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THE EFFECT OF THE CHOICE OF $\pi\pi$-SCATTERING PHASE PAPAMETRIZATION ON $\gamma\gamma \to \pi\pi$ AND $\gamma\gamma \to \gamma\gamma$ REACTION CROSS SECTIONS

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THE EFFECT OF THE CHOICE OF $\pi\pi$-SCATTERING PHASE PAPAMETRIZATION ON $\gamma\gamma \to \pi\pi$ AND $\gamma\gamma \to \gamma\gamma$ REACTION CROSS SECTIONS

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Влияние выбора параметризации фаз $\pi\pi$-рассеяния на сечения реакции $\gamma+\gamma\to\pi+\pi$ и $\gamma+\gamma\to\gamma+\gamma$

В работе получены сечения процессов $\gamma+\gamma\to\pi+\pi$ и $\gamma+\gamma\to\gamma+\gamma$ для фаз $\pi\pi$-рассеяния, выходящих в асимптотике на π. Указано на важность использования в решении дисперсионного интегрального уравнения общего решения однородного уравнения.

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The Effect of the Choice of the $\pi\pi$-Scattering Phase Parametrization on the $\gamma+\gamma\to\pi+\pi$ and $\gamma+\gamma\to\gamma+\gamma$ Reaction Cross Sections

The cross sections of the processes $\gamma+\gamma\to\pi+\pi$ and $\gamma+\gamma\to\gamma+\gamma$ for the case, when the $\pi\pi$ phase shifts are going to π in the asymptotic regions, have been obtained in the work. The importance of the use of the general solution of the homogeneous integral equation for obtaining the solution of the integral dispersion relation has been pointed out.

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Putting into operation and use of accelerators on colliding 2-5 GeV electron-positron beams make it possible to study $\gamma+\gamma$-hadron interactions [1,2]. At these energies the two-photon processes become very essential in the reaction of colliding beam interactions with production of other particles, i.e., those two-photon processes are important in which the final particles are generated by a virtual photon pair.

From this point of view the interactions of two γ-quanta with π-meson pair production are of interest for experimental investigation. Studying this process allows one to obtain some useful information on $\pi\pi$-interactions (on $\pi\pi$-scattering phases, possible σ-, f- and other resonances) and verify the validity of various theoretical predictions for this reaction.

In our work [3] a detailed theoretical analysis of the $\gamma+\gamma\to\pi+\pi$ reaction at low energies has been performed by the dispersion relation method. The dispersion singular integral equations for partial waves of the above process have been derived by using the amplitude properties in the direct channel variable $t=(K+K')^2$ (K and K' are four-momenta of the initial photons), and the two-particle unitarity condition (two-pion intermediate state). These equations were solved by the method of Muskhelishvili-Gakhov [4] through reducing to the Riemann boundary value problem. The solution of a similar problem for the particular case of zero-th asymptotics of the $\pi\pi$-scattering phases, $\delta_\ell^{(T)}(t)$, (at $t\to\infty$), is unique and is written in the form [4]:

$$T^{(T)}_{\alpha\beta}(t)^{(1)}_{\ell} = X^{(+)}(t)\, A^{+}_{\alpha\beta}(t) + B^{(T)}_{\alpha\beta}(t)_{\ell}$$

$$X^{(+)}(t) = e^{i\delta_\ell^{(T)}(t)} \cdot \exp\left\{ \frac{t}{\pi} P \int \frac{\delta_\ell^{(T)}(t')}{t'(t'-t)}\, dt' \right\} \tag{1}$$

$$A^{+}_{\lambda\beta}(t) = \frac{t}{\pi}\int_{4\mu_\pi^2}^{\infty} \frac{e^{i\delta_\ell^{(T)}(x)}\cdot \sin\delta_\ell^{(T)}(x)\cdot B^{(T)}_{\lambda\beta}(x)_\ell}{x(x-t-i\varepsilon)\,X^{(+)}(x)}\,dx\,. \quad (1)$$

Here $T^{(T)}_{\lambda\beta}(t)^{(1)}_\ell$ is the partial ℓ- wave amplitude of the $\gamma\gamma\to\pi\pi$ reaction with a given isospin T, $B^{(T)}_{\lambda\beta}(t)_\ell$ is the contribution of π,ω,ρ exchange diagrams to the process. Eq. (1) is, in fact, a particular solution of the Riemann nonhomogeneous boundary value problem.

The investigations [3] have revealed the essential role of $\pi\pi$-interaction in the process under consideration. Experiment and the phase shift analysis give the two-valued solution for the S-wave phase of $\pi\pi$-scattering, $\delta^0_S(t)$ (down - and down-up sets of experimental points). In the work [3] the S- partial wave cross section of the $\gamma\gamma\to\pi\pi$ process has been calculated by making use of the analytic expressions, which have been agreed well with the down- and down-up sets, at the known experimental values ($\sqrt{t} \lesssim 1$ GeV), for the δ^0_S phase

The main peculiarities of the cross section behaviour depend considerably on the kind of parametrization(down,down-up) for the δ^0_S scattering phase $\pi\pi$. The calculations show that the contribution from inelastic processes at high energies ($\sqrt{t} > 1$ GeV) as well as the rate of decrease of the phases to zero-th asymptotical value do not essentially change the low-energy behaviour of the obtained cross sections (10-20%). On the other hand, the changes of the phases in low-energy and threshold regions result in the proportional changes of the $\gamma\gamma\to\pi\pi$ cross sections in these energy regions. Recall that the results have been found for the phases $\delta^T_\ell(t)$ going asymptotically to zero (this corresponds to the zero-th index of the Riemann boundary value problem). The phases $\delta^0_S(t)_{down-up}$ and $\delta^0_d(t)$ resonant at $M_\sigma \simeq 730$ MeV and $M_f \simeq 1260$ MeV, respectively, pass for the second time through $90°$ in the region of M_{S^*} and $M_{f'}$ under the chosen asymptotics, and at high energies they become practically zero.

The present work deals with the other possibility: the phases $\delta^0_S(t)_{down-up}$ and $\delta^0_d(t)$ in asymptotics tend to $180°$. This case differs in principle from the above one as for it the index $\varkappa$ of the Riemann problem is equal to unity; the solution of singular integral equation is nonunique and in addition to the particular solution of (1), contains also the general solution of homogeneous equation with an arbitrary constant (a polynomial of the order of $\varkappa$ -1) :

$$T^{(T)}_{\lambda\beta}(t)^{(2)}_\ell = T^{(T)}_{\lambda\beta}(t)^{(1)}_\ell + e^{i\delta^T_\ell(t)}\cdot e^{\frac{t}{\pi}P\int_{4\mu_\pi^2}^{\infty}\frac{\delta_\ell^{(T)}(t')}{t'(t'-t)}dt'}\cdot\alpha\,, \quad (2)$$

where α is an arbitrary constant.

The behaviour of the function $y(x) = \exp\left\{\frac{x}{\pi}P\int_1^\infty \frac{\delta_\ell^{(T)}(x')}{x'(x'-x)}dx'\right\}$ in eqs. (1), (2) is different for various parametrizations of the $\pi\pi$-scattering phases. Fig.1 shows the function $y(x)$, corresponding to different cases of the $\pi\pi$- scattering phases. For the cases $\delta^0_S(\infty)_{down-up} = 0$ and $\delta^0_S(\infty)_{down-up} = \pi$ both the curves have the resonance behaviour in the range of σ meson ($x \simeq 7.3$) but differ in magnitude.

In paper [5] the dispersion analysis of the $\eta \to \pi\pi$ process has been carried out under the assumption that the function changes smoothly ($y(x) \approx$ const). As follows from the curves in Fig.1, this assumption may be justified only for the smooth (nonresonance) $\pi\pi$ - scattering phases (for instance, for δ_s^2). For the resonance phases this assumption is not valid.

The solutions (1) and (2) possess another difference of principle. Using the known identity, $\frac{1}{x+i\varepsilon} = P\left(\frac{1}{x}\right) - i\pi\delta(x)$ we now rewrite eq. (1) in the following form

$$T_{\alpha\beta}^{(T)}(t)_\ell^{(1)} = X^{(+)}(t)\cdot\frac{t}{\pi}P\int_{4\mu_\pi^2}^{\infty}\frac{e^{i\delta_\ell^{(T)}(x)}\cdot\sin\delta_\ell^{(T)}(x)\cdot B_{\alpha\beta}^{(T)}(x)_\ell}{x(x-t)X^{+}(x)}dx + \tag{3}$$

$$+ B_{\alpha\beta}^{(T)}(t)_\ell\cdot\cos\delta_\ell^{(T)}(t)\cdot\exp\left(i\delta_\ell^{(T)}(t)\right).$$

The second term in (3) vanishes at the resonance point $\delta_\ell^T = \frac{\pi}{2}$. The integrand of the principal value integral ($B_{\alpha\beta}^{(T)}(t)_\ell$ is a smooth function) is equal approximately to

$$\frac{\sin\delta_\ell^T(x)\cdot B_{\alpha\beta}^{(T)}(x)_\ell}{x(x-t)\cdot y(x)} \simeq Const\ \frac{\sin\delta_\ell^T(x)}{y(x)}\cdot\frac{1}{x(x-t)} \equiv \tag{4}$$

$$\equiv \frac{Const}{x(x-t)}\cdot F(x).$$

In the case of zero-th asymptotical behaviour of the phase $\delta_s^0(t)$ $\left(\delta_d^0(t)\right)$ the function $F(x)$ has two maxima which correspond to the σ meson (f meson) and to passing of the phase through $\frac{\pi}{2}$ when it goes back from π to zero. The second maximum (being in the region of energies higher than 1 GeV) enters into the principal value integral in (3) as a large positive back-

ground. The first term in (3) is nonzero at the resonance point $M_\sigma \approx 730$ MeV ($x = 7.3$) and the partial amplitude is a resonance one due to the function $y(x)$.

Consider now the phase $\delta_\ell^{(T)}$ having asymptotic value equal to π ($\varkappa = 1$) . The particular solution of nonhomogeneous integral equation (see the eq. (1)) is not valid for this case. The correct solution (2) includes now also the general solution of the homogeneous integral equation with an arbitrary constant. At the resonance point $\delta_s^0(M_\sigma^2)_{down-up} = \frac{\pi}{2}$ (e.g., for $\delta_s^0(t)$ phase) the second term of the expression for $T_{\alpha\beta}^{(T)}(t)_\ell^{(1)}$ becomes zero, as well ($\cos\delta_s^0 = 0$). In this case the function $F(x)$ is nearly symmetric with respect to this point and has the maximum value at it. The principal value integral over the expression (4) will approximately be zero at the resonance point and the partial amplitude $T_{\alpha\beta}^{(T)}(t)_\ell^{(1)}$ will possess in the region of σ meson a minimum instead of a peak.

Thus the particular solution (eq. (1)) alone of nonhomogeneous integral equation does not result in the expected resonance behaviour at the point $x = 7.3$ ($M_\sigma = 730$ MeV), for the case of $\delta_s^0(\infty)_{down-up} = \pi$. In particular, the authors of [6] have employed in their calculations the asymptotic behaviour of the $\pi\pi$-scattering resonance phases, $\delta(\infty) = \pi$, but have allowed for the particular solution only. Let us note that the general solution of the homogeneous equation entering into (2) has just required resonance behaviour. An arbitrary constant α of this solution can be found by norming the resonance maximum to its two-photon width. In the preceding paper [3] the constant $g_{\rho\pi\gamma}$ in the expression $B_{\alpha\beta}^{(T)}(t)_\ell$ was taken to correspond to the width

$\Gamma_{\rho\to\pi+\gamma} \approx 0.5$ MeV of the ρ meson electromagnetic decay ($\Gamma_{\rho\to\pi+\gamma} \simeq \frac{g^2_{\rho\pi\gamma}}{3\pi} M_\rho^3 \left(\frac{M_\rho^2 - m_\pi^2}{M_\rho^2}\right)^3$). The experimental data for this width lie in the wide interval 0.1 - 0.7 MeV (see, e.g., the data presented in the work by L.D.Soloviev [7]). In the present work the value $\Gamma_{\rho\to\pi+\gamma} \simeq 0.1$ MeV has been taken for calculation. This value was also used to calculate for comparison the curves of S - wave cross sections for the down-parametrization of the phase $\delta_s^0(t)$.
The behaviour of the s-wave $\gamma\gamma \to \pi\pi$ cross section for the phases $\delta_s^0(t)_{down}$ and $\delta_s^0(\infty)_{down-up} = \pi$ are given in Figs.2a and 2b, respectively (the dash-dotted lines are the contributions of the ω, ρ, π exchange diagrams representing the crossing cuts). In Fig.2b the simplest Breit-Wigner approximation of σ- meson is given by the dashed line. In Fig.3 the d- wave $\gamma\gamma \to \pi\pi$ cross section corresponding to the phase $\delta_d^0(\infty) = \pi$ is shown (the Breit-Wigner curve for f meson is given by the dashed line). The d-wave at the maximum of f resonance gives a contribution to the total cross section $(\sigma_{\gamma\gamma\to\pi\pi} \simeq \sigma^s_{\gamma\gamma\to\pi\pi} + \sigma^d_{\gamma\gamma\to\pi\pi})$ comparable with the S- wave cross section. Comparison with the earlier results [3] reveals that all qualitative features of the cross section behaviour do not change when the contribution from the ω, ρ exchange diagrams and phase asymptotics change.
On the other hand, the values of the cross sections are sensitive to similar changes. The selection of correct solutions will be possible after comparison with experiment.

From the $\gamma\gamma \to \pi\pi$ amplitudes found in ref. [3] the partial S- and d- waves have been calculated later for the scattering of γ- quanta on γ- quanta via the two-pion state [8], as well as for the processes $\gamma\gamma \to K\bar{K}$ ($T = 0$) and $\gamma\gamma \to K\bar{K}(T=0) \to \gamma\gamma$

(i.e., the γ- quanta scattering via the $K\bar{K}\,(T=0)$ intermediate state) [9].

In our work the partial cross sections of the listed processes have been obtained, too. Fig.4 presents the graphs of the S- wave $\gamma\gamma \to \pi\pi \to \gamma\gamma$ cross sections which correspond to the down (Fig.2a) and down-up (Fig.2b) variants of $\gamma\gamma \to \pi\pi$ amplitudes. The d-wave partial cross section for scattering of γ quanta on γ quanta via the two-pion state is shown in Fig.5. Note that if the corresponding total cross section of photon scattering $(\sigma_{\gamma\gamma\to\gamma\gamma} = \sigma_s(t) + \sigma_d(t))$ is compared with the electrodynamical cross section for the scattering of γ quanta on γ quanta (via the electron-positron pairs) [10] then the first one will provide the main contribution in the wide energy range, 300-1200 MeV.

The S and d wave cross sections for the $\gamma\gamma \to K\bar{K}\,(T=0)$ reaction found from the partial waves of the $\gamma\gamma \to \pi\pi$ process (Fig.2b and 3) by a method given in the work [9], are represented in Fig.6. The curves obtained are analogous by form with those given earlier [9] but differ from them in magnitude. The partial cross sections for photons scattered via the $K\bar{K}$ (T=0) state, which have been derived from the amplitudes of the $\gamma\gamma \to K\bar{K}$ (T=0) reactions, are also analogous by form with those given in the work [9] and provide a contribution to $\gamma\gamma \to \gamma\gamma$ process negligible small as compared with that from the two-pion state.

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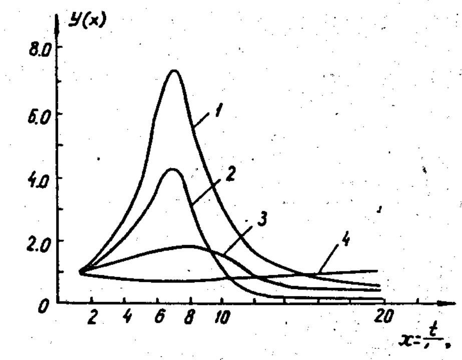


Fig.1. The plot of the function $y(x) = \exp\left\{\frac{x}{\pi}\int_1^{\infty}\frac{\delta_{\ell}^{T}(x')\,dx'}{x'(x'-x)}\right\}$ for the following parametrizations of the $\pi\pi$ scattering phases (see the corresponding curves in work [3]):

$1 - \delta_s^0(\infty)_{down-up} = \pi$; $2 - \delta_s^0(\infty)_{down-up} = 0$;

$3 - \delta_s^0(t)_{down}$; $4 - \delta_s^2(t)$.

a)

b)

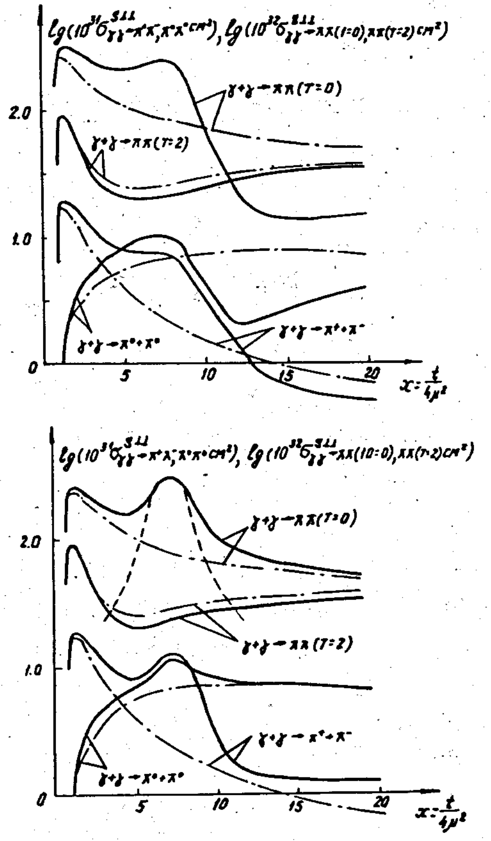


Fig. 2.

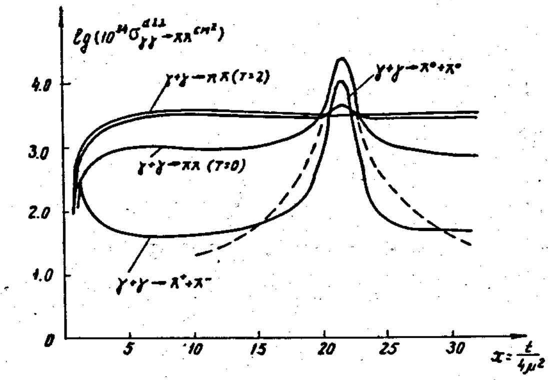


Fig. 3.

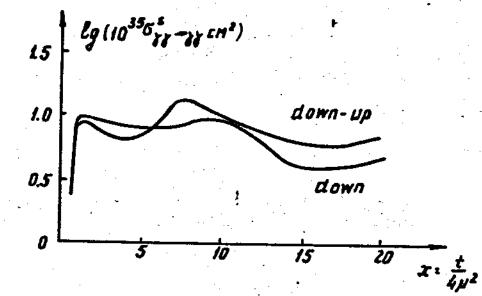
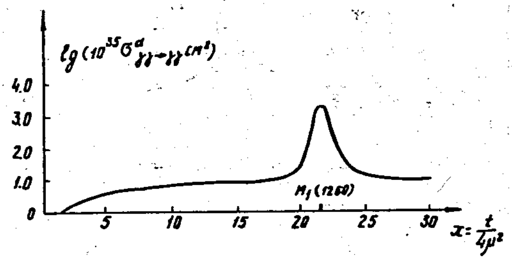
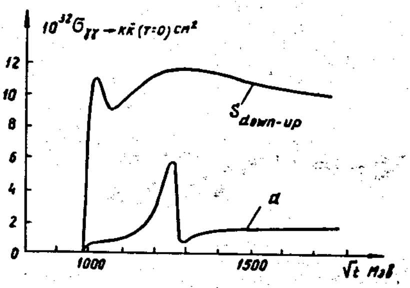


Fig. 4.



lg (10^35 σ^d_γγ→γγ (cm²))
4.0
3.0
2.0
1.0
0
5
10
15
20
25
30
x = t/4μ²
M_f (1260)

Fig. 5 .



10^32 σ_γγ→KK̄ (T=0) cm²
12
10
8
6
4
2
0
1000
1500
√t MeV
S_down-up
d

Fig. 6.