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NONZERO MASSES OF π, K, AND κ MESONS AND ALGEBRAIC REALIZATION OF SU(3) × SU(3) CHIRAL SYMMETRY

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1. Introduction

One of the main results of algebraic realization [1] of chiral symmetry (ARCHS) $SU(3)\times SU(3)$ with linearization on $SU(2)\times Y$ subgroup [2] is the fact, that the particles of any given helicity must be classified according to an irreducible or reducible representation of $SU(3)\times SU(3)$ group. Further, the squared mass operator can contain besides the invariant term (in agreement with the hypothesis of Gell-Mann, Oakes, Renner (GMOR [3]) also an admixture of zeroth and eighth components of $(3,3^*)+(3^*,3)$ representation, i.e. $m^2 = m^2_{inv} + u_0 + c\, u_8$. We note that this relation has been obtained without any assumption of chiral symmetry breaking and therefore it has a meaning for all particles excluding π, K, η and κ mesons. (They are Goldstone bosons in the theory). But in fact those mesons have masses and these are not small. In this work we attempt to introduce masses of π, K, η and κ mesons by means of the explicit breaking of the symmetry.

Analyzing a reasonable asymptotic behaviour of the scattering amplitude in the tree graph approximation we consider nonzero masses of pseudoscalar π, K, η and scalar κ mesons in virtual states and we can neglect (in comparison with their energy ω, $\omega\to\infty$) masses of these bosons in initial and final states. All basic relations of ref. [2] are valid in such approach but besides them new relations arise, from which the parameter c (earlier introduced into the mass operator- is determined). This parameter is not estimated in the scheme of Ogievetsky [2]. The value of parameter c ($c \approx -1.17$) thus obtained is very close to that predicted by GMOR ($c \simeq -1.25$).

2. Effective Lagrangian of $SU(3)\times SU(3)$

Because of the invariance of the Lagrangian [2,4]

$$\mathcal{L}_{inv} = V^{i}_{\mu}(x)[F_{\pi}^{-2} f_{ijk}\pi_j \overset{\leftrightarrow}{\partial}_{\mu}\pi_k + F_K^{-2} K_a \overset{\leftrightarrow}{\partial}_{\mu} K_b + F_{\kappa}^{-2}\kappa_a \partial_{\mu}\kappa_b) f_{iab}] +$$

$$+ V^{8}_{\mu}(x)\,[\,F_K^{-2} K_a \overset{\leftrightarrow}{\partial}_{\mu} K_b + F_{\kappa}^{-2}\kappa_a \overset{\leftrightarrow}{\partial}_{\mu}\kappa_b\,]\, f_{8ab} + \qquad (2.1)$$

$$+ F_{\pi}^{-1} V^{i}_{\mu 5}(x)\, D_{\mu}\pi_i + F_K^{-1} V^{a}_{\mu 5}(x) D_{\mu} K_a + F_{\kappa}^{-1} V^{a}_{\mu}(x)\, D_{\mu}\kappa_a +$$

$+ F_{\eta}^{-1} V^{8}_{\mu 5}(x)\, D_{\mu}\eta$ +(interactions with larger number of covariant derivatives of π , K , η , κ mesons) the Goldstone bosons have zero masses. To equip them with the nonzero mass we explicitly break the symmetry by

$$\mathcal{L}_{break} = -\frac{1}{2} m_{\pi}^2 (\pi_i)^2 - \frac{1}{2} m_K^2 (K_a)^2 - \frac{1}{2} m_{\kappa}^2 (\kappa_a)^2 - \frac{1}{2} m_{\eta 8}^2 (\eta_8)^2 + \qquad (2.2)$$

$$+ A\, f_{aib}\,\kappa_a \pi_i K_b + B\, f_{a8b}\kappa_a \eta_8 K_b + (\text{multilinear terms}),$$

where A and B are free parameters, $i = 1,2,3$, $a,b = 4,5,6,7$. Further in (2.2) we shall take into account only the bilinear and trilinear terms, which are sufficient for the incorporation of the nonzero masses of π , K , η and κ mesons in our scheme. We would like to note that mainly trilinear couplings of Goldstones will be important in what follows. The occurence of these terms is a specific feature of the algebraic realization of $SU(3)\times SU(3)/SU(2)\times Y$ unlike $SU(2)\times SU(2)/SU(2)$ and $SU(3)\times SU(3)/SU(3)$ [5].

3. Asymptotic Behaviour of Tree Graphs

Following Ogievetsky [2] we consider the forward scattering process

$$M(q) + \alpha(p,\lambda) \to M'(q') + \beta(p',\lambda') \qquad (3.1)$$

in collinear coordinate system in which

$$q_\mu = n_\mu \omega, q'_\mu = n_\mu \omega'; \; p_\mu = (p_0, -\vec{n}p); \; p'_\mu = (p'_0, -\vec{n}p')$$

$$n_0 = |\vec{n}| = 1$$

α, β are arbitrary target particles or resonances and M, M' are Goldstone bosons.

Energy and momentum conservations yield the relations

$$p_0 + |\vec{p}| = p'_0 + |\vec{p}\,'| \equiv E$$
$$\omega' = \omega + \frac{m_\alpha^2 - m_\beta^2}{2E} \tag{3.2}$$

and the angular momentum conservation tells us that the helicity is conserved. The invariant Mandelstam variables s, t and u become

$$s = m_\alpha^2 + 2\omega E$$
$$t = 0. \tag{3.3}$$
$$u = m_\alpha^2 - 2\omega' E$$

Simple calculations give the explicit expressions for constant terms /2/ of the M^+ amplitude with the tree graph contributions only. We write down only those in which we shall be interested further.

$$M^+_{\beta K_b, \alpha K_a; 0}(\lambda) = -2F_K^{-1} F_K^{-1} \{[T^a, [T^{b5}, m^2]] + [T^{b5}, [T^a, m^2]]\}_{\beta\alpha} +$$

$$+ 2iA F_\pi^{-1} m_\pi^{-2} f_{aib} [T^{i5}, m^2]_{\beta\alpha} + 2iB F_\eta^{-1} m_\eta^{-2} f_{a8b} [T^{8,5}, m^2]_{\beta\alpha} \tag{3.4a}$$

$$M^+_{\beta\pi_i, \alpha K_a; 0}(\lambda) = -2F_K^{-1} F_\pi^{-1} \{T^{a5} [T^{i5}, m^2]] + [T^{i5} [T^{a5}, m^2]]\}_{\beta\alpha} +$$

$$+2iA\,F_{\kappa}^{-1}\,m_{\kappa}^{-2}\,f_{aib}\,[T^{a},m^{2}]_{\beta\alpha} \qquad (3.4b)$$

$$M^{+}_{\beta\pi_i,\alpha\kappa_a;0}(\lambda) = -2F_{\kappa}^{-1}F_{\pi}^{-1}\{[T^{a},[T^{i5},m^{2}]]+[T^{i5},[T^{a},m^{2}]]\}_{\beta\alpha}+$$

$$+2iA\,F_{K}^{-1}\,m_{K}^{-2}f_{aib}\,[T^{b5},m^{2}]_{\beta\alpha} \qquad (3.4c)$$

$$M^{+}_{\beta\eta,\alpha\kappa_a;0}(\lambda) = -2F_{\kappa}^{-1}F_{\eta}^{-1}\{[T^{a},[T^{8,5},m^{2}]]+[T^{8,5},[T^{a},m^{2}]]\}_{\beta\alpha}+$$

$$+2iB\,F_{K}^{-1}\,m_{K}^{-2}\,f_{a8b}\,[T^{b5},m^{2}]_{\beta\alpha} \qquad (3.4d)$$

$$M^{+}_{\beta\eta,\alpha K_a;0}(\lambda) = -2F_{K}^{-1}F_{\eta}^{-1}\{[T^{a5},[T^{8,5},m^{2}]]+[T^{8,5}[T^{a5},m^{2}]]\}_{\beta\alpha}+$$

$$+2iB\,F_{\kappa}^{-1}m_{\kappa}^{-2}f_{a8b}\,[T^{8,5}m^{2}]_{\beta\alpha}\,, \qquad (3.4e)$$

where for simplicity we have omitted the λ -dependence; $T^{i,a,8;5}_{(\lambda)}$, $T^{a}(\lambda)$ are form factors and m^{2} is the diagonal mass matrix defined by $m^{2}_{\gamma\alpha} = m^{2}_{\alpha}\ \delta_{\gamma\alpha}$. The explicit forms of the constant terms of M^{-} amplitudes and the remaining constant terms of M^{+} are the same as in the work of Ogievetsky /2/.

Now, demanding that the asymptotic behaviour of $M^{-}_{\beta M',\alpha M;0}$ amplitudes were not worse than would be expected from Regge Theory, we have to put $M_{\beta M',\alpha M;0}$ equal to zero. Using this procedure the algebra of $SU(3)\times SU(3)$ arises /2/. In the case of $M^{-}_{\beta M',\alpha M;0}$ amplitudes (requiring Regge asymptotic behaviour) we put only those constant terms $M^{+}_{\beta M',\alpha M;0}$ equal zero for which the intercepts of leading trajectories $\alpha_{T,Y}(0)$ are negative.

Table

Reaction	Amplitude	$I^G(J^P)$	Y	Leading trajectory	Intercept $\alpha_{T,Y}(0)$
$\kappa \leftrightarrow K$	M^+	1^- (A) *	0	π	<0
$\kappa \leftrightarrow K$	M^+	0^+ (A)	0	η	<0
$\pi \leftrightarrow \eta$, $\eta \leftrightarrow K$	M^+	1/2 (N)	± 1	K^* (not κ)	0.3 (>0)
$\pi \leftrightarrow \kappa$, $\eta \leftrightarrow \kappa$	M^+	1/2 (A)	± 1	K	<0

* N means normal series $J^P = 0^+, 1^-, 2^+, \ldots$

A means abnormal series $J^P = 0^-, 1^+, 2^-, \ldots$

In this paper we consider the nonzero masses of π , K , η and κ mesons in virtual states and therefore the intercepts of leading trajectories of these mesons (excluding κ) are negative (see table).

As it is seen from the table, the intercepts of leading trajectories of amplitudes (3.4b) and (3.4e) are positive. For this reason we cannot obtain any physical information from these amplitudes.

So, if we take into account the second important consequence of ARCHS of $SU(3)\times SU(3)$ /2/, i.e. that the mass matrix m^2 must be expressed as a sum of u_0 and u_8 component of $(3,3^*)+(3^*,3)$ representation of $SU(3)\times SU(3)$ group

$$m^2 = m^2_{inv} + u_0 + c u_8 \tag{3.5}$$

and the fact that

$$[T^{\sigma}, m^2_{inv}] = 0, \quad [T^{\sigma}, u_{\tau}] = i f_{\sigma\tau\delta}\, u_{\delta}; \quad [T^{\sigma}, v_{\tau}] = i f_{\sigma\tau\delta}\, v_{\delta}$$

$$[T^{\sigma 5}, m^2_{inv}] = 0, \quad [T^{\sigma 5}, u_{\tau}] = -i d_{\sigma\tau\delta}\, v_{\delta}; \quad [T^{\sigma 5}, v_{\tau}] = i d_{\sigma\tau\delta}\, u_{\delta}, \tag{3.6}$$

where u_{τ} , v_{τ} ($\tau = 0, 1, \ldots, 8$) are components of the multiplet $(3,3^*)+(3^*,3)$ we get for the $\pi K \kappa$ vertex constant A ;

$$A = \frac{F_{\pi}\; m^2_{\pi}}{F_K\; F_{\kappa}} \; \frac{2c - \sqrt{2}}{c + \sqrt{2}} , \tag{3.7a}$$

$$A = \frac{F_K\; m^2_K}{F_{\pi}\; F_{\kappa}} \; \frac{2\sqrt{2} + 5c}{2\sqrt{2} - c} . \tag{3.7b}$$

Comparing (3.7a) with (3.7b) we obtain quadratic equation for parameter c , the solution of which is

$$c_{1,2} = -\sqrt{2}\ \{ \frac{(7a-5) \pm 3\sqrt{a^2-14a+1}}{2\,(5a+2)} \}, \quad (3.8)$$

where $a = (\frac{F_K m_K}{F_\pi m_\pi})^2$. Using the values of $F_K/F_\pi \approx 1.18$; $m_\pi^2 = 0.02\ (GeV)^2$; $m_K^2 = 0.24\ (GeV)^2$ we find two values of

$$\begin{aligned} c_1 &\approx -1.17 \\ c_2 &\simeq -0.75\,. \end{aligned} \quad (3.9)$$

The first value is very close to the GMOR value of $c\ (c \sim -1.25)$ which was obtained in a different way (see ref. /3/). Further for $\kappa\ \eta\ K$ vertex constant B we get

$$B = \frac{F_\eta m_\eta^2}{F_K F_\kappa}\ , \quad (3.10a)$$

$$B = \frac{F_K m_K^2}{F_\eta\ F_\kappa}\ \frac{2\sqrt{2} - 3c}{2\sqrt{2} - c}\ , \quad (3.10b)$$

$$B = \frac{F_\eta m_\eta^2}{F_K F_\kappa}\ \frac{\sqrt{2}}{c - \sqrt{2}}\ . \quad (3.10c)$$

Comparing (3.10a) with (3.10b) and putting the value $c = c_1$ and $m_\eta^2 = 0.3\ (GeV)^2$ we get $F_\eta/F_\pi \sim 1.14 F_K/F_\pi$. If we compare (3.10c) with (3.10a) and (3.10b) we get inconsistent results. We conjecture that the difficulty arises due to the fact, that we have ''a priori'' no mixing between η and η' in our scheme.

In conclusion we would like to note the following:

1. The basic result (3.9) confirms that our scheme of introduction of nonzero masses of π , K, η and κ mesons in virtual states is reasonable.
2. The problem of scattering of Goldstones on Goldstones can be incroporated, in principle, into ARCHS $SU(3)\times SU(3)$ as in ARCHS of $SU(2)\times SU(2)$ /6/ but this problem is beyond our object.

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