D and K_μ and find the Cartan differential forms:

$$\bar{\omega}^P_\mu(d) = e^{\sigma(x)} dx_\mu \quad (a)$$

$$\bar{\omega}^k_\mu(d) = e^{-\sigma(x)} (d\phi_\mu(x) + \phi^2(x) dx_\mu - 2(\phi(x) dx)\phi_\mu(x)) \quad (b)$$

$$\bar{\omega}^D(d) = d\sigma(x) - 2dx_\mu \phi_\mu(x) \quad (c)$$

$$\bar{\omega}^L_{\mu\nu}(d) = 2(dx_\mu \phi_\nu(x) - dx_\nu \phi_\mu(x)), \quad (d) \tag{38}$$

The covariant derivative of field $\sigma(x)$ is defined as

$$\bar{\nabla}_\mu \sigma(x) = \frac{\bar{\omega}^D(d)}{\bar{\omega}^P(d)} = e^{-\sigma(x)} (\partial_\mu \sigma(x) - 2\phi_\mu(x)). \tag{39}$$

Let us make now an important observation. From (39) it follows that the Goldstone field, $\phi_\mu(x)$, is nonessential and may be dropped out. Really, let us put the covariant derivative $\bar{\nabla}_\mu \sigma(x)$ eq. (39), be zero, which is a covariant operation because $\bar{\nabla}_\mu \sigma(x)$ transforms like the Lorentz vector (with parameters $\bar{u}_{\mu\nu}$). The field $\phi_\mu(x)$ turns then into the gradient of field $\sigma(x)$ *.

$$\bar{\nabla}_\mu \sigma(x) = 0 \rightarrow \phi_\mu(x) = \frac{1}{2} \partial_\mu \sigma(x) \tag{40}$$

* If $\bar{\nabla}_\mu \sigma(x)$ is not put zero then $\phi_\mu = \frac{1}{2}\partial_\mu \sigma + e^\sigma v_\mu$ where $v_\mu(x)$ transforms according to (37) like any other vector field and, consequently, is not the Goldstone field.

$(\bar{\omega}^k_\mu(d)/\bar{\omega}^P_\nu(d)$ gives the Lorentz tensor composed of $\partial_\mu \partial_\nu \sigma(x)$, $\partial_\mu \sigma(x)$, $\sigma(x))$.

The covariant derivative of an arbitrary field, $\Psi(x)$ transformed according to the representation of the Lorentz group with generators $L^\Psi_{\mu\nu}$ is constructed with the help of the Cartan form (38d)

$$\bar{\nabla}_\lambda \Psi(x) = \frac{d\Psi(x) + \frac{i}{2}\bar{\omega}^L_{\mu\nu} L_{\mu\nu}\Psi(x)}{\bar{\omega}^P_\lambda(x)} =$$
$$= e^{-\sigma(x)} (\partial_\lambda \Psi(x) + 2i\phi_\nu L_{\lambda\nu} \Psi(x)). \tag{41}$$

Or, on substituting $\phi_\nu = \frac{1}{2}\partial_\nu \sigma$

$$\bar{\nabla}_\lambda \Psi(x) = e^{-\sigma(x)} (\partial_\lambda \Psi(x) + i\partial_\nu \sigma(x) L^\Psi_{\lambda\nu} \Psi(x)). \tag{42}$$

For instance, for a tensor field, $h_{\alpha\beta}(x)$

$$\bar{\nabla}_\lambda h_{\alpha\beta}(x) = e^{-\sigma(x)} \{\partial_\lambda h_{\alpha\beta} + \partial_\tau \sigma(x)(\delta_{\alpha\lambda} h_{\tau\beta}(x) +$$
$$+ \delta_{\beta\lambda} h_{\alpha\tau}(x) - \delta_{\alpha\tau} h_{\lambda\beta}(x) - \delta_{\beta\tau} h_{\alpha\lambda}(x)\}. \tag{43}$$

The scalar volume element, $d\bar{V}'(x') = dV(x)$ is written as

$$d\bar{V}(x) = -i\bar{\omega}^P_1 \wedge \bar{\omega}^P_2 \wedge \bar{\omega}^P_3 \wedge \bar{\omega}^P_4 = e^{4\sigma(x)} d^4x. \tag{44}$$

4. Simultaneous Realizations of the Affine and Conformal Symmetries

Let us require now the simultaneous invariance with respect to conformal and affine groups. Then, as was discussed above in Introduction, from the theorem proven by one of the authors/5/ it follows that invariance under the general covariance group has to emerge and we arrive at the Einstein general relativity. Let us look into this in detail.

Obviously, the trace of affine generator $R_{\mu\nu}$ is connected with the generator of scale transformations D via the relation * $R_{\mu\mu}=2D$. Hence, we should identify the trace of affine Goldstone field $h_{\mu\nu}$ with conformal Goldstone field $\sigma(x)$

$$\sigma(x)=\frac{1}{4}h_{\mu\mu}(x) \qquad (45)$$

and put

$$h_{\mu\nu}(x)=\bar h_{\mu\nu}(x)+\delta_{\mu\nu}\sigma(x). \qquad (46)$$

Then affine volume element (30) coincides with that conformal (44). As we have seen before, in the dynamical affine symmetry the covariant derivative (28) includes free parameters c_1, c_2, c_3. We will prove now that for definite values of these parameters the expression (28) will be the covariant derivative not only for the affine symmetry but simultaneously for the conformal symmetry also.

The conformal covariant derivative, $\bar\nabla_\lambda \bar h_{\mu\nu}$ is given by Eq. (43). Let us rewrite now the affine covariant derivative, $\nabla_\lambda h_{\mu\nu}$, in terms of $\bar\nabla_\lambda \bar h_{\mu\nu}(x)$, $\sigma(x)$ and $\partial_\nu\sigma(x)$. Making use of (43) we rewrite (26) for $\nabla_\lambda h_{\mu\nu}$ in the following form

$$\nabla_\lambda h_{\mu\nu}=\frac{1}{2}(e^{-\bar h})_{\lambda\gamma}\, e^{-\sigma}\{e^{-\bar h},\partial_\gamma e^{\bar h}\}_{\mu\nu}+$$

$$+e^{-\sigma}(e^{-\bar h})_{\lambda\tau}\,\partial_\tau\sigma\,\delta_{\mu\nu}=\frac{1}{2}(e^{-\bar h})_{\lambda\gamma}\{e^{-\bar h},\bar\nabla_\gamma e^{\bar h}\}_{\mu\nu}+$$

$$+\frac{1}{2}\partial_\tau\sigma e^{-\sigma}[(e^{-\bar h})_{\mu\tau}\delta_{\lambda\nu}-(e^{-2\bar h})_{\mu\lambda}(e^{\bar h})_{\tau\nu}+(e^{-\bar h})_{\lambda\tau}\delta_{\mu\nu}+$$

$$+(\mu\to\nu)]. \qquad (47)$$

We shall rewrite $v^{min}_{\mu\nu,\lambda}$ (27) analogously

$$v^{min}_{\mu\nu,\lambda}=\frac{1}{2}r^{-1}_{\lambda\gamma}[r^{-1},\partial_\gamma r]_{\mu\nu}=\frac{1}{2}e^{-\sigma}(e^{-\bar h})_{\lambda\gamma}[e^{-\bar h},\partial_\gamma e^{\bar h}]_{\mu\nu}=$$

$$=\frac{1}{2}(e^{-\bar h})_{\lambda\gamma}[e^{-\bar h},\bar\nabla_\gamma e^{\bar h}]_{\mu\nu}+\frac{1}{2}\partial_\tau\sigma e^{-\sigma}[(e^{-\bar h})_{\mu\tau}\delta_{\lambda\nu}+$$

$$+2(e^{-\bar h})_{\mu\lambda}\delta_{\tau\nu}+(e^{-2\bar h})_{\nu\lambda}(e^{\bar h})_{\mu\tau}-(\mu\to\nu)]. \qquad (48)$$

Finally we represent $r^{-1}_{\lambda\tau}\partial_\tau\Psi(x)$ in (29) in the form

$$r^{-1}_{\lambda\tau}\partial_\tau\Psi=(e^{-\bar h})_{\lambda\tau}\;{}_\tau\Psi-i\partial_\nu\sigma e^{-\sigma}(e^{-\bar h})_{\lambda\mu}L^{\Psi}_{\mu\nu}\Psi. \qquad (49)$$

Let us substitute (47), (48) and (49) into (29). As we have discussed above our requirement is that $\nabla_\lambda\Psi$ must depend on $\sigma(x)$ and $\partial_\nu\sigma(x)$ only via the conformal covariant operations $\bar\nabla$. This requirement appears to be realizable and gives four equations for the parameters c_1, c_2, c_3 from which it follows that $c_1=-1$, $c_2=c_3=0$. Thus, for affine and conformal symmetries simultaneously the covariant derivative of any field, $\Psi(x)$, is:

$$\nabla_\lambda\Psi(x)=r^{-1}_{\lambda\tau}\partial_\tau\Psi(x)+\frac{i}{2}V_{\mu\nu,\lambda}L^{\Psi}_{\mu\nu}\Psi(x), \qquad (50)$$

where the connection $V_{\mu\nu,\lambda}$ is defined uniquely as:

$$V_{\mu\nu,\lambda}=\frac{1}{2}\left(r^{-1}_{\lambda\gamma}[r^{-1},\partial_\gamma a]_{\mu\nu}-r^{-1}_{\mu\gamma}\{r^{-1},\partial_\gamma r\}_{\lambda\nu}+r^{-1}_{\nu\gamma}\{r^{-1},\partial_\gamma r\}_{\lambda\mu}\right) \qquad (51)$$

At the same time one can be convinced that no combinations of $\nabla_\lambda h_{\mu\nu}$, $\nabla_\sigma h_{\tau\tau}$, $\nabla_\tau h_{\sigma\tau}$ and Kronecker symbols can be expressed only through the conformal covariant derivative $\bar\nabla$. It follows therefore that within

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* For example, in space-time itself
$R_{\mu\nu}=-i(x_\mu\partial_\nu+x_\nu\partial_\mu)$, $D=-ix_\lambda\partial_\lambda$

the simultaneous realization of affine and conformal symmetries the covariant derivative disappears not only of the field $\sigma(x)$ but of the whole Goldstone tensor field, $h_{\mu\nu}$, as well. A covariant expression can be nevertheless obtained which includes the second derivatives of the tensor field $h_{\mu\nu}$ in addition to $h_{\mu\nu}, \partial_\lambda h_{\mu\nu}$. The most easy way to do this is to consider the commutator of covariant derivatives of an arbitrary field, $\Psi(x)$. We have:

$$(\nabla_\lambda \nabla_\rho - \nabla_\rho \nabla_\lambda)\Psi = \frac{i}{2}\mathcal{R}_{\mu\nu,\lambda\rho} L^{\Psi}_{\mu\nu}\Psi , \qquad (52)$$

where

$$\mathcal{R}_{\mu\nu,\lambda\rho} = r^{-1}_{\lambda\gamma}\partial_\gamma V_{\mu\nu,\rho} + V_{\mu\nu,\gamma} V_{\rho\gamma,\lambda} + V_{\mu\gamma,\rho} V_{\nu\gamma,\lambda} - (\lambda \to \rho). \qquad (53)$$

Under the action of affine and conformal groups $\mathcal{R}_{\mu\nu,\lambda\rho}$ transforms as a tensor under the Lorentz transformations with parameters $u_{\mu\nu}$ (10) and $\bar{u}_{\mu\nu}$ (36), respectively. Now

$$\mathcal{R} = \mathcal{R}_{\mu\nu,\mu\nu} = 2r^{-1}_{\mu\gamma}\partial_\gamma V_{\mu\nu,\nu} + V_{\mu\nu,\gamma} V_{\nu\gamma,\mu} - V_{\mu\gamma,\mu} V_{\nu\gamma,\nu} \qquad (54)$$

(summation over Lorentz indices is implied) is a scalar with respect to the affine and conformal groups. It is evident that any expression $L(\Psi, \nabla_\mu\Psi, \mathcal{R}_{\mu\nu,\lambda\rho})$ composed of different fields, Ψ, their covariant derivatives, $\nabla_\mu\Psi$ and $\mathcal{R}_{\mu\nu,\lambda\rho}$, is a scalar under the affine and conformal groups simultaneously if it is invariant under the Lorentz group. The invariant action can be obtained by integrating such an expression over the scalar volume dV (30). A minimal interaction with the field $h_{\mu\nu}$ is described by the action integral

$$\int [\mathcal{L}(\Psi, \nabla_\mu\Psi) + \frac{1}{4f^2}\mathcal{R}] \det r \, d^4x , \qquad (55)$$

where $\mathcal{L}(\Psi, \nabla_\mu\Psi)$ is derived from the free Lagrangian of Ψ replacing usual derivatives $\partial_\mu\Psi(x)$ by covariant ones $\nabla_\mu\Psi(x)$ (50) and the term $(2f)^{-2}\mathcal{R}$ describes the

self-interaction of the field $h_{\mu\nu}$. To ensure the correct dimension, the universal coupling constant f having the dimension of length $(\hbar = c = 1)$ should be introduced, and the replacement $h_{\mu\nu} \to f h_{\mu\nu}$ should be performed throughout (just as in chiral dynamics the constant F_π is introduced [3]). Let us stress that the Goldstone field $h_{\mu\nu}$ itself can enter into a Lagrangian only via the covariant derivatives of different fields $\nabla_\mu\Psi$, $R_{\mu\nu,\lambda\rho}$ and $\det r$ in the scalar volume.

5. Identification with the Theory of Gravitational Field

Let us now verify that the theory constructed in this way coincides with the Einstein gravity theory (see, e.g., ref [12]). We have seen above (see eq. (19)) that for fields of integer spin $a_{\mu\nu}$ ... the linearly transformed covariant and contravariant quantities can be introduced by multiplying by $r_{\mu\nu}$ or $r^{-1}_{\mu\nu}$ over each index, respectively. For instance, $A_\mu = r_{\mu\bar\nu} a_{\bar\nu}$ will be a covariant vector, $A_{\mu\nu} = r_{\mu\bar\mu} r_{\nu\bar\nu} a_{\bar\mu\bar\nu}$ a covariant two-tensor, $A_\mu^\nu = r_{\mu\bar\mu} r^{-1}_{\nu\bar\nu} a_{\bar\mu\bar\nu}$ a mixed two-tensor, and so on. An analogous operation can be made also with covariant derivatives, and the linearly transformed covariant derivative of a covariant vector A_μ can be defined as

$$D_\lambda A_\mu = r_{\lambda\bar\lambda} r_{\mu\bar\mu} \nabla_{\bar\lambda} a_{\bar\mu} \qquad (56')$$

the one for a contravariant vector

$$D_\lambda A^\mu = r_{\lambda\bar\lambda} r^{-1}_{\mu\bar\mu} \nabla_{\bar\lambda} a_{\bar\mu} \qquad (56'')$$

and for a covariant tensor of second rank:

$$D_\lambda A_{\mu\nu} = r_{\lambda\bar\lambda} r_{\mu\bar\mu} r_{\nu\bar\nu} \nabla_{\bar\lambda} a_{\bar\mu\bar\nu} ,$$

and so on. After some easy calculations one may be convinced that these definitions are exactly the same as the standard ones of gravity theory, For instance,

$$D_\lambda A_\mu = r_{\lambda\bar\lambda} r_{\mu\bar\mu} \nabla_{\bar\lambda} a_{\bar\mu} = r_{\lambda\bar\lambda} r_{\mu\bar\mu} [r^{-1}_{\bar\lambda\tau} \partial_\tau (r_{\bar\mu\sigma} A_\sigma + $$

$$+ V_{\bar\mu,\beta,\bar\lambda} r^{-1}_{\beta\sigma} A_\sigma] = \partial_\lambda A_\mu - \Gamma^\sigma_{\lambda\mu} A_\sigma , \qquad (57)$$

where $\Gamma^\sigma_{\mu\lambda}$ is the Christoffel symbol:

$$\Gamma^\sigma_{\mu\lambda} = -(r \partial_\lambda r^{-1})_{\mu\sigma} - r_{\mu\bar\mu} r_{\lambda\bar\lambda} V_{\bar\mu\bar\sigma,\bar\lambda} r^{-1}_{\bar\sigma\sigma} \equiv$$

$$\equiv \frac{1}{2} g^{\sigma\gamma} (\partial_\mu g_{\gamma\lambda} + \partial_\lambda g_{\mu\gamma} - \partial_\gamma g_{\mu\lambda}) . \qquad (58)$$

Note should be made, that vanishing of the covariant derivative D_λ of metric tensor is tightly connected with that of the covariant derivative ∇_μ of constant tensor $\delta_{\mu\nu}$ (or $\eta_{\mu\nu}$ if $\mu = 1,2,3,0$)

$$D_\lambda g_{\mu\nu} = r_{\lambda\bar\lambda} r_{\mu\bar\mu} r_{\nu\bar\nu} \nabla_{\bar\lambda} (g_{\sigma\tau} r^{-1}_{\sigma\bar\mu} r^{-1}_{\tau\bar\nu}) =$$

$$= r_{\lambda\bar\lambda} r_{\mu\bar\mu} r_{\nu\bar\nu} \nabla_{\bar\lambda} \delta_{\bar\mu\bar\nu} = 0 . \qquad (59)$$

The covariant derivative of a spinor, $D_\lambda \Psi = r_{\lambda\bar\lambda} \nabla_{\bar\lambda} \Psi$ is defined in the same way as in the vierbein formalism in the relativistically symmetric gauge of vierbeins[10]. Further, the covariant curvature tensor $R_{\mu\nu,\lambda\rho}$ is expressed in terms of $\mathfrak{R}_{\mu\nu,\lambda\rho}$

$$\mathfrak{R}_{\mu\nu\lambda\rho} = r_{\mu\bar\mu} r_{\nu\bar\nu} r_{\lambda\bar\lambda} r_{\rho\bar\rho} \mathfrak{R}_{\bar\mu\bar\nu\bar\lambda\bar\rho} . \qquad (60)$$

For the Ricci tensor we have

$$\mathfrak{R}_{\mu\lambda} = r_{\mu\bar\mu} r_{\lambda\bar\lambda} \mathfrak{R}_{\bar\mu\nu\bar\lambda\nu} \qquad (61)$$

and the curvature R coincides with the complete contraction

$$R = \mathfrak{R} . \qquad (62)$$

The rule stated above as to form scalars is identical with the corresponding one in the Riemannian geometry. For example

$$a_\mu a_\mu = A^\mu A_\mu \quad \nabla_\mu \phi \, \nabla_\mu \phi = g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

$$\nabla_\mu a_\nu \nabla_\mu a_\nu = g^{\sigma\tau} D_\sigma A^\mu D_\tau A_\mu .$$

Also the expression for scalar volume coincide, for $\det r = \sqrt{\det g}$. Finally, the coupling constant f with the field $h_{\mu\nu}$ is connected with the Newton gravitational constant k ($k = 6.67 . 10^{-8}$ cm^{-3} g^{-1} sec^{-2}) by the relation:

$$\frac{1}{4\pi} f^2 = k . \qquad (63)$$

Now we have identified completely the theory obtained with the Einstein general relativity. The invariant interval is constructed basing on the differential forms $\omega^p_\lambda(d)$ (23a)

$$ds^2 = \omega^p_\lambda(d)\, \omega^p_\lambda(d) =$$

$$= r_{\mu\lambda} dx^\mu r_{\nu\lambda} dx^\nu = g_{\mu\nu} dx^\mu dx^\nu . \qquad (64)$$

It can be observed that we have arrived at the gravity theory in the vierbein formalism (see, e.g., ref. [13]), using the relativistically symmetric gauge * employed

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* Vierbein $L_{\mu a}$($L_{\mu a} L_{\nu a} = g_{\mu\nu}$) is determined up to an orthogonal matrix. In the polar decomposition we have $L_{\mu\tau} = r_{\mu\nu} (e^{\Omega})_{\nu\tau}$ where $r_{\mu\nu}$ is definite symmetric matrix, $\Omega_{\nu\tau}$ an arbitrary antisymmetric matrix. Under an action of the Weil gauge group $SL(2,C)$, $r_{\mu\nu}$ remains the same and $\Omega_{\nu\tau}$ changes arbitrarily. We define the relativistically symmetric gauge according to $SL(2,C)$ as the gauge in which Ω simply vanishes, $\Omega_{\mu\nu} = 0$. Then the vierbein ($r_{\mu\nu}$ for us) is symmetric and has ten components as the metric tensor itself.

for the first time by Polubarinov and one of the present authors in 1965 [10].

6. Conclusion

It has been proven that the theory of simultaneous nonlinear realizations of affine and conformal symmetries is the Einstein gravity theory. These symmetries are spontaneously broken up to the Poincare group and are dynamical. Gravitons appear to be the appropriate Goldstone particles (as known, they are at the same time the gauge fields for the general covariance group).

The fine analogy found between the general relativity and much more simple theories of nonlinear realizations of internal symmetries (chiral, unitary, etc.) looks rather promising. This analogy suggests new ways for searching links of the gravity theory and elementary particle theory. Thus, in the theories of nonlinear realizations of internal symmetries the asymptotic algebraic symmetry emerges if one requires that tree diagrams be of good behaviour at high energies [15]: particles should be classified over linear representations of the initial symmetry $SU(2)\times SU(2)$ in chiral symmetry, $SU(3)$ in unitary symmetry and so on), and the mass operator has to possess simple transformation properties which for all cases prove to be reasonable [15]. Under an analogous requirement in the gravity theory the algebraization of affine and conformal symmetries can be expected. Because of absence of the finite-dimensional spinor representations of the group $SL(4,R)$ the infinite-dimensional representations of $SL(4,R)$ have to appear. This group contains $SL(3,R)$ as the three-dimensional subgroup. Note that the use of $SL(3,R)$ (and of its infinite-dimensional representations) as the generating spectra algebra was advocated by Gell-Mann et al. [16] for describing the hadron orbital excitations. Recently Biedeharn [17] has demonstrated that the primitive infinite-dimensional representations reproduce the Regge sequences of hadrons. Under the condition of reasonable interactions of elementary particles with gravitons the use of $SL(3,R)$ and even $SL(4,R)$ will be justified. It seems to be natural that the gravitation for which the charge is the mass can also define some qualitative aspects of particle mass variety. Note should be made that here nonminimal gravitational couplings arise describing the transitions $s_1 \to s_2 +$ gravitons where s_1 and s_2 are different particles * with the coupling constants fixed. Apparently, studying the algebra of affine and conformal currents (cr. the chiral current algebra) is of further importance. Note that the tree graphs of the gravitational theory depend on momenta like those of the nonlinear realization theory of chiral symmetry (the coupling constants have the same dimensionality) and unlike those of renormalizable gauge theories. One may imagine of the Einstein gravity theory as an effective Lagrangians theory (cr. effective Lagrangians in the chiral dynamics) which is valid for the description of classical effects and in the long-wave limit. In analogy with the chiral symmetry one may think about constructing the possible σ-model of the theory of gravitation. The problems emerging here though being complicated are noteworthy.

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* We think that such couplings are essential for analysis of the one-loop divergences, which has been performed recently by G.Hooft and M.Veltman [18], and these may help improving the situation with renormalization of gravitational interaction of a scalar field in the one-loop approximation.

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