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A RELATIVISTIC MODEL OF COMPOSITE MESONS

The principal features of high energy hadronic reactions suggest a simple picture of hadrons as composite systems of a finite spatial dimension $R \sim 1$ (fermi) built up of point-like constituents* . Here we discuss a simplified dynamical realization of such a picture for mesons, describing them as composed of a fermion C and antifermion $\bar{C}$, glued by a relativistic potential, which is the kernel of the Euclidean Bethe-Salpeter equation (BSE) [2,3]. An appealing idea is to treat such a potential as a bootstrap potential, i.e., to consider it as determined by the exchange of resonances R which are built of the same constituents C glued by the same potential. Usually the potential is approximated by several low-lying resonances (see, e.g. [4]). However, if the mass spectrum is asymptotically exponential as proposed by R.Hagedorn [5], then this is a poor approximation, and a better approximation can be obtained by taking into account an average contribution of infinitely

*For a recent discussion and references see, e.g. [1,2].

massive resonances. We try to do this by writing the potential in the form

$$U(r,s) = \int_{m_0^2}^{\infty} dm^2 \rho_{ef}(m^2, s) \Delta_F(m^2, r) , \qquad (1)$$

where $r^2 = \vec{r}^2 + r_4^2$, $\Delta_F(r)$ is the Feynman propagator. $E = s^{1/2}$ is the CM energy of constituents and the dependence on s is introduced to allow complex potentials for $s > s_0 > 0$.

If the potential is defined by the exchange of infinitely narrow scalar resonances, then $\rho_{ef} = \sum_n g^2(R_n)\, \delta(m^2 - m_n^2)$ which is a scalar function of m^2 approximately proportional to the mass distribution of resonances (the dependence of g^2 on R_n is supposed to be weak enough). For finite width resonances, ρ_{ef} is a smooth function of m^2 assuming that the average mass difference is much less than the average resonance width. If exchanged resonances R have non-zero spins J_R the summation over J_R is necessary. To do this the knowledge of the resonance spin distribution and of the average dependence of g on J_R is required. We suppose that J_R vanishes exponentially (or faster) with $m \to \infty$ * and that g is weakly dependent on J_R. Then, as was shown in ref./6/, the potential has the form (1), with ρ_{ef} being a spinorial matrix, i.e.,

$$\rho_{ef} = \sum_i \Gamma_C^{(i)} \times \Gamma_{\bar{C}}^{(i)} \rho_{ef}^{(i)} , \quad i = S, V, A, P, T . \qquad (2)$$

For small m (large r) the main contribution in Eq. (2) is due to pseudoscalar (P) and vector (V)

* This is true for the statistical and dual models/5/.

resonances. But for large m (small r) all terms in Eq. (2) are, a priori, of the same order.

The local approximation (1) for the potential is certainly incorrect for $r \ll R$. However, as was shown in ref. [6], the potential corresponding to $\rho_{ef} \sim cm^{b}\exp(ma)$ has an infinitely high barrier (IB) at $r=a$ (where a is of order R) and is finite for $r \to 0$ *. Then the characteristic features of our model are defined by IB and are correctly incorporated in the local approximation.

The maximal radius of $U(r)$ is due to the π-exchange, and so, $m_0 \geq m_\pi$. In the crudest approximation we assume that $m_0 = m_\pi$ and use the asymptotic form of ρ_{ef} for $m_\pi \leq m < \infty$. The next approximation, in which the π-exchange is considered separately and m_0 is $\geq 2m_\pi$ can be developed only after the pion is constructed from C and $\bar{C}$. It is this problem that is shortly treated in the present paper. We solve it with the simplest IB potential (IBP), corresponding to $\rho_{ef} \sim m^{-3/2}\exp(ma)$ and using zero pion mass approximation [7] **.

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* Note that in this case Eq. (1) does not define U for $r < a$ and to find U for all r another representations are used (see [6] and what follows).

** Note that in our case $U(r)$ is finite for $r \to 0$ and there is no Goldstein difficulty.

In this case BSE for pions is reduced to the equation

$$\frac{d^2u}{dr^2} - [M^2 + \frac{3}{4} r^{-2} + U_\pi(r)]\, u(r) = 0, \qquad (3)$$

where BS wave function $X_\pi(r) = \gamma_5 [2\pi^2 r^3]^{-1/2} u(r)$ is normalized according to the condition

$$\int d^4r\, |X_\pi(r)|^2 = \int_0^\infty dr\, |u(r)|^2 = 1. \qquad (4)$$

The potential $U_\pi(r)$ in Eq. (3) depends on all terms in Eq. (2). As we are interesetd here only in pion states, we simply suppose that $U_\pi(r)$ is determined by $\rho_{ef}(m^2) \sim c\, m^b e^{ma}$. The singularity point is defined by a, the analytic structure of the singularity depends on b and the coupling constant is proportional to c. The simplest and most interesting potential, corresponding to $b = -3/2$ has the singularity of the form $\underset{r\to a}{\sim}(r-a)^{-1}$. We call this IB "semipenetrable" barrier because defining the singularity as $V.P.(r-a)^{-1}$ we find that the wave function $u(r)$ is continuous at $r = a$ and satisfies the condition

$$[u'(a+\epsilon) - u'(a-\epsilon)] \xrightarrow[\epsilon \to 0]{} 0 \qquad (5)$$

which provides the unique correspondence between the solutions for $r < a$ and $r > a$. If $U_\pi(r) \underset{r\to a+0}{\sim}(r-a)^{-1}$ and $U_\pi(r)$ is finite for $r \to a-0$, then there is no unique correspondence and the barrier is impenetrable. It is impenetrable for $b > -3/2$ and penetrable for $b < -3/2$. Impenetrable IBP can keep the quarks inside hadrons and are consistent with the quantum field theory axioms (except, possibly, the strict localizability). They are also useful for qualitative estimation of the effects of penetrable IBP. However, statistical and bootstrap models suggest a less hard barrier [5,6] and so we investigate the simplest IBP.

First we give a definition of U_π for all r and of its Fourier transform into momentum space. The equation (1) is the Kallen-Lehmann representation for the effective propagator, describing resonance exchanges between C and $\bar{C}$ and $\rho_{ef}(-p^2), p^2 = -m^2$, is its imaginary part on the cut $-\infty < p^2 < -m_\rho^2$. Therefore, to find the potential is to find the propagator with the exponentially rising imaginary part. This problem was solved in the context of non-polynomial field theories and such propagators are usually called superpropagators (see [6,8]) where there are further references).

To formulate accurately the receipt for constructing the potential identified with the superpropagator we introduce a scalar gluon field ϕ interacting with ψ_C and $\bar{\psi}_C$ through a non-polynomial Lagrangian

$$L = \tilde{f} \sum_{n=0}^{\infty} d_n (n!)^{-1} (g\phi)^n . \qquad (6)$$

Then, in $\tilde{f}^2$-approximation U_π is formally defined as

$$U_\pi(r) = \tilde{f}^2 \sum_{n=1}^{\infty} c_n [g^2 \Delta_F(r)]^n , \; c_n = d_n^2 (n!)^{-1} . \qquad (7)$$

The potential (7) has IB if $c_n^{1/n} \underset{n\to\infty}{\to} 1$ the singularity point a being defined by the equation $g^2 \Delta_F(a) = 1$ and the analytic structure of IB by the dependence of c_n on n for $n \to \infty$. For $c_n \equiv 1$ this structure is of the "Coulomb" form $_{r\to a}$ $(r-a)^{-1}$. As was shown in ref.[8] with massless ϕ the coordinate and momentum representations of U_π are

$$U_\pi(r) = \text{V.P.}\{ f^2 (r^2 - a^2)^{-1} \}, \quad \tilde{U}_\pi(p) = -f^2 \frac{\pi a}{2p} Y_1(ap),$$

$$a = \frac{g^2}{4\pi^2}, \quad f^2 = 4\pi \tilde{f}^2. \tag{8}$$

The imaginary part of $\tilde{U}_\pi(p)$ on the cut $p^2 = -m^2 < 0$ is asymptotically proportional to $m^{-3/2} \exp(ma)$. By properly choosing d_n, f and g one can find a superpropagator corresponding to exponential ρ_{ef} with arbitrary a, b and c.

Now, BSE for pions may be solved in both the coordinate and momentum spaces. Here we consider the coordinate equation (3), and solve it by WKBJ method (for some exact solutions and further details see [6]). Then the eigenvalue equation for massless pions is [6]

$$\int_{r_0}^{a} dr [M^2 + r^{-2} + f^2 (r^2 - a^2)^{-1}]^{1/2} = N\frac{\pi}{2}, N=1,3,5\ldots, \tag{9}$$

where r_0 is the turning point and the pion solution corresponds to $N = 1$. For $M = 0$ this condition gives the exact result $f^2 = N(N+2)$[6]. For $Ma \gg 1$ and $Ma \ll 1$ it follows from (9) that

$$f^2 \simeq 2MaN - \frac{1}{2}N(N-2) + O((Ma)^{-1}), \quad Ma \gg 1,$$

$$f^2 = N(N+2) + \frac{1}{2}(Ma)^2 N(N+2)[N(N+2)+1]^{-1} + \tag{10}$$
$$+ O((Ma)^4), \quad Ma \ll 1.$$

In the statistical and dual models a is of order which is consistent with the empirical mass spectrum[5]. Very rough estimate of the pion trajectory slope α'_π near $s=0$ suggests that α'_π is of order $\sim 1\ GeV^{-1}$ if $a = (4.5 \div 5.5)\ GeV^{-1}$ and $M \le 1$ GeV. This gives some support to our model as the experimental data are consistent with $\alpha'_\pi \sim 1\ GeV^{-1}$ [9]. Another test is provided by $\pi \to \mu\nu$ decay. In our approximation one can easily find the following simple expression for f_π assuming usual V-A $\bar{\psi}_C \psi_C \mu \nu$ vertex:

$$f_\pi = 2M \int_0^\infty dr\, r^2\, u(r)\, M\, K_1(Mr) . \tag{11}$$

Using WKBJ expression for $M \to \infty$ and exact solution for $M \to 0$ we find

$$\begin{aligned} f_\pi \underset{Ma > r}{\simeq} & \frac{3}{4\sqrt{\pi}} M (Ma)^{1/2} \exp(-Ma + 2) , \\ f_\pi \underset{Ma < 1/2}{\simeq} & \frac{M}{2\sqrt{2}} [7 - 3Ma] . \end{aligned} \tag{12}$$

Now, from the experimental value $f_\pi \simeq m_\pi$ we obtain two possible solutions for M. If $4\ GeV^{-1} < a < 6\ GeV^{-1}$ the two solutions vary correspondingly in the intervals $62\ MeV < M < 68\ MeV \sim \frac{1}{2} m_\pi$, $0.9\ GeV > M > 0.5$ GeV. One has not to consider these numbers too seriously due to the approximations made and the special potential used. But, the existence of two solutions, one with a small mass of C and the other with a much higher mass, seems to be of more general nature. The vanishing of f_π for $M \to 0$ follows from symmetry considerations if we assume V-A $\bar{\psi} \psi \mu \nu$

vertex and use the most general BS equation for the bound state pion. The second solution exists for all potentials giving $M^{-1}\chi_\pi(0) \underset{M\to\infty}{\to} 0$. This condition is essentially dynamical and is naturally satisfied for IBP, which concentrates the wave function near the surface $r = a$. Here we do not consider the process $\pi^0 \to \gamma\gamma$ and $e^+e^+ \to \gamma\gamma$ $e^+e^+ \to \pi^0 e^+ e^+$ which are very good tests (especially the last one, with virtual photons) of composite models. Using the considerations given above one can easily estimate these processes, but the result depends crucially on the number of C particles (3 quarks, 8 baryons, 9 quarks, etc.) on approximation for $\bar\psi\psi \to \gamma\gamma$ transition and on hadronic symmetry.

Instead, we mention a direct test of IBP prediction. Consider the pion form factor $F_\pi(q^2)$ for $q^2 = \vec{q}^{\,2} - q_0^2 \to +\infty$. Using the non-relativistic static approximation and neglecting spins, we write $F_\pi(q^2)$ in the form $(r_0 = 0;\ r = |\vec{r}|)$

$$F_\pi(q^2) = 2\int_0^\infty dr\,(qr)^{-1}\sin(qr/2)u^2(r). \quad (13)$$

Then for $q^2 \to +\infty$ we have

$$F_\pi(q^2) = F_0(q^2) + \gamma(qa)^{-\beta}\cos(qa/2) + \dots,$$

where F_0 is a smoothly decreasing function of q^2, determined by the behaviour of $u(r)$ near $r = 0$ and the second term depends on IB*. For "Coulomb" barrier $\beta = 3$ and γ is of order 4-5 (for $M \sim 1$GeV). This asymptotic expansion is valid

*For potentials with a marginal singularity or for regular potentials there is no such oscillating term (see, e.g. /1-3,9/).

for $\frac{1}{2} qa \gg 1$ or $q \gg 0.5$ GeV. The static nonrelativistic approximation is of course very crude but the principal qualitative fact, the existence of oscillations with the period of order 1 GeV, survives evidently in better approximation. The experimental discovery of such oscillations would be a very serious evidence in favour of the composite model discussed above.

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