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GENERALIZED SUM RULES FROM LIGHT-CONE CURRENT ANTICOMMUTATORS

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1. Introduction

This work is an extension and a generalization of a former paper (Ref. [1] , hereafter denoted by A) devoted to the derivation of so-called "wrong-signature" sum rules for forward virtual Compton scattering, i.e. sum rules constructed from structure functions with crossing properties opposite to those of the usual causal amplitudes.

We have shown in A that the use, to that end, of current anticommutators restricted to a light-like hyperplane $x^0+x^3=0$ combined with a sum rule derivation method proposed by Dicus, Jackiw and Teplitz [2] provides, to all appearances, a more general framework than the quark-parton model approach [3] (which, for instance, led to the Gottfried sum rule [4] for electroproduction) and other methods based on the infinite momentum frame techniques (see, e.g. Ref. [5] and Pavkovic's work [6]).

Our purpose is to exploit further the method of A in order to extract additional information on the structure functions of the lepton-hadron scattering from the canonical null-plane current anticommutation relations postulated in A within the framework of a free, massive quark field theory model.

First, it appears to be a natural development of our null-plane anticommutator investigation to inquire into the algebra of the moments of the currents, i.e. to consider derivatives of the Fourier transform of the anticommutator function. The specific possibilities of this technique have been established for a long time within the framework of the ordinary equal-time current algebra [7] and more recently for light-cone commutators too [8,9] .

It permits us especially to increase the model-independence of sum rules derived in A and to deduce several high-energy asymptotic constraints for the structure functions.

Second, we deal not only with the diagonal matrix elements of the anticommutators of two conserved vector currents as in A but extend our considerations to nondiagonal matrix elements and nonconserved axial-vector currents. This enables us to obtain new sum rules for nonforward virtual Compton scattering (and, among them, a $t \neq 0$ generalization of the Gottfried sum rule [4]) and neutrinoproduction amplitudes (involving chiral-symmetry breaking structure functions).

The paper is organized as follows. In Section 2 we review briefly the choice of the null-plane anticommutators and its implications on our results. The moment anticommutator technique is examined in Section 3 and the generalizations to the nonforward direction and nonconserved currents are considered, respectively in Sections 4 and 5. Concluding remarks are provided in Section 6.

2. Null-plane current anticommutators

We recapitulate briefly in this section the essential features concerning the definition of the current anticommutators which have been already discussed in A.

The null-plane current anticommutators are abstracted from a free, massive quark field theory model. In this model, the vector and axial-vector currents and their bilocal generalizations are given by:

$$V^\mu_a(x) = :\bar{\psi}(x)\,\gamma^\mu \tfrac{1}{2}\lambda_a \psi(x): \tag{1a}$$

$$A^\mu_a(x) = i:\bar{\psi}(x)\,\gamma^\mu \gamma_5 \tfrac{1}{2}\lambda_a \psi(x): \tag{1b}$$

and

$$V^\mu_a(x|y) = :\bar{\psi}(x)\,\gamma^\mu \tfrac{1}{2}\lambda_a \psi(y): \tag{2a}$$

$$A^\mu_a(x|y) = i:\bar{\psi}(x)\,\gamma^\mu \gamma_5 \tfrac{1}{2}\lambda_a \psi(y): \tag{2b}$$

where $:\ \ :$ denotes the normal product of the field operators.

We postulate the following null-plane anticommutators of vector and axial-vector currents which emerge from canonical manipulations (it is understood that only the operator part of the anticommutators is included):

$$\begin{aligned}\{V^\mu_a(x), V^\nu_b(0)\}_{x^+=0} \hat{=} & -\partial_\alpha\Big\{D^{(1)}(x)\Big[f_{abc}\,(S^{\mu\nu\alpha\beta}V_{c\beta}(x|0) + \epsilon^{\mu\nu\alpha\beta}\bar{a}_{c\beta}(x|0)) \\ & + d_{abc}\,(S^{\mu\nu\alpha\beta}\bar{V}_{c\beta}(x|0) - \epsilon^{\mu\nu\alpha\beta}a_{c\beta}(x|0))\Big]\Big\} \\ & + 2g^{\mu\alpha}D^{(1)}(x)\,\partial_\alpha\,\big[f_{abc}\,V^\nu_c(x|0) + d_{abc}\,\bar{V}^\nu_c(x|0)\big]\quad ,\end{aligned} \tag{3a}$$

$$\begin{aligned}\{V^\mu_a(x), A^\nu_b(0)\}_{x^+=0} \hat{=} & -\partial_\alpha\Big\{D^{(1)}(x)\Big[f_{abc}\,(S^{\mu\nu\alpha\beta}a_{c\beta}(x|0) + \epsilon^{\mu\nu\alpha\beta}\bar{V}_{c\beta}(x|0)) \\ & + d_{abc}\,(S^{\mu\nu\alpha\beta}\bar{a}_{c\beta}(x|0) - \epsilon^{\mu\nu\alpha\beta}V_{c\beta}(x|0))\Big]\Big\} \\ & + 2g^{\mu\alpha}D^{(1)}(x)\,\partial_\alpha\big[f_{abc}\,a^\nu_c(x|0) + d_{abc}\,\bar{a}^\nu_c(x|0)\big]\quad ,\end{aligned} \tag{3b}$$

$$\{A^\mu_a(x), A^\nu_b(0)\}_{x^+=0} \hat{=} \{V^\mu_a(x), V^\nu_b(0)\}_{x^+=0}\ , \tag{3c}$$

where $v^{\mu}_{a}(x|0)$ and $a^{\mu}_{a}(x|0)$ ($\bar{v}$ and $\bar{a}$) are the hermitian (antihermitian) parts of the bilocal currents (2), f_{abc} and d_{abc} are the structure constants of the λ-matrix commutators and anticommutators, $S^{\mu\nu\alpha\beta} = g^{\mu\alpha}g^{\nu\beta} + g^{\mu\beta}g^{\nu\alpha} - g^{\mu\nu}g^{\alpha\beta}$ and $\epsilon^{\mu\nu\alpha\beta}$ is the fully antisymmetric tensor. The singular function $D^{(1)}(x)$ reads, in the notations of Bogolubov and Shirkov [10] :

$$-i D^{(1)}(x) = D^{(+)}(x) - D^{(-)}(x) , \tag{4}$$

where $D^{(\pm)}(x)$ are the positive (+) and negative (-) frequency parts of the Pauli-Jordan commutation function. The symbol $\hat{=}$ indicates that we have retained only the two most singular (leading and next-to-leading) terms of the current anticommutators near the light-cone because only these terms can be expressed through the bilocal currents (2).

Eqs. (3) have therefore an approximate character and the sum rules derived on their basis are asymptotic relations valid in principle only in the limit of infinite current masses $(-q^2 \to \infty)$. However, we expect that they might be well verified for not too large values of q^2 ($-q^2 \gtrsim 1(\mathrm{GeV})^2$) as suggested by the "precocious scaling phenomena" observed in e-p inelastic scattering[11].

Finally, note that the specific q^2- dependence which might occur in the sum rules due to the noncausal nature of the light-cone singularity is removed under the crucial assumption that the form factors describing the matrix elements of the bilocal currents (2) are smooth-behaved near the light-cone.

3. Derivation of sum rules from moment anticommutators

We consider the forward anticommutator function of two conserved vector currents defined in A:

$$
\begin{aligned}
A^{\mu\nu}_{ab}(p,q) &\equiv \int d^4x\, e^{iqx} \langle p,s | \{ V^\mu_a(x), V^\nu_b(0) \} | p,s \rangle = \\
&= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) A^{ab}_L(\nu,q^2) + \left[p^\mu p^\nu - \frac{\nu}{q^2}(p^\mu q^\nu + p^\nu q^\mu) + \frac{\nu^2}{q^2} g^{\mu\nu} \right] A^{ab}_2(\nu,q^2) \\
&\quad + i\, \epsilon^{\mu\nu\alpha\beta} s_\alpha q_\beta\, A^{ab}_3(\nu,q^2) + i\,(q\cdot s)\, \epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta\, A^{ab}_4(\nu,q^2),
\end{aligned}
\tag{5}
$$

where the vector $S^\alpha = i\,\bar{u}(p)\gamma^\alpha\gamma_5 u(p)$ describes the spin states, a and b are isotopic spin indices and $\nu = p\cdot q$.

The $A_i^{ab}(\nu,q^2)$ verify the crossing-symmetry relations:

$$
\begin{aligned}
A^{ab}_i(\nu,q^2) &= A^{ba}_i(-\nu,q^2) \qquad i = L,2,3 \\
A^{ab}_4(\nu,q^2) &= -A^{ba}_4(-\nu,q^2)
\end{aligned}
\tag{6}
$$

and are connected to the conventional structure functions $2W_i^{ab}(\nu,q^2)$ of the absorptive part of the forward virtual Compton scattering amplitude by:

$$
A^{ab}_i(\nu,q^2) = W^{ab}_i(\nu,q^2), \qquad \nu > 0. \tag{7}
$$

We will also need the decomposition of $A^{ab}_i(\nu,q^2)$ into parts symmetric $\left(A^{(ab)}_i\right)$ and antisymmetric $\left(A^{[ab]}_i\right)$ under interchange of a and b:

$$
A^{ab}_i(\nu,q^2) = A^{(ab)}_i(\nu,q^2) + i\, A^{[ab]}_i(\nu,q^2) \tag{8}
$$

whose crossing properties are fixed by Eq. (6).

Now, following Dicus and Palmer [9], we differentiate (5) with respect to q_α $(\alpha = -,1,2)$, integrate over q^- and then set $q^+ \equiv \frac{q^0+q^3}{\sqrt{2}} = 0$. This procedure leads to the general equality:

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty} dq^- \frac{\partial}{\partial q_{\alpha_1}} \cdots \frac{\partial}{\partial q_{\alpha_n}} A^{\mu\nu}_{ab}(p,q)\Big|_{q^+=0} = (i)^n \int d^4x\, e^{-i\vec{q}_\perp \vec{x}_\perp} x^{\alpha_1} \cdots x^{\alpha_n} \langle p,s | \{ V^\mu_a(x), V^\nu_b(0) \} \delta(x^+) | p,s \rangle . \quad (9)$$

Sum rules are then deduced from Eq. (9) with the help of Eqs. (3a), (5)-(8) and of the usual definitions of the bilocal form-factors:

$$\begin{aligned} \langle p,s | v^\mu_a(x|0) | p,s \rangle &= p^\mu V^1_a(x^2, x\cdot p) + x^\mu V^2_a(x^2, x\cdot p) \\ \langle p,s | a^\mu_a(x|0) | p,s \rangle &= s^\mu A^1_a(x^2, x\cdot p) + p^\mu (x\cdot s) A^2_a(x^2, x\cdot p) + \\ &\quad + x^\mu (x\cdot s) A^3_a(x^2, x\cdot p) \end{aligned} \quad (10)$$

and of their light-cone Fourier transforms:

$$\tilde{V}^i_a(k) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\alpha\, e^{-ik\alpha}\, V^i_a(0,\alpha) \qquad i = 1,2 . \quad (10')$$

Relations analogous to (10) and (10') hold respectively for $\bar{v}^\mu_a$, $\bar{a}^\mu_a$ and $\bar{V}^i_a$ $(i=1,2)$, $\bar{A}^i_a$, A^i_a ($i = 1,2,3$). The derivatives $\frac{\partial}{\partial q_\alpha}$ are rewritten with respect to the scalars (ν, q^2) by means of:

$$\frac{\partial W(\nu,q^2)}{\partial q_\alpha} = p^\alpha \frac{\partial W(\nu,q^2)}{\partial \nu} + 2q^\alpha \frac{\partial W(\nu,q^2)}{\partial q^2} .$$

The sum rules and high-energy asymptotic relations emerging from the components $(\mu\nu) = (++)$ and $(+-)$ of Eq. (9) with $n = 1,2$ are given in Tables I and II.

We did not list all of them but only the really new results in comparison with those given in A. So, for example, with $(\mu\nu) = (++)$, $n = 1$ we find also that the first derivative of the Gottfried integral (see A, Eq. (I.1)) with respect to q^2 is equal to zero, but it is a direct consequence of the Gottfried sum rule itself which has been derived with the same (++) anti-commutator (for $n=0$).

In the same manner, the (+-) components ($n=1$) constrain the derivatives with respect to q^2 of the left-hand sides of Eqs. (I.2-6) of A to vanish. Analogous results are deduced for the second derivatives with $n=2$. In addition, the second moments give the q^2- derivatives of the $n=1$ relations (I.1,3-6) and (II.1,4-6).

The (++) relations deserve a particular attention because they are the least model-dependent ones, as is well known [12]. For instance, Eq. (I.1) which implies that

$$\int_0^{+\infty} d\nu \, \frac{\nu}{-q^2} \, W_2^{[ab]}(\nu,q^2) = \mathrm{Const.}$$

is a less model-dependent statement about the integral than the (+-) sum rule Eq. (I.5) of A. Similarly, Eqs. (I.3-6) are less model-dependent and more general relations than the Eqs. (I.7-10) of A which proceeded from the "bad" anticommutators $(\mu\nu) = (ij)$ and $(i-)$ under the additional assumption that $\frac{M^2}{q^2} \to 0$ ($M \equiv$ mass of the nucleon). Eqs. (I.3-6) appear here without any special requirement of this type. Eqs. (I.7-10) are new sum rules involving higher powers of ν which could not have been derived with the method of A.

The high-energy asymptotic relations of Table II are specific consequences of the use of the moment algebra. They originally emerge from Eq. (9) in a form which can be symbolically represented by:

$$\int_{-\infty}^{+\infty} d\nu \frac{\partial}{\partial \nu} f(\nu, q^2) = g(q^2) . \tag{11}$$

Equations of the type (11) are then transformed into statements about the odd parts of the functions $f(\nu, q^2)$ under the exchange $\nu \to -\nu$:

$$\left[f^{(odd)}(\nu, q^2) \right]_{\nu = \infty} = \frac{1}{2} g(q^2) . \tag{12}$$

It is worth noting that all our moment anticommutator asymptotic conditions of Table II deal with structure functions entering light-cone commutator sum rules [2], e.g. Eq. (II.1) is a requirement about the high-energy behaviour of the integrand of the Fubini-Dashen-Gell-Mann sum rule [13]. Inversely, the asymptotic conditions derived from the moment commutators should involve structure functions appearing in the anticommutator or "wrong-signature" sum rules of A. We shall return to this point in the next Section.

Table I

	$(\mu\nu)$	moment	Sum rules for forward virtual Compton scattering from light-cone moment anti-commutators
1	(++)	1	$\frac{\partial}{\partial q^2}\int_0^\infty d\nu\,\nu\,W_2'^{[al]}(\nu,q^2) - \frac{1}{q^2}\int_0^\infty d\nu\,\nu\,W_2'^{[al]}(\nu,q^2) = 0$
2	(++)	2	$2\frac{\partial^2}{\partial(q^2)^2}\int_0^\infty d\nu\,\nu^2 W_2'^{(al)}(\nu,q^2) - 4\frac{\partial}{\partial q^2}\frac{1}{q^2}\int_0^\infty d\nu\,\nu^2 W_2'^{(al)}(\nu,q^2) - \frac{1}{q^2}\int_0^\infty d\nu\, W_L'^{(al)}(\nu,q^2) = 0$
3	(+-)	1	$2\frac{\partial}{\partial q^2}\int_0^\infty d\nu\,\nu\,W_L'^{[al]}(\nu,q^2) - \frac{1}{q^2}\int_0^\infty d\nu\,\nu\,W_L'^{[al]}(\nu,q^2) = 2\pi i f_{alr}\int_0^\infty dK\,\frac{\tilde{V}_c^2(K)}{K^2}$
4	(+-)	1	$2\frac{\partial}{\partial q^2}\frac{1}{q^2}\int_0^\infty d\nu\,\nu^2 W_2'^{(al)}(\nu,q^2) + 2\frac{\partial}{\partial q^2}\int_0^\infty d\nu\, W_L'^{(al)}(\nu,q^2) - \frac{1}{q^2}\int_0^\infty d\nu\, W_L'^{(al)}(\nu,q^2) = \pi i d_{alr}\int_0^\infty dK\,\frac{\tilde{V}_c^1}{K^2}$
5	(+-)	1	$\frac{\partial}{\partial q^2}\int_0^\infty d\nu\,\nu\,W_3'^{[al]}(\nu,q^2) = -\frac{\pi i}{2} f_{alr}\int_0^\infty dK\,\frac{\tilde{A}_c^1(K)}{K^2}$
6	(+-)	1	$\frac{\partial}{\partial q^2}\int_0^\infty d\nu\,\nu^2 W_4'^{[al]}(\nu,q^2) = \pi f_{alr}\int_0^\infty dK\,\frac{\tilde{A}_c^2(K)}{K^3}$
7	(+-)	2	$\frac{\partial^2}{\partial(q^2)^2}\int_0^\infty d\nu\,\nu^2 W_L'^{(al)}(\nu,q^2) - \frac{\partial}{\partial q^2}\frac{1}{q^2}\int_0^\infty d\nu\,\nu^2 W_L'^{(al)}(\nu,q^2) = \pi d_{alr}\int_0^\infty dK\,\frac{\tilde{V}_c^2(K)}{K^3}$
8	(+-)	2	$\frac{\partial^2}{\partial(q^2)^2}\frac{1}{q^2}\int_0^\infty d\nu\,\nu^3 W_2'^{[al]}(\nu,q^2) + 2\frac{\partial^2}{\partial(q^2)^2}\int_0^\infty d\nu\,\nu\,W_L'^{[al]}(\nu,q^2) - 2\frac{\partial}{\partial q^2}\frac{1}{q^2}\int_0^\infty d\nu\,\nu\,W_L'^{[al]}(\nu,q^2) = -\frac{\pi}{2} f_{alr}\int_0^\infty dK\,\frac{\tilde{V}_c^1}{K^3}$
9	(+-)	2	$\frac{\partial^2}{\partial(q^2)^2}\int_0^\infty d\nu\,\nu^2 W_3'^{(al)}(\nu,q^2) = \frac{\pi}{2} d_{alr}\int_0^\infty dK\,\frac{\tilde{A}_c^1(K)}{K^3}$
10	(+-)	2	$\frac{\partial^2}{\partial(q^2)^2}\int_0^\infty d\nu\,\nu^3 W_4'^{(al)}(\nu,q^2) = -\frac{3\pi}{2} d_{alr}\int_0^\infty dK\,\frac{\tilde{A}_c^2(K)}{K^4}$

Table II

	$(\mu\nu)$	moment	High energy asymptotic conditions for forward virtual Compton scattering from light-cone moment anticommutators
1	$(++)$	1	$\left[W_2^{[ab]}(\nu,q^2) \right]_{\nu=\infty} = 0$
2	$(++)$	2	$\left[\frac{\partial}{\partial\nu} W_2^{(ab)}(\nu,q^2) \right]_{\nu=\infty} = 0$
3	$(++)$	2	$\left[\frac{\partial}{\partial q^2} \nu W_2^{(ab)}(\nu,q^2) - \frac{\nu}{q^2} W_2^{(ab)}(\nu,q^2) \right]_{\nu=\infty} = 2\frac{\partial}{\partial q^2} \int_0^\infty d\nu\, W_2^{(ab)}(\nu,q^2) = 0$
4	$(+-)$	1	$\left[\frac{\nu}{q^2} W_2^{(ab)}(\nu,q^2) \right]_{\nu=\infty} = 0$
5	$(+-)$	1	$\left[W_L^{[ab]}(\nu,q^2) \right]_{\nu=\infty} = \pi f_{abc} \int_0^\infty dK \frac{\tilde{V}_c^1(K)}{K} + \int_0^\infty d\nu \frac{\nu}{q^2} W_2^{[ab]}(\nu,q^2)$
6	$(+-)$	1	$\left[W_3^{[ab]}(\nu,q^2) \right]_{\nu=\infty} = 0,\ \left[W_4^{(ab)}(\nu,q^2) \right]_{\nu=\infty} = 0,\ \left[\nu W_4^{[ab]}(\nu,q^2) \right]_{\nu=\infty} = 0$
7	$(+-)$	2	$\left[\frac{\partial}{\partial\nu} \frac{\nu}{q^2} W_2^{[ab]}(\nu,q^2) \right]_{\nu=\infty} = 0$
8	$(+-)$	2	$\left[\frac{\partial}{\partial\nu} W_L^{(ab)}(\nu,q^2) - 2\frac{\nu}{q^2} W_2^{(ab)}(\nu,q^2) \right]_{\nu=\infty} = 0$
9	$(+-)$	2	$\left[2\frac{\partial}{\partial q^2} \nu W_L^{(ab)}(\nu,q^2) - \frac{\nu}{q^2} W_L^{(ab)}(\nu,q^2) \right]_{\nu=\infty} =$ $= -\pi i d_{abc} \int_0^\infty dK \frac{\tilde{V}_c^1(K)}{K^2} + 2\frac{\partial}{\partial q^2} \int_0^\infty d\nu \frac{\nu^2}{q^2} W_2^{(ab)}(\nu,q^2) + 2\frac{\partial}{\partial q^2} \int_0^\infty d\nu\, W_L^{(ab)}(\nu,q^2) - \frac{1}{q^2} \int_0^\infty d\nu\, W_L^{(ab)}(\nu,q^2)$
10	$(+-)$	2	$\left[2\frac{\partial}{\partial q^2} \frac{\nu^2}{q^2} W_2^{[ab]}(\nu,q^2) + 2\frac{\partial}{\partial q^2} W_L^{[ab]}(\nu,q^2) - \frac{1}{q^2} W_L^{[ab]}(\nu,q^2) \right]_{\nu=\infty} =$ $= 4\frac{\partial}{\partial q^2} \int_0^\infty d\nu \frac{\nu}{q^2} W_2^{[ab]}(\nu,q^2) = 4\left[\frac{\partial}{\partial q^2} W_L^{[ab]}(\nu,q^2) \right]_{\nu=\infty}$
11	$(+-)$	2	$\left[\frac{\partial}{\partial\nu} W_3^{(ab)}(\nu,q^2) \right]_{\nu=\infty} = 0,\ \left[\frac{\partial}{\partial\nu} W_4^{[ab]}(\nu,q^2) \right]_{\nu=\infty} = 0,\ \left[\frac{\partial}{\partial\nu} \nu W_4^{(ab)}(\nu,q^2) \right]_{\nu=\infty} = 0$
12	$(+-)$	2	$\left[\frac{\partial}{\partial q^2} \nu W_3^{(ab)}(\nu,q^2) \right]_{\nu=\infty} = \frac{\partial}{\partial q^2} \int_0^\infty d\nu\, W_3^{(ab)}(\nu,q^2),\ \left[\frac{\partial}{\partial q^2} \nu^2 W_4^{(ab)}(\nu,q^2) \right]_{\nu=\infty} = 0$

4. Nonforward spin-independent virtual Compton scattering sum rules

The results of A and of the previous section are now extended to the nonforward spin-independent virtual Compton scattering amplitudes. Apart from a more complicated kinematical situation, the previous methods apply automatically.

We first define the spin-averaged nondiagonal matrix element of the anticommutator of two conserved vector currents between fermion states with 4-momenta p_1 and p_2 :

$$A^{\mu\nu}_{ab}(p_2 q_2; p_1 q_1) \equiv \int d^4x\, e^{i(q_1+q_2)\frac{1}{2}x} \langle p_2 | \{ V^{\mu}_{a}(\tfrac{x}{2}), V^{\nu}_{b}(-\tfrac{x}{2}) \} | p_1 \rangle . \qquad (13)$$

Following Gross [14], we decompose this nonforward anticommutator function into invariants:

$$\begin{aligned} A^{\mu\nu}_{ab}(p_2 q_2; p_1 q_1) = \mathcal{O}^{\mu\alpha}(q_1)\, \mathcal{O}^{\nu\beta}(q_2) \Big[& -g_{\alpha\beta} A^{ab}_1(\nu, Q^2, t, \delta) + P_\alpha P_\beta A^{ab}_2(\nu, Q^2, t, \delta) \\ & + (P_\alpha \Delta_\beta - P_\beta \Delta_\alpha) A^{ab}_3(\nu, Q^2, t, \delta) + (P_\alpha \Delta_\beta + P_\beta \Delta_\alpha) A^{ab}_4(\nu, Q^2, t, \delta) + \Delta_\alpha \Delta_\beta A^{ab}_5(\nu, Q^2, t, \delta) \Big] \end{aligned} \qquad (14)$$

where the projection operators $\mathcal{O}^{\mu\nu}(q) \equiv \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right)$ ensure current conservation,

$$P = \tfrac{1}{2}(p_1 + p_2), \quad Q = \tfrac{1}{2}(q_1 + q_2), \quad \Delta = q_2 - q_1 = p_1 - p_2 \qquad (15a)$$

and

$$\nu = P\cdot Q, \quad t = \Delta^2, \quad \delta = Q\cdot\Delta = \tfrac{1}{2}(q_2^2 - q_1^2) \qquad (15b)$$

The invariant functions $A_i^{ab}(\nu, Q^2, t, \delta)$ possess the crossing-symmetry properties:

$$A_i^{ab}(\nu,Q^2,t,\delta) = A_i^{ba}(-\nu,Q^2,t,-\delta) \qquad i = 1,2,4,5$$
$$A_3^{ab}(\nu,Q^2,t,\delta) = - A_3^{ba}(-\nu,Q^2,t,-\delta) . \tag{16}$$

By definition, for $\nu > 0$ $A_{ab}^{\mu\nu}(p_2 q_2 ; p_1 q_1)$ is proportional to the imaginary part of the nonforward Compton scattering amplitude, i.e.

$$A_i^{ab}(\nu,Q^2,t,\delta) = W_i^{ab}(\nu,Q^2,t,\delta) \ , \ \nu > 0, \tag{17}$$

where the W_i^{ab} are the structure functions used in Ref. [14]. The sum rules will be most conveniently written in terms of combinations of the functions W_i^{ab}:

$$2W_{i(\pm)}^{ab}(\nu,Q^2,t,\delta) = W_i^{ab}(\nu,Q^2,t,\delta) \pm W_i^{ba}(\nu,Q^2,t,-\delta). \tag{18}$$

The nondiagonal matrix elements of the bilocal currents, summed over the spin , are expressed in terms of form-factors by the relations [14]:

$$\langle p_2 | V_a^\mu(x|0) | p_1 \rangle = P^\mu V_a^1(x^2, x\cdot P, x\cdot\Delta, t) + x^\mu V_a^2(x^2, x\cdot P, x\cdot\Delta, t) + \\ + i\Delta^\mu V_a^3(x^2, x\cdot P, x\cdot\Delta, t) \tag{19a}$$

$$\langle p_2 | a_a^\mu(x|0) | p_1 \rangle = i\,\epsilon^{\mu\nu\alpha\beta} P_\nu \Delta_\alpha x_\beta A_a(x^2, x\cdot P, x\cdot\Delta, t) \tag{19b}$$

and similar expressions hold for $\overline{V}^{r}_{a}(x|0)$ and $\overline{a}^{r}_{a}(x|0)$.

Sum rules for the functions $W^{ab}_{i}(\nu, Q^2, t, \delta)$ are now easily derived from the basic equality of the method of Dicus et al.[2]:

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty} dq^- A^{\mu\nu}_{ab}(p_2q_2, p_1q_1)\Big|_{q_1^+ = q_2^+ = 0} = \int d^4x\, e^{-i\vec{q}_{2\perp}\cdot\vec{x}_\perp} \langle p_2|\{V^{\mu}_{a}(x), V^{\nu}_{b}(0)\}\delta(x^+)|p_1\rangle, \tag{20}$$

where $q^{\pm} = \frac{q^0 \pm q^3}{\sqrt{2}}$, $\vec{q}_\perp = (q_1, q_2)$.

Eq. (20) is evaluated by means of Eqs. (3a), (13)-(19) and by using the light-cone Fourier transforms of the bilocal form-factors (19):

$$\tilde{V}^{i}_{a}(k, t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\alpha\, e^{-ik\alpha}\, V^{i}_{a}(0, \alpha, 0, t), \tag{21}$$

where $\alpha \equiv x\cdot P$.

The resulting sum rules are given in Table III. Eqs. (III.1, 2,5) are $t \neq 0$ generalizations of the Gottfried sum rule, Eq. (I.1) and of Eqs. (I.5,6) of A respectively.

We have also summarized in Table IV some relations which can be obtained by applying the moment anticommutator technique to the (++) components of Eq.(13). Note in addition that the first and second derivatives with respect to Q^2 and δ of the generalized Gottfried integral (III.1) are also constrained to vanish. Eqs. (IV.1-6) generalize Eqs. (I.1,2) and (II.1-3) to the nonforward direction.

The high-energy asymptotic constraints (IV.1,3,4) are obtained as in the forward case (see Eqs. (11), (12)) from general equations of the type:

$$\int_{-\infty}^{+\infty} d\nu \frac{\partial}{\partial\nu} f(\nu, Q^2, t, \delta) = g(Q^2, t, \delta) \tag{22}$$

which imply that the odd parts of the functions $f(\nu, Q^2, t, \delta)$ under the exchange $\nu \rightarrow -\nu$ verify the relation:

$$\left[f^{(odd)}(\nu, Q^2, t, \delta) \right]_{\nu=\infty} = \frac{1}{2}\, g(Q^2, t, \delta) \,. \tag{23}$$

We would like to point out here that we may not agree with Dicus and Palmer [9] who deduce systematically opposite conclusions from the same Eqs. (22) occuring in the light-cone moment commutator case, i.e. that Eq. (23) should hold for the even parts of $f(\nu, Q^2, t, \delta)$ instead of the odd ones.

Hence they claim in Ref. [9] that the first moments of the (++) commutator imply (with our notations):

$$\left[W_{2(-)}^{a\ell}(\nu, Q^2, t, \delta) \right]_{\nu=\infty} = 0 \tag{24}$$

while our conclusion is:

$$\left[W_{2(+)}^{a\ell}(\nu, Q^2, t, \delta) \right]_{\nu=\infty} = 0 \,. \tag{25}$$

Eq. (24) is actually a moment anticommutator result as indicated by Eq. (II.1). In this connection the remarks made at the end of Section 3 should be extended to the nonforward direction too.

Table III

	$(\mu\nu)$	Sum rules for nonforward virtual Compton scattering from light-cone current anticommutators
1	$(++)$	$\int_0^\infty d\nu\, W^{ab}_{2(+)}(\nu,Q^2,t,\delta) = 2d_{abc}\int_0^\infty d\alpha\, \frac{\bar{V}^1_c(0,\alpha,0,t)}{\alpha}$
2	$(+-)$	$\int_0^\infty d\nu\, \frac{\nu}{q_1\cdot q_2}\, W^{ab}_{2(-)}(\nu,Q^2,t,\delta) = -i\pi f_{abc}\int_0^\infty dK\, \frac{\tilde{V}^1_c(K,t)}{K}$
3	$(+-)$	$\int_0^\infty d\nu\, W^{ab}_{3(-)}(\nu,Q^2,t,\delta) = i\pi f_{abc}\int_0^\infty dK\, \frac{\tilde{V}^1_c(K,t)}{K}$
4	$(+-)$	$\int_0^\infty d\nu\, W^{ab}_{4(+)}(\nu,Q^2,t,\delta) = 0$
5	$(+-)$	$2\int_0^\infty d\nu\, W^{ab}_{1(+)}(\nu,Q^2,t,\delta) + \int_0^\infty d\nu\, \frac{\nu}{q_1^2 q_2^2}\Big\{(q_1^2+q_2^2)\nu\, W^{ab}_{2(+)}(\nu,Q^2,t,\delta) + \left[2\delta^2-\frac{1}{2}t(q_1^2+q_2^2)\right] W^{ab}_{3(+)}(\nu,Q^2,t,\delta) + 2q_1q_2\delta\, W^{ab}_{4(-)}(\nu,Q^2,t,\delta)\Big\} = i\pi d_{abc}\, t\int_0^\infty dK\, \frac{\tilde{A}_c(K,t)}{K^2}$
6	$(+-)$	$\int_0^\infty d\nu\, \frac{\nu}{q_1^2 q_2^2}\Big\{2\delta\nu\, W^{ab}_{2(+)}(\nu,Q^2,t,\delta) + 2q_1q_2\delta\, W^{ab}_{3(+)}(\nu,Q^2,t,\delta) + \left[2\delta^2-\frac{1}{2}t(q_1^2+q_2^2)\right] W^{ab}_{4(-)}(\nu,Q^2,t,\delta)\Big\} = 2i\pi d_{abc}\,\delta\int_0^\infty dK\, \frac{\tilde{A}_c(K,t)}{K^2}$

Table IV

	moment	Sum rules and high-energy asymptotic conditions for nonforward virtual Compton scattering from the light-cone moment anticommutator (++)
1	1	$\left[W^{ab}_{2(-)}(\nu,Q^2,t,\delta) \right]_{\nu=\infty} = 0$
2	1	$2\frac{\partial}{\partial Q^2}\int_0^\infty d\nu\, \nu\, W^{ab}_{2(-)}(\nu,Q^2,t,\delta) - \int_0^\infty d\nu\, W^{ab}_{(-)}(\nu,Q^2,t,\delta) = 0$
3	2	$\left[\frac{\partial}{\partial\nu} W^{ab}_{2(+)}(\nu,Q^2,t,\delta)\right]_{\nu=\infty} = 0$
4	2	$\left[\frac{\partial}{\partial Q^2} W^{ab}_{2(-)}(\nu,Q^2,t,\delta)\right]_{\nu=\infty} = 0 \quad , \left[\frac{\partial}{\partial \delta} W^{ab}_{2(-)}(\nu,Q^2,t,\delta)\right]_{\nu=\infty} = 0$
5	2	$\left[2\frac{\partial}{\partial Q^2}\nu W^{ab}_{2(+)}(\nu,Q^2,t,\delta) - W^{ab}_{(+)}(\nu,Q^2,t,\delta)\right]_{\nu=\infty} = 2\frac{\partial}{\partial Q^2}\int_0^\infty d\nu\, W^{ab}_{2(+)}(\nu,Q^2,t,\delta)$
6	2	$\frac{1}{q_1^2 q_2^2}\int_0^\infty d\nu \Big\{ (q_1^2+q_2^2-q_1 q_2) W^{ab}_{1(+)}(\nu,Q^2,t,\delta) + \nu^2 W^{ab}_{2(+)}(\nu,Q^2,t,\delta) - t\nu W^{ab}_{3(+)}(\nu,Q^2,t,\delta)$ $+ 2\delta\nu W^{ab}_{4(-)}(\nu,Q^2,t,\delta) + (\delta^2 - \frac{1}{4}t^2) W^{ab}_{5(+)}(\nu,Q^2,t,\delta)\Big\} +$ $+ 2\frac{\partial^2}{\partial(Q^2)^2}\int_0^\infty d\nu\, \nu^2 W^{ab}_{2(+)}(\nu,Q^2,t,\delta) - 2\frac{\partial}{\partial Q^2}\int_0^\infty d\nu\, \nu W^{ab}_{(+)}(\nu,Q^2,t,\delta) = 0$
		with: $W^{ab}_{(\pm)}(\nu,Q^2,t,\delta) \equiv \frac{1}{q_1^2 q_2^2}\Big\{(q_1^2+q_2^2)\nu W^{ab}_{2(\pm)}(\nu,Q^2,t,\delta) + \left[2\delta^2 - \frac{1}{2}t(q_1^2+q_2^2)\right] W^{ab}_{3(\pm)}(\nu,Q^2,t,\delta)$ $+ 2q_1 q_2 \delta\, W^{ab}_{4(\mp)}(\nu,Q^2,t,\delta)\Big\}$

5. Neutrinoproduction sum rules

Until now we have dealt only with conserved currents although it is not especially required by the method. We would like to generalize our results to the case when nonconserved axial-vector currents are present, i.e. to the structure functions of the neutrino-hadron interaction. We shall restrict ourselve to a brief survey of the most significant relations which emerge from the study of the spin-independent forward amplitudes.

The basic object of our investigation reads:

$$\bar{A}^{\mu\nu}_{ab}(p,q) \equiv \int d^4x\, e^{iqx} \langle p | \{ J^{\mu}_{a}(x), J^{\nu}_{b}(0) \} | p \rangle , \qquad (26)$$

where $J^{\mu}_{a}(x) = V^{\mu}_{a}(x) - A^{\mu}_{a}(x)$ is the usual V-A weak current, a and b are SU_3 indices and the spins have been summed over. In the quark field theory model the vector and axial-vector currents are given by Eqs. (1).

The function $\bar{A}^{\mu\nu}_{ab}(p,q)$ has the tensor structure:

$$\bar{A}^{\mu\nu}_{ab}(p,q) = \left(-g^{\mu\nu} + \frac{q^{\mu}q^{\nu}}{q^2}\right) \bar{A}^{ab}_{L}(\nu,q^2) + \Big[p^{\mu}p^{\nu} - \frac{\nu}{q^2}(p^{\mu}q^{\nu} + p^{\nu}q^{\mu}) + \frac{\nu^2}{q^2} g^{\mu\nu} \Big] \bar{A}^{ab}_{2}(\nu,q^2) + i\, \epsilon^{\mu\nu\alpha\beta} p_{\alpha} q_{\beta}\, \bar{A}^{ab}_{3}(\nu,q^2) + q^{\mu}q^{\nu} \bar{A}^{ab}_{4}(\nu,q^2) + (p^{\mu}q^{\nu} + p^{\nu}q^{\mu}) \bar{A}^{ab}_{5}(\nu,q^2) \qquad (27)$$

$\bar{A}^{ab}_{3}$ and $\bar{A}^{ab}_{4,5}$ are respectively the parity-violating and chiral-symmetry breaking structure functions. Crossing implies that:

$$\bar{A}_i^{ab}(\nu,q^2) = \bar{A}_i^{ba}(-\nu,q^2) \qquad i = 1,2,3,4$$
$$\bar{A}_5^{ab}(\nu,q^2) = -\bar{A}_5^{ba}(-\nu,q^2) \tag{28}$$

$\bar{A}^{\mu\nu}_{ab}(p,q)$ is connected to the absorptive part of the causal current-hadron interaction amplitude

$$\bar{W}^{\mu\nu}_{ab}(p,q) \equiv \int d^4x\, e^{iqx} \langle p| \left[J_a^\mu(x), J_b^\nu(0) \right] |p\rangle . \tag{29}$$

If one decomposes $\bar{W}^{\mu\nu}_{ab}$ into structure functions W_i^{ab} in the same way as $\bar{A}^{\mu\nu}_{ab}$ (see Eq. (27)) one has:

$$\bar{A}_i^{ab}(\nu,q^2) = \bar{W}_i^{ab}(\nu,q^2) \quad , \quad \nu > 0 . \tag{30}$$

Their parts symmetric and antisymmetric with respect to a and b are defined as in Eq. (8).

Now, applying the same methods as in A and in Section 3 of the present work, we are able to derive a set of sum rules and asymptotic relations for the weak structure functions $\bar{W}_i^{ab}$ on the basis of the postulated null-plane anticommutators Eqs. (3).

The most interesting of them are listed in Tables V and VI.

6. Concluding remarks

We have formally derived in the present work some consequences of the light-cone structure postulated for the current anticommutators. The obtained relations require a further analysis and the divergent expressions must be regularized in order to be compared with experiment. Indeed, we have written "divergent" sum rules as formal equalities, requiring only that the criterion of self-consistency proposed by Dicus et al. [2] is satisfied, i.e. that the sum rules are true in free-field theory. The regularization of the divergent expressions may be performed by employing the well-known techniques [16] for converting them into, e.g. finite energy sum rules by means of the subtraction of the leading Regge-pole contributions. One may also combine different structure functions in such a way that these leading contributions cancel (for example, the Gottfried sum rule for the ep-en difference converges because of the cancellation of the Pomeranchukon exchanges).

It should be especially interesting, in our opinion, to investigate further the possible violation of the scaling behaviour of the "wrong-signature" sum rules in the Bjorken region which might occur if the matrix elements of the bilocal currents relevant to the real world, unlike the situation discussed in this paper, appeared to be not sufficiently regular in the vicinity of the light-cone.

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Table V

	$(\pm)$	moment	Neutrinoproduction sum rules from light-cone anticommutators of currents and their moments
1	(++)	0	$\int_0^\infty d\nu\, \overline{W}_2^{(al)}(\nu,q^2) = 4 d_{alr} \int_0^\infty d\alpha \frac{\overline{V}_c^1(0,\alpha)}{\alpha}$
2	(++)	1	$\frac{\partial}{\partial q^2} \int_0^\infty d\nu\, \nu\, \overline{W}_2^{[al]}(\nu,q^2) + \int_0^\infty d\nu \left[\frac{\nu}{-q^2} \overline{W}_2^{[al]}(\nu,q^2) + \overline{W}_5^{[al]}(\nu,q^2)\right] = 0$
3	(++)	2	$2\frac{\partial^2}{\partial (q^2)^2} \int_0^\infty d\nu\, \nu^2 \overline{W}_2^{(al)}(\nu,q^2) + 4\frac{\partial}{\partial q^2} \int_0^\infty d\nu\, \nu \left[\frac{\nu}{-q^2} \overline{W}_2^{(al)}(\nu,q^2) + \overline{W}_5^{(al)}(\nu,q^2)\right] + \int_0^\infty d\nu \left[\frac{1}{q^2} \overline{W}_L^{(al)}(\nu,q^2) + \overline{W}_4^{(al)}(\nu,q^2)\right] = 0$
4	(+-)	0	$\int_0^\infty d\nu\, \overline{W}_3^{(al)}(\nu,q^2) = -2\pi d_{alr} \int_0^\infty dK \frac{\tilde{V}_c^1(K)}{K}$
5	(+-)	0	$\int_0^\infty d\nu \left[\frac{\nu}{-q^2} \overline{W}_2^{[al]}(\nu,q^2) + \overline{W}_5^{[al]}(\nu,q^2)\right] = 2\pi f_{alr} \int_0^\infty dK \frac{\tilde{V}_c^1(K)}{K}$
6	(+-)	0	$\int_0^\infty d\nu \left[-\overline{W}_L^{(al)}(\nu,q^2) + \nu\, \overline{W}_5^{(al)}(\nu,q^2)\right] = 0$
7	(+-)	1	$\frac{\partial}{\partial q^2} \int_0^\infty d\nu\, \nu\, \overline{W}_3^{[al]}(\nu,q^2) = \pi i f_{alr} \int_0^\infty dK \frac{\tilde{V}_c^1(K)}{K^2}$
8	(+-)	1	$2\frac{\partial}{\partial q^2} \int_0^\infty d\nu \left[-\overline{W}_L^{(al)}(\nu,q^2) + \frac{\nu^2}{-q^2} \overline{W}_2^{(al)}(\nu,q^2) + 2\nu W_5^{(al)}(\nu,q^2)\right] + \int_0^\infty d\nu \left[\frac{1}{q^2} \overline{W}_L^{(al)}(\nu,q^2) + \overline{W}_4^{(al)}(\nu,q^2)\right] = -2\pi i d_{alr} \int_0^\infty dK \frac{\tilde{V}_c^1(K)}{K^2}$
9	(+-)	1	$2\frac{\partial}{\partial q^2} \int_0^\infty d\nu\, \nu \left[-\overline{W}_L^{[al]}(\nu,q^2) + \nu \overline{W}_5^{[al]}(\nu,q^2)\right] + \int_0^\infty d\nu\, \nu \left[\frac{1}{q^2} \overline{W}_L^{[al]}(\nu,q^2) + \overline{W}_4^{[al]}(\nu,q^2)\right] = -4\pi i f_{alr} \int_0^\infty dK \frac{\tilde{V}_c^2(K)}{K^2}$
10	(+-)	2	$\frac{\partial^2}{\partial (q^2)^2} \int_0^\infty d\nu\, \nu^2 \overline{W}_3^{(al)}(\nu,q^2) = -\pi i d_{alr} \int_0^\infty dK \frac{\tilde{V}_c^1(K)}{K^3}$
11	(+-)	2	$\frac{\partial^2}{\partial (q^2)^2} \int_0^\infty d\nu\, \nu \left[-2\overline{W}_L^{[al]}(\nu,q^2) + \frac{\nu^2}{-q^2} \overline{W}_2^{[al]}(\nu,q^2) + 3\nu \overline{W}_5^{[al]}(\nu,q^2)\right] + 2\frac{\partial}{\partial q^2} \int_0^\infty d\nu\, \nu \left[\frac{1}{q^2} \overline{W}_L^{[al]}(\nu,q^2) + \overline{W}_4^{[al]}(\nu,q^2)\right] = \pi f_{alr} \int_0^\infty dK \frac{\tilde{V}_c^1(K)}{K^3}$
12	(+-)	2	$\frac{\partial^2}{\partial (q^2)^2} \int_0^\infty d\nu\, \nu^2 \left[-\overline{W}_L^{(al)}(\nu,q^2) + \nu \overline{W}_5^{(al)}(\nu,q^2)\right] + \frac{\partial}{\partial q^2} \int_0^\infty d\nu\, \nu^2 \left[\frac{1}{q^2} \overline{W}_L^{(al)}(\nu,q^2) + \overline{W}_4^{(al)}(\nu,q^2)\right] = -2\pi d_{alr} \int_0^\infty dK \frac{\tilde{V}_c^2(K)}{K^3}$

Table VI

	$(\mu\nu)$	moment	High-energy asymptotic conditions for neutrinoproduction structure functions from light-cone moment anticommutators
1	(++)	1	$\left[\bar{W}_1^{[ab]}(\nu,q^2)\right]_{\nu=\infty} = 0$
2	(++)	2	$\left[\frac{\partial}{\partial\nu}\bar{W}_2^{(ab)}(\nu,q^2)\right]_{\nu=\infty} = 0$
3	(++)	2	$\left[\frac{\partial}{\partial q^2}\nu\bar{W}_2^{(ab)}(\nu,q^2) - \frac{\nu}{q^2}\bar{W}_2^{(ab)}(\nu,q^2) + \bar{W}_5^{(ab)}(\nu,q^2)\right]_{\nu=\infty} = \frac{\partial}{\partial q^2}\int_0^\infty d\nu\, \bar{W}_2^{(ab)}(\nu,q^2) = 0$
4	(+-)	1	$\left[-\bar{W}_L^{[ab]}(\nu,q^2) + \nu\bar{W}_5^{[ab]}(\nu,q^2)\right]_{\nu=\infty} = -2\pi f_{abc}\int_0^\infty dK\,\frac{\tilde{V}_c^1(K)}{K} + \int_0^\infty d\nu\left[\frac{\nu}{-q^2}\bar{W}_2^{[ab]}(\nu,q^2) + \bar{W}_5^{[ab]}(\nu,q^2)\right]$
5	(+-)	1	$\left[\frac{\nu}{-q^2}\bar{W}_2^{(ab)}(\nu,q^2) + \bar{W}_5^{(ab)}(\nu,q^2)\right]_{\nu=\infty} = 0$
6	(+-)	1	$\left[\bar{W}_3^{[ab]}(\nu,q^2)\right]_{\nu=\infty} = 0$
7	(+-)	2	$\left[\frac{\partial}{\partial\nu}\left(-\bar{W}_L^{(ab)}(\nu,q^2) + \nu\bar{W}_5^{(ab)}(\nu,q^2)\right) - 2\left(\frac{\nu}{-q^2}\bar{W}_2^{(ab)}(\nu,q^2) + \bar{W}_5^{(ab)}(\nu,q^2)\right)\right]_{\nu=\infty} = 0$
8	(+-)	2	$\left[\frac{\partial}{\partial\nu}\left(\frac{\nu}{-q^2}\bar{W}_2^{[ab]}(\nu,q^2) + \bar{W}_5^{[ab]}(\nu,q^2)\right)\right]_{\nu=\infty} = 0$
9	(+-)	2	$\left[2\frac{\partial}{\partial q^2}\nu\left(-\bar{W}_L^{(ab)}(\nu,q^2) + \nu\bar{W}_5^{(ab)}(\nu,q^2)\right) + \nu\left(\frac{1}{q^2}\bar{W}_L^{(ab)}(\nu,q^2) + \bar{W}_4^{(ab)}(\nu,q^2)\right)\right]_{\nu=\infty} =$ $= 2\pi i d_{abc}\int_0^\infty dK\,\frac{\tilde{V}_c^1(K)}{K^2} - \frac{2\partial}{\partial q^2}\int_0^\infty d\nu\left[\bar{W}_L^{(ab)}(\nu,q^2) + \frac{\nu^2}{q^2}\bar{W}_2^{(ab)}(\nu,q^2) - 2\nu\bar{W}_5^{(ab)}(\nu,q^2)\right] + \int_0^\infty d\nu\left[\frac{1}{q^2}\bar{W}_L^{(ab)} + \bar{W}_4^{(ab)}\right.$
10	(+-)	2	$\left[2\frac{\partial}{\partial q^2}\left(\bar{W}_L^{[ab]}(\nu,q^2) + \frac{\nu^2}{q^2}\bar{W}_2^{[ab]}(\nu,q^2) - 2\nu\bar{W}_5^{[ab]}(\nu,q^2)\right) - \left(\frac{1}{q^2}\bar{W}_L^{[ab]}(\nu,q^2) + \bar{W}_4^{[ab]}(\nu,q^2)\right)\right] =$ $= 4\frac{\partial}{\partial q^2}\int_0^\infty d\nu\left[\frac{\nu}{q^2}\bar{W}_2^{[ab]}(\nu,q^2) - \bar{W}_5^{[ab]}(\nu,q^2)\right] - 4\left[\frac{\partial}{\partial q^2}\left(\bar{W}_L^{[ab]}(\nu,q^2) - \nu\bar{W}_5^{[ab]}(\nu,q^2)\right)\right]_{\nu=\infty}$
11	(+-)	2	$\left[\frac{\partial}{\partial\nu}\bar{W}_3^{(ab)}(\nu,q^2)\right]_{\nu=\infty} = 0$
12	(+-)	2	$\left[\frac{\partial}{\partial q^2}\nu\bar{W}_3^{(ab)}(\nu,q^2)\right]_{\nu=\infty} = \frac{\partial}{\partial q^2}\int_0^\infty d\nu\,\bar{W}_3^{(ab)}(\nu,q^2)$

References:

1. S.B.Gerasimov, J.Moulin. JINR E2-7247, Dubna (1973).
2. D.A.Dicus, R.Jackiw, V.L.Teplitz. Phys.Rev.,D4, 1733 (1971).
3. R.P.Feynman, Phys.Rev.Lett.,23, 1415 (1969).
   J.D.Bjorken, E.A.Paschos. Phys.Rev.,185, 1975 (1969).
4. K.Gottfried. Phys.Rev.Lett.,18, 1174 (1967).
5. S.B.Gerasimov. JINR E2-4295, Dubna (1969).
6. M.I.Pavkovic. Nuovo Cimento,15A, 127 (1973).
7. N.Cabibbo, L.Radicati. Phys.Lett.,19, 697 (1966).
   K.Kawarabayashi, M.Suzuki. Phys.Rev.,150, 1181 (1966).
   M.Hosoda, K.Yamamoto. Progr.Theor.Phys.,36, 425 (1966).
   A.Pietti. Phys.Rev.,142, B1258 (1966).
8. D.A.Dicus, D.R.Palmer. Phys.Rev.,D6, 720 (1972).
9. D.A.Dicus, D.R.Palmer. Ann.of Phys.,79, 68 (1973).
10. N.N.Bogolubov, D.V.Shirkov. Introduction to the theory of quantized fields (Interscience, New York, 1959).
11. E.D.Blom, F.J.Gilman. Phys.Rev.,D4,2901 (1971).
12. Y.Georgelin, J.Stern, J.Jersak. Nucl.Phys.,B27, 493 (1971).
    D.L.Palmer. Phys.Rev.,D5, 2575 (1972).
13. S.Fubini. Nuovo Cimento,43A, 475 (1966).
    R.F.Dashen, M.Gell-Mann, in Proceedings of the 3rd Coral Gables Conference on Symmetry Principles at High Energy (San Francisco, Cal. 1966).
14. D.J.Gross. Phys.Rev.,D4, 1130 (1971).
15. D.A.Dicus, V.L.Teplitz. Phys.Rev.,D6, 2262 (1972).
16. See for example: D.J.Broadhurst, J.F.Gunion and R.L.Jaffe, Ann.of Phys., 81, 88 (1973) and references quoted therein.

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