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SUPERPROPAGATOR REGULARIZATION OF THE S - MATRIX FOR CHIRAL LAGRANGIAN

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1. Introduction

Chiral-invariant effective Lagrangians proposed as a compact means of describing low-energy theorems[1], are successfully applied now for constructing dynamical quantum field theory[2].

Calculations of contributions from one-loop diagrams made in works[3,4,5] provide a satisfactory fit to experimental data on $\pi\text{-}\pi$ -scattering[3,4] and pion form factor[5]. In these works[3,4,5] the superpropagator (S.P.) method[6] was used for calculations.

The present paper is devoted to the construction of the regularized S -matrix by the superpropagator method in the Henerkamp covariant perturbation theory[7].

The second section deals with brief sketch of the covariant perturbation theory; a Lagrangian is calculated and transition to the S -matrix with normal-ordered Lagrangian is performed.

The third section presents an analysis of the standard renormalization theory for the one-loops diagrams, and basing on this a regularization of the normal-ordered Lagrangian is carried out by the S.P. method. An experience stored in exploiting the S.P. method for calculations of diagrams with an arbitrary number of vertices, allows one to suppose that the S -matrix constructed contains neither infrared nor ultraviolet divergences to all orders.

All cumbersome computations are removed to Appendices A, B, C and D.

In Appendix A the Honerkamp Lagrangian is calculated. The normal-ordered Lagrangian is obtained in Appendix B. In Appendix C the renormalization of the π-π-scattering amplitude is given. In Appendix D the calculation results are presented for π-π-scattering amplitude in the one-loop approximation[4) by the S.P. method and superpropagator with derivatives is calculated by rules proposed by A.Salam et al.[8)].

2. Covariant perturbation theory.

To calculate a usual one-loop diagram by the superpropagator method one has to know the sum of all two-vertex diagrams. Within conventional perturbation theory the sum of diagrams with fixed number of vertices depends on the choice of pion field coordinates, and the procedure of calculating superpropagator is noncovariant.

However, the chiral Lagrangian has such a specific property that it contains a field derivative squared. Within conventional perturbation theory, pion lines can be contracted transferring two derivatives at vertices into one propagator, and in so doing a diagram with n vertices may be converted into a sum of "noncontractible" diagrams with smaller amount of vertices[+)].

+) Let $\mathcal{D}(n)$ be the expression corresponding to the diagram with n vertices. Then for the "contractible" diagram $\mathcal{D}(n)=\sum_{k=1}^{n}\bar{\mathcal{D}}(k)C(k)$; the diagram $\mathcal{D}(n)$ is "noncontractible", if $C(n)=1$ and $C(k\neq n)=0$.

Therefore that part of two-vertex diagrams which must be summed up to apply the S.P. method is present in all diagrams with an arbitrary number of vertices.

The main advantage of covariant perturbation theory[7)] equivalent to the standard one[9)] is that the sum of all diagrams with fixed set of vertices is now independent of the choice of the pion field coordinates. On the other hand, covariant theory deals just with "noncontractible" diagrams.

This work rests on the covariant perturbation theory[7)], so it makes sense to present here its short formulation.

The action and Lagrangian are considered in the following form:

$$W=\int d^4x\,\mathcal{L}(\pi)\quad;\quad \mathcal{L}(\pi)=\frac{1}{2}g_{ij}(\pi)\,\partial_\mu\pi^i\partial^\mu\pi^j, \tag{1}$$

where $g_{ij}(\pi)$ is the metric in a curved isospace of constant curvature F_π^{-2} , $F_\pi = 92$ MeV pion decay constant.

S-matrix as a functional of asymptotic field is written in the form of functional integral[2)]:

$$S(\pi^{(in)})=N^{-1}\int\prod_i\prod_x d\pi^i(x)\sqrt{g}\,\exp\left\{iW-i\int d^4x\,\partial_\mu\pi^i\partial^\mu\pi_i^{(in)}\right\}, \tag{2}$$

where N is a normalization constant, $g=\det g_{ij}$.

Covariant perturbation theory by Honerkamp[7] consists in expanding the action of eq.(2) in powers of derivative of geodetic interval with respect to the classical field φ_i. The geodetic interval is taken between the points φ^i and integration variable π^i in a curved isospace. We would like to remind how to derive this expansion.

Let $\xi^i(\lambda)$ be the geodesics from $\xi^i(\lambda=0)=\varphi^i$ to $\xi^i(\lambda=1)=\pi^i$ where the parameter λ measures the length of the curve. The classical field φ^i is a solution of the equation

$$\frac{\delta}{\delta\varphi^i} W = -\Box\pi_i^{(in)} \quad ; \quad g_{ik}\left(\Box\varphi^k + \Gamma^k_{\ell j}\,\partial_\mu\varphi^\ell\partial^\mu\varphi^j\right) = \Box\pi_i^{(in)}, \tag{3}$$

where $\Gamma^k_{\ell j} = \frac{1}{2} g^{k\tau}\left(\partial_\ell g_{\tau j} + \partial_j g_{\tau\ell} - \partial_\tau g_{\ell j}\right)$ is the Christoffel symbol and $\partial_\tau = \frac{\partial}{\partial\varphi^\tau}$. The equations of the geodesics $\xi^i(\lambda)$ and the geodetic interval I are the following

$$\frac{d^2\xi^i}{d\lambda^2} + \Gamma^i_{k\ell}\frac{d\xi^k}{d\lambda}\frac{d\xi^\ell}{d\lambda} = 0 \ ; \ \xi^i(0)=\varphi^i \ ; \ \xi^i(1)=\pi^i \tag{4}$$

$$I(\pi,\varphi) = \int_{0,\varphi^i}^{1,\pi^i} d\lambda\, \frac{1}{2}\, g_{ij}(\xi(\lambda))\frac{d\xi^i}{d\lambda}\frac{d\xi^j}{d\lambda}. \tag{5}$$

Let us expand the action $W(\xi^i(\lambda))$, in powers of the parameter λ around the point $\lambda=0$

$$W(\xi^i(\lambda)) = W(\xi(0)) + \lambda\left(\frac{d\xi^i}{d\lambda}\frac{\partial}{\partial\xi^i}W(\xi^i)\right)_{\lambda=0} + \cdots$$

Making use of eq.(4) and writting the derivative of geodesics

$\xi^i(\lambda)$ with respect to the parameter λ via the derivative of interval with respect to the classical field (by virtue of the Jacobi equation),

$$\partial_i I(\pi,\varphi) = \left(\frac{d\xi^i}{d\lambda}\right)_{\lambda=0} \equiv L^i(\pi,\varphi)$$

we get for $\lambda=1$:

$$W(\pi) = W(\varphi) + \sum_{n=2}^{\infty}\frac{1}{n!}W_{;\ell_1;\ell_2;\ldots;\ell_n}(\varphi)L^{\ell_1}L^{\ell_2}\ldots L^{\ell_n}\,, \tag{6}$$

where $W_{;\ell_1;\ldots;\ell_n}$ stand for covariant derivatives

$$W_{;\ell_1} = \partial_{\ell_1}W \quad ; \quad W_{\ell_1;\ell_2} = \partial_{\ell_2}W_{;\ell_1} - \Gamma^n_{\ell_1\ell_2}W_{;n} \ ; \ \ldots$$

Taking new integration variables

$$\Gamma^a = e^a_i(\varphi)L^i(\pi,\varphi) \tag{7}$$

(where e^a_i are drei-bein fields : $e^a_i e^{a j} = \delta_{ij}$; $e^a_i e^a_j = g_{ij}$), we obtain for the functional S-matrix the following expression[6]

$$S(\pi^{(in)}) = e^{iW_{tree}(\varphi)}N^{-1}\int\prod_a\prod_x d\Gamma^a_{(x)}\sqrt{\frac{g(\varphi)}{g(\pi)}}\det\frac{\partial\pi^i}{\partial L^j(\pi,\varphi)}\, e^{iW(\varphi|\Gamma)}, \tag{8}$$

where

$$iW_{tree}(\varphi) = iW(\varphi) - i\int\partial_\mu\varphi^k\partial^\mu\pi_k^{(in)}$$

is the generating functional describing tree graphs:

$$W(\varphi|\Gamma)=\sum_{n=2}^{\infty}\frac{1}{n!}\left[W_{;\ell_1;\dots;\ell_n}(\varphi)e^{a_1\ell_1}(\varphi)\cdots e^{a_n\ell_n}\right]\Gamma^{a_1}\cdots\Gamma^{a_n} \quad (9)$$

As is shown in ref.[10],all higher covariant derivatives of the action W are expressed through the metric tensor and Christoffel symbol because of the constant curvature of isospace. In Appendix A we obtain the explicit form of Lagrangian in the covariant theory:

$$W(\Gamma|\varphi)=\int d^4x\,\mathcal{L}(\Gamma|\varphi) = \int d^4x\,\frac{1}{2}(\partial_\mu\Gamma)^2 + W^I(\Gamma|\varphi) \quad (10)$$

$$\mathcal{L}(\Gamma|\varphi)=\frac{1}{2}\left(\partial_\mu\Gamma^a+\rho_\mu^{ac}(\varphi)\Gamma^c\right)\left(\partial_\mu\Gamma^b+\rho_\mu^{bc'}(\varphi)\Gamma^{c'}\right)g^G_{ab}(z)+$$
$$+e_i^a(\varphi)\partial_\mu\varphi^i\left[(\partial_\mu\Gamma^b+\rho_\mu^{bc}\Gamma^c)(1-g(z))\left(\delta_{ab}-\frac{\Gamma^a\Gamma^b}{\Gamma^2}\right)\right]+$$
$$+\frac{1}{2}e_i^a(\varphi)\partial_\mu\varphi^i e_j^b(\varphi)\partial_\mu\varphi^j\frac{f(z)}{F_\pi^2}\left(\Gamma^a\Gamma^b-\Gamma^2\delta_{ab}\right) \quad (11)$$

by means of a calculation technique somewhat different from that used in the work[10] +), namely, by expanding the Lagrangian (1) itself. Here $g^G_{ab}(z)$ is the metric tensor in Gursey coordinates

$$g^G_{ab}(z) = \delta_{ab}f(z) - (f(z)-1)\frac{\Gamma^a\Gamma^b}{\Gamma^2}$$

$$f(z)=\left(\frac{\sin z}{z}\right)^2 \;;\quad g(z)=\frac{\sin 2z}{2z} \;;\quad z=\sqrt{\frac{\Gamma^2}{F_\pi^2}}$$

$$\rho_\mu^{ab}= e_n^a\partial_\mu e^{bn} + e_n^a\Gamma_{k,m}^n\partial_\mu\varphi^m e^{bk}=\frac{1}{2F_\pi^2}\left(\varphi_a\partial_\mu\varphi_b-\varphi_b\partial_\mu\varphi_a\right)+O(\varphi^4) \quad (12)$$

+) In the work[10] the expansion of action is made.

In the Honerkamp Lagrangian the classical fields enter covariantly, while the part of the Lagrangian (11) dependent on internal fields only, is the chiral Lagrangian in terms of the Gursey coordinates. This is not astonishing, as the Gursey coordinates are the only ones for which "contractible" diagrams without external legs are absent and in this case the covariant perturbation theory coincides by form with the conventional one.*

A theory with the Lagrangian (12) is localizable. From the Honerkamp covariant perturbation theory it follows that the local properties of theory do not depend upon the choice of pion field coordinates and are defined rather by the transformation group under which the Lagrangian is invariant.

Quantizing the theory with Lagrangian (11) results in the S -matrix

$$S(\pi^{(in)}) = e^{iW_{tree}}\,N^{-1}\int\prod_a\prod_x d\Gamma^a\sqrt{g^G}\;e^{iW(\Gamma|\varphi)}. \quad (13)$$

Hence comparing (8) and (13) we find:

$$\sqrt{\frac{g(\varphi)}{g(\pi)}}\,\det\frac{\partial\pi^i}{\partial L^j(\pi,\varphi)} = \sqrt{g^G}.$$

To calculate concrete physical processes it is convenient to represent the S -matrix (13) in the form of vacuum expectation value of T^* -product of the exponent of the normal-ordered action.

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* There is similar effect in tree approximation (see D.V.Volkov, ЭЧАЯ, 4,3,1972).

$$S(\varphi) = \langle 0|T_w^* \, e^{iN\bar{W}^I(\Gamma|\varphi)}|0\rangle \, e^{iW_{tree}(\varphi)} \tag{14}$$

$$\bar{W}^I(\Gamma|\varphi) = \exp\left\{\frac{i}{2}\int d^4x\, d^4y\, \frac{\delta}{\delta\Gamma^a(x)}\Delta^c(x-y)\frac{\delta}{\delta\Gamma^a(y)}\right\} W^I(\Gamma|\varphi) \tag{15}$$

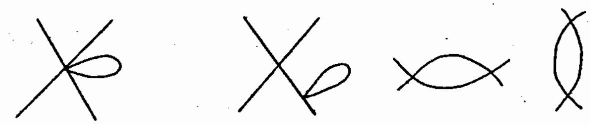
The normal-ordered Lagrangian is calculated in Appendix B. When calculating the Lagrangian (15) only the contracted products+) of fields without derivatives are taken, because the contracted products of fields with derivatives are compensated by determinant $\sqrt{g^G}$ in (13), and those of fields with derivatives and fields without derivatives equal zero.

3. Regularization of the S-matrix

It is known that the constraints due to chiral invariance are not sufficient to derive convergent results even from the one-loop diagrams:

+) The contracted product of two operators means:

$$\overline{AB} = \langle 0|T(AB)|0\rangle$$



(a) (b) (c)

Fig.1

The quadratic divergences in diagrams (a), (b) and (c) can be eliminated by standard renormalization of charge and wave function (see Appendix C). Some quadratic divergences in the diagrams (a) and (b) and those of the diagrams (c) cancel. To rule out the logarithmic divergences one needs to introduce two arbitrary parameters, and the renormalization procedure becomes nonconsistent (See the paper by Ecker and Honerkamp[2]).

Calculations of the diagrams (c) by the S.P. method result in the convergent expression and fix these arbitrary constants[4]. However, as was pointed above, nonregularized expressions for these diagrams contain quadratic divergences which cancel some divergences in the diagrams (a) and (b).

Thus, on the one hand, the standard renormalization method removes the quadratic divergences but cannot eliminate in a self-consistent way the logarithmic divergences, requiring arbitrary parameters. On the other hand, the S.P. method of calculations of the diagrams (c) fixes these constants but changes the standard method of renormalization of quadratic divergences. A natural way out of this situation consists in construction of regularization of the diagrams (a) and (b) which is adequate to that of the diagrams (c) by the S.P. method.

To calculate by the S.P. method it is necessary to know the sum of all one-loop diagrams with an arbitrary number of closed meson lines (Fig.2).

Fig.2.

To achieve this one should make a transition to the S matrix with normal-ordered Lagrangian (15), which leads to expressions of the type

$$\mathcal{J} = \sum_{n=1}^{\infty} C(n) \left(\Delta^c(0)\right)^n \quad ; \; \Delta^c(0) = \frac{1}{(2\pi)^4} \int \frac{d^4k}{-k^2 - i\varepsilon} \tag{16}$$

We suggest to calculate such expression by a method close as much as possible to the S.P.method[4] in calculating the

diagrams (c):+)

$$\mathcal{J}_{S.P} = g_{\mu\nu} \lim_{q_\mu \to 0} \int d^4x \, e^{iqx} \sum_{n=1}^{\infty} \left(\Delta^c(x)\right)^n \partial_\mu \partial_\nu \Delta^c(x) \, C(n) \tag{17}$$

(See Appendix D , formulae(D.3), (D.5), where the superpropagators with derivatives for the diagrams (c) of Fig.1 are given.)

For coefficients $C(n)$ defined by the transition to the normal-ordered Lagrangian (see B.4) the calculation of (17) results in (see (D.5)):

$$\mathcal{J}_{S.P} = 0$$

and, consequently, for (15) we have

$$\left[\overline{W}^{I}(\Gamma/\varphi)\right]_{Sp} = W^{I}(\Gamma/\varphi). \tag{18}$$

Therefore the S.P.regularization of the quadratic divergences leads to the following expression for the S -matrix functional:

+) An analogous regularization method is considered in the work by G.Lazarides and A.Patani[12].

$$S(\pi^{(n)}) = e^{iW_{tree}(\varphi)} \left[\langle 0 | T^* e^{iNW^I(r|\varphi)} | 0 \rangle \right]_{S.P} \qquad (19)$$

Subscript "S.P." means that we must apply the S.P.method when calculating the physical quantities.

We expand the S-matrix (19) in powers of W^I. The zeroth-order expansion gives all tree graphs. The first order equals zero due to the normal product. The second order results are convergent[4] and are given with slight modifications in Appendix D. Works[13] are devoted to a study of the highest-order expansions in analogical theories. In particular M.K.Volkov in his work [13] has proved the convergence of theory of any order. Results of these works[13] support the assumption that the S-matrix (19) contains no divergences, and applying the Honerkamp perturbation theory guarantees the calculated physical quantities be independent of the choice of pion field coordinates.

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Appendix A

The usual and covariant expansions of the Lagrangian $\mathcal{L}(\varphi+L)$ eq.(1) in powers of the variable L are denoted, respectively, as:

$$\mathcal{L}(\varphi+L) = \mathcal{L}^{(0)}(\varphi) + \mathcal{L}^{(1)}(\varphi|L) + \sum_{n=2}^{\infty} \mathcal{L}^{(n)}(\varphi|L)\frac{1}{n!}, \qquad (A.1)$$

$$\mathcal{L}_{x}(\varphi(+)L) = \mathcal{L}^{(0)}(\varphi) + \mathcal{L}^{(1)}(\varphi|L) + \sum_{n=2}^{\infty} \mathcal{L}_{x}^{(n)}(\varphi|L)\frac{1}{n!}, \qquad (A.2)$$

where $\mathcal{L}^{(0)}(\varphi) = \mathcal{L}(\varphi)$

$$\mathcal{L}^{(1)}(\varphi|L) = g_{ie_1}\partial_\mu\varphi^i\partial_\mu L^{e_1} + \frac{1}{2}\partial_\mu\varphi^i\partial_\mu\varphi^j\partial_{e_1}g_{ij}L^{e_1}$$

and the following relation holds:

$$\mathcal{L}^{(n)}(\varphi+L|L) = \mathcal{L}^{(n)}(\varphi|L) + \mathcal{L}^{(n+1)}(\varphi|L) + \dots \qquad (A.3)$$

Allowing for the equality

$$\partial_e g_{ij} = \Gamma_{i,ej} + \Gamma_{j,ei} \qquad (A.4)$$

we represent $\mathcal{L}^{(1)}(\varphi/L)$ in the form

$$\mathcal{L}^{(1)}(\varphi/L) \equiv \mathcal{L}_{,e_1} L^{e_1} = g_{ij}\,\partial_\mu\varphi^j (\mathcal{D}^\mu L)^i, \tag{A.5}$$

$$(\mathcal{D}^\mu L)^i = \partial_\mu L^i + \mathcal{T}_{e_1\,m}^{\;i}\,\partial^\mu\varphi^m L^{e_1}, \tag{A.6}$$

where $\mathcal{L}_{,e_1}$ is the first-order differential operator.
When calculating $\mathcal{L}^{(2)}$ we employ the eq.(A.3)

$$\mathcal{L}^{(2)}(\varphi/L) = (\mathcal{D}^\mu L)^i(\mathcal{D}_\mu L)^j g_{ij} + g_{ij}\,\partial_\mu\varphi^i\partial^\mu\varphi^m\Big[\partial_{e_2}\mathcal{T}_{e_1 m}^{\;i} +$$
$$+ \mathcal{T}_{e_2 k}^{\;i}\,\mathcal{T}_{e_1 m}^{\;k}\Big] L^{e_1}L^{e_2} + \partial_\mu\varphi^j\,\mathcal{T}_{j,e_1e_2}\,\partial^\mu(L^{e_1}L^{e_2}) . \tag{A.7}$$

The second term of the covariant expansion (A.2) $\mathcal{L}_{\mathcal{X}}^{(2)}(\varphi/L)$ is expressed via $\mathcal{L}^{(2)}$ in the following way:

$$\mathcal{L}_{\mathcal{X}}^{(2)}(\varphi/L) = \mathcal{L}^{(2)}(\varphi/L) - \mathcal{L}_{,k}\big[\mathcal{T}_{e_1e_2}^{\;k} L^{e_1}L^{e_2}\big] =$$
$$= \mathcal{L}^{(2)}(\varphi/L) - g_{ij}\,\partial_\mu\varphi^j\partial^\mu\varphi^m\Big[\partial_m\mathcal{T}_{e_1e_2}^{\;i} + \mathcal{T}_{km}^{\;i}\,\mathcal{T}_{e_1e_2}^{\;m}\Big]L^{e_1}L^{e_2} - \tag{A.8}$$
$$- \partial_\mu\varphi^j\,\mathcal{T}_{j,e_1e_2}\,\partial_\mu(L^{e_1}L^{e_2}) .$$

From (A.8) we get for a space of constant curvature the following expression:

$$\mathcal{L}_{\mathcal{X}}^{(2)}(\varphi) = (\mathcal{D}_\mu L)^i(\mathcal{D}^\mu L)^j g_{ij} + F_\pi^{-2}\big(g_{me_1}g_{je_2} - g_{e_1e_2}g_{mj}\big)\partial_\mu\varphi^j\partial^\mu\varphi^m L^{e_1}L^{e_2} \tag{A.9}$$

Here the following relation:

$$g_{ij}\big[\partial_m\mathcal{T}_{e_1e_2}^{\;i} - \partial_{e_2}\mathcal{T}_{e_1m}^{\;i} + \mathcal{T}_{km}^{\;i}\mathcal{T}_{e_1e_2}^{\;k} - \mathcal{T}_{e_2k}^{\;i}\mathcal{T}_{e_1m}^{\;k}\big] =$$
$$= R_{m,e_1,je_2} = F_\pi^{-2}\big[g_{e_1e_2}g_{mj} - g_{me_1}g_{je_2}\big] \tag{A.10}$$

has been used.

Calculations of subsequent terms of the covariant expansion are simplified significantly if one exploits the formula (A.3) and the fact that the covariant derivative of metric tensor is zero :

$$\mathcal{L}_{\mathcal{X}}^{(2n)}(\varphi/L) = (-1)^n\left(\frac{2}{F_\pi}\right)^{2(n-1)}\Big\{(\mathcal{D}_\mu L)^i(\mathcal{D}_\mu L)^j\big[g_{ij}(\varphi)(\tilde{L}^2)^{n-1} -$$
$$- g_{ik}L^k g_{je}L^e(\tilde{L}^2)^{n-2}\big] + \frac{1}{F_\pi^2}\partial_\mu\varphi^j\partial^\mu\varphi^i\big[g_{ik}L^k g_{je}L^e - g_{ij}(\tilde{L}^2)\big](\tilde{L}^2)^{n-1}\Big\}$$

$$\mathcal{L}_{\mathcal{X}}^{2n+1}(\varphi/L) = (-1)^n\left(\frac{2\pi}{F_\pi}\right)^{2n}(\mathcal{D}_\mu L)^i\partial^\mu\varphi^j\big[g_{ik}L^k g_{je}L^e - g_{ij}(\tilde{L}^2)\big](\tilde{L}^2)^{n-1}, \tag{A.11}$$

where

$$\tilde{L}^2 = L^k g_{ke}(\varphi)L^e .$$

Now changing variables (7) and summing all terms in the expansion (A.2) we obtain the formulae (11) and (12).

Appendix B

To calculate $\overline{W}^{I} = \int d^4x\, \overline{\mathcal{L}}^{I}(r|\varphi)$ (see eq.(15)) it is convenient to expand the functions f and g in the series:

$$f = \sum_{k=0}^{\infty} C_f(k)\, r^{2k} \quad ; \quad g = \sum_{k=0}^{\infty} C_g(k)\, r^{2k}. \tag{B.1}$$

Then it suffices to compute the expressions

$$\overline{\Lambda_\ell(r)\, r^{2k}} = \left(\frac{d}{dx}\right)^n \left[\overline{e^{x r^2} \Lambda_\ell(r)}\right]_{x=0} \equiv e^{\frac{1}{2i}\frac{\partial}{\partial r_i}\Delta^c(0)\frac{\partial}{\partial r_i}} \Lambda_\ell(r)\, r^{2k} \tag{B.2}$$

$$\Lambda_1 = 1 \quad ; \quad \Lambda_2 = r^a \quad ; \quad \Lambda_3 = r^a r^b .$$

The expression $\overline{e^{x r^2} \Lambda_\ell(r)}$ is calculated by the Fourier transformation $e^{x r^2}\Lambda_\ell(r)$ with respect to the variable r :

$$\overline{e^{x r^2}\Lambda_\ell(r)} = \exp\left[x r^2 \frac{\beta}{\beta - x}\right] \left(\frac{\beta}{\beta - x}\right)^{3/2} \Phi^\ell(x,\beta,r)$$

$$\Phi^1 = 1 \quad ; \quad \Phi^2 = r^a \frac{\beta}{\beta - x} \quad ; \quad \Phi^3 = r^a r^b \left(\frac{\beta}{\beta - x}\right)^2 + \frac{\delta_{ab}}{2\beta}\left(\frac{\beta}{\beta - x}\right)$$

$$\beta = \frac{i}{2\Delta^c(0)} \quad ; \quad \Delta^c(0) = \frac{1}{(2\pi)^4}\int \frac{d^4k}{-k^2 - i\varepsilon} . \tag{B.3}$$

And finally, we get:

$$\overline{f} = \sum_{k=0}^{\infty} r^{2k} C_f(k) \sum_{n=0}^{\infty} \frac{\alpha^n}{n!} \frac{k+1}{k+n+1} \quad ; \quad \overline{g} = e^{\alpha} g$$

$$\overline{r^2 f} = -C_f(-1) + \sum_{k=0}^{\infty} r^{2k} C_f(k-1) \sum_{n=0}^{\infty} \frac{\alpha^n}{n!} \frac{2(k+n)+1}{2k+1}$$

$$\overline{f r^a} = r^a \sum_{k=0}^{\infty} r^{2k} C_f(k) \sum_{n=0}^{\infty} \frac{\alpha^n}{n!} \frac{k+1}{k+n+1} \frac{2(k+n)+3}{2k+3} \tag{B.4}$$

$$\overline{g r^a} = r^a \sum_{k=0}^{\infty} r^{2k} C_g(k) \sum_{n=0}^{\infty} \frac{\alpha^n}{n!} \frac{2(k+n)+3}{2k+3}$$

$$\overline{\frac{f-1}{r^2} r^a r^b} = \sum_{k=0}^{\infty} r^{2k} C_f(k+1) \sum_{n=0}^{\infty} \frac{\alpha^n}{n!} \left[r^a r^b \frac{2(k+n)+5}{2k+5} \frac{(k+2)(k+1)}{(k+n+2)(k+n+1)} + \frac{\delta_{ab}}{2\beta} \frac{(2k+4)(k+1)}{(2(k+n)+4)(k+n+1)} \right]$$

$$\overline{f r^a r^b} = \sum_{k=0}^{\infty} r^{2k} C_f(k) \sum_{n=0}^{\infty} \frac{\alpha^n}{n!} \left[r^a r^b \frac{2(k+n)+5}{2k+5} \frac{2(k+n)+3}{2k+3} \frac{k+1}{k+n+1} + \frac{\delta_{ab}}{2\beta} \frac{2(k+n)+3}{2k+3} \frac{k+1}{k+n+1} \right]$$

$$\overline{\frac{g-1}{r^2} r^a r^b} = \sum_{k=0}^{\infty} r^{2k} C_g(k) \sum_{n=0}^{\infty} \frac{\alpha^n}{n!} \left[r^a r^b \frac{2(k+n)+5}{2k+5} \frac{k+1}{k+n+1} + \frac{\delta_{ab}}{2\beta} \frac{k+1}{k+n+1} \right]$$

where

$$\alpha = \frac{2i\Delta^c(0)}{F_\pi^2} .$$

Appendix C

Let us here renormalize the $\pi\pi$ -scattering amplitude in the one-loop approximation. In calculations we will use the formulae (14) and (15). The Lagrangian (15) in the one-loop approxiamtion has the form:

$$\mathcal{L} = \left[\frac{1}{2}(\partial_\mu \varphi)^2 + \frac{1}{4F_\pi^2}\pi^2(\partial_\mu \pi)^2\right]\left(1 + \frac{2i\Delta^c(0)}{F_\pi^2}\right) - \partial_\mu \varphi^i \partial^\mu \pi_i + \frac{1}{4F_\pi^2}\pi^2(\partial_\mu \pi)^2 \frac{3\Delta^c(0)}{2i}$$

$$+ \frac{1}{2F_\pi^2}(\pi_a \partial_\mu \pi_b - \pi_b \partial_\mu \pi_a) r^a \partial^\mu r^b + \frac{1}{2F_\pi^2}\partial_\mu \pi^a \partial_\mu \pi^b (r^a r^b - r^2 \delta_{ab}), \tag{C.1}$$

where π is the asymptotic field, $\Delta^c(0)$ the quadratically divergent expression (B.3). From the equations of motion (3) we have:

$$\frac{1}{2}(\partial_\mu \varphi)^2 = \frac{1}{2}(\partial_\mu \pi)^2 - 2\frac{\pi^2(\partial_\mu \pi)^2}{4F_\pi^2} + O(\pi^6),$$

$$\partial_\mu \varphi_i \partial_\mu \pi_i = -(\partial_\mu \pi)^2 + 2\frac{\pi^2(\partial_\mu \pi)^2}{4F_\pi^2} + O(\pi^6).$$

From eq.(C.1) we obtain for the functional $S(\pi)$ the following expressions:

$$S(\pi) = S_0 + S_1^{(1)} + S_1^{(2)} + S_1^{(3)} + O(\pi^6) + O\left(\frac{1}{F_\pi^6}\right), \qquad (C.2)$$

$$S_0 = -\frac{i}{2}\int d^4x\, (\partial_\mu \pi)^2 \left(1 - \frac{2i\Delta^c(0)}{F_\pi^2}\right), \qquad (C.3)$$

$$S_1^{(1)} = \frac{i}{4F_\pi^2}\int d^4x\, \pi^2 (\partial_\mu \pi)^2 \left(1 - \frac{15}{2}\frac{\Delta^c(0)i}{F_\pi^2}\right), \qquad (C.4)$$

$$S_1^{(2)} = \frac{i}{4F_\pi^4}\int d^4x\, d^4y\, \pi_a(x)\partial_\mu \pi_b(x)\left[\pi_a(y)\partial_\nu \pi_b(y) - \pi_b(y)\partial_\nu \pi_a(y)\right] \otimes$$

$$\otimes \left[\partial_\mu \Delta^c(x-y)\partial_\nu \Delta^c(x-y) - \Delta^c(x-y)\partial_\mu\partial_\nu \Delta^c(x-y)\right], \qquad (C.5)$$

$$S_1^{(3)} = \frac{1}{2F_\pi^4}\int d^4x\, d^4y \left[(\partial_\mu \pi)^2(\partial_\nu \pi(y))^2 + \left(\partial_\mu \vec{\pi}(x)\partial_\nu \vec{\pi}(y)\right)^2\right] \Delta^{c\,2}(x-y). \qquad (C.6)$$

The formula (C.3) defines the renormalized wave function

$$(\pi)_{renorm} = \left(1 + \frac{i\Delta^c(0)}{F_\pi^2}\right)\pi. \qquad (C.7)$$

The expressions (C.5) and (C.6) correspond to the two-vertex one-loop diagrams and have the quadratic divergence

$$\left[\Delta S_1^{(2)}\right]_{quad.div} = i\int d^4x\, \frac{\pi^2(\partial_\mu \pi)^2}{4F_\pi^2}\left(\frac{3}{4}\frac{\Delta(0)i}{F_\pi^2}\right) \qquad (C.8)$$

Hence it is obvious that all the quadratic divergences in the functional $S(\pi)$ are eliminated by renormalization of wave function (C.7) and charge:

$$\left(\frac{1}{F_\pi^2}\right)_{renorm} = \frac{1}{F_\pi^2}\left(1 - \frac{11}{4}\frac{\Delta(0)i}{F_\pi^2}\right). \qquad (C.9)$$

Appendix D

Here we would like to present the results for the π-π-scattering amplitude calculated by the S.P. method in the one-loop approximation, making use also of the rules proposed in the work[8].

The second-order expansion of the functional $S(\pi)$ (19) in powers of W has the form

$$S(\pi)=\int d^4x_1\, d^4x_2 \Big\{\frac{1}{2}\Big[(\vec{\pi}(1)\vec{\pi}(2))(\partial_\mu\vec{\pi}(1)\partial_\nu\vec{\pi}(2)) - (\vec{\pi}(1)\partial_\nu\vec{\pi}(2))(\vec{\pi}(2)\partial_\mu\vec{\pi}(1))\Big] i\mathcal{Z}^{(3)}_{\mu\nu}(x_1-x_2)$$

$$+ (\partial_\mu\pi(1))^2(\partial_\nu\pi(2))^2\, i\mathcal{Z}_2(x_1-x_2) + (\partial_\mu\vec{\pi}(1)\,\partial_\nu\vec{\pi}(2))^2\, i\mathcal{Z}_1(x_1-x_2)\Big\}, \qquad \text{(D.1)}$$

where

$$\mathcal{Z}_k(x) = -i\frac{1}{2F_\pi^4}\Delta^{c^2}(x)\sum_{n=0}^{\infty}\left(\frac{-4i\Delta^c(x)}{F_\pi^2}\right)^{2n} C_k(n) \;;\quad k=1,2$$

$$C_1=\frac{(8n+5)(2n+3)}{(2n+2)(2n+2)!}\,\frac{4}{15} \;;\quad C_2=\frac{(6n+5)(2n+3)}{(2n+2)(2n+2)!}\,\frac{4}{15} \qquad \text{(D.2)}$$

$$\mathcal{Z}^{(3)}_{\mu\nu}(x)=\frac{-i}{2F_\pi^4}\left(\partial_\mu\Delta(x)\partial_\nu\Delta(x)-\partial_\mu\partial_\nu\Delta(x)\cdot\Delta(x)\right)\sum_{n=0}^{\infty}\left(-\frac{4i\Delta^c(x)}{F_\pi^2}\right)^{2n}\left[\frac{4(2n+3)}{3(2n+2)(2n+2)!}\right]. \qquad \text{(D.3)}$$

The Fourier transforms $\tilde{\mathcal{Z}} = \int e^{iqx}\mathcal{Z}(x)\,d^4x$ calculated by the S.P. method are of the form[4,8]:

$$\tilde{\mathcal{Z}}_k = -\frac{1}{32\cdot 4F_\pi^4\pi^2}\,\frac{\pi}{2i}\int_{-i\infty-\gamma}^{+i\infty-\gamma} dz\,\frac{1}{\sin^2\pi z}\left(\frac{-q^2-i\varepsilon}{4\pi^2F_\pi^2}\right)^{2z}\frac{(2z+2)C_k(z)}{\Gamma(2z+1)\Gamma(2z+3)} =$$

$$= -\frac{1}{16\pi^2F_\pi^4}\,\frac{1}{4}\left[\ln\left(\frac{-q^2-i\varepsilon}{4\pi^2F_\pi^2}\right) + 3(c-1)+a_k\right] + O(q^2) \qquad \text{(D.4)}$$

$$a_1=\frac{8}{15} \;;\quad a_2=\frac{14}{15}$$

$$\tilde{\mathcal{Z}}^{(3)}_{\mu\nu} = -\frac{1}{96F_\pi^4\pi^2}\,\frac{\pi}{2i}\int_{-i\infty-\gamma}^{+i\infty-\gamma} dz\left(\frac{-q^2-i\varepsilon}{(2\pi F_\pi)^2}\right)^{2z}\frac{\left[(2z+1)(q_\mu q_\nu - g_{\mu\nu}q^2) + g_{\mu\nu}q^2 2(z)^M\right]}{\Gamma(2z+2)\,(\Gamma(2z+3))^2}\Bigg|_{M>1} =$$

$$= \frac{1}{(4\pi F_\pi^2)^2}\,\frac{1}{4}\,\frac{1}{3}(q^2g_{\mu\nu}-q_\mu q_\nu)\left[\ln\left(-\frac{q^2-i\varepsilon}{4\pi^2F_\pi^2}\right)+3(c-1)\right] + O(q^4) \qquad \text{(D.5)}$$

and C is the Euler constant.

Calculating the amplitude by the formula

$$\int\prod_{i=1}^{4} d^4x_i\, e^{ip_1x_1+ip_2x_2-ip_3x_3-ip_4x_4}\,\frac{\delta^4}{\delta\pi_{i_1}\,\delta\pi_{i_2}\,\delta\pi_{i_3}\,\delta\pi_{i_4}}\,S(\pi) =$$

$$= i(2\pi)^4\delta(p_1+p_2-p_3-p_4)\big[A(s,t,u)\delta_{i_1i_2}\delta_{i_3i_4} + A(t,s,u)\delta_{i_1i_3}\delta_{i_2i_4} + \qquad \text{(D.6)}$$

$$+ A(u,t,s)\,\delta_{i_1i_4}\delta_{i_3i_2}\big]$$

$$s=(p_1+p_2)^2 \;;\quad t=(p_1-p_3)^2 \;;\quad u=(p_1-p_4)^2$$

we obtain for $A(s,t,u)$ the following expression

$$A(s,t,u) = \frac{s}{F_\pi^2} - \frac{1}{16\pi^2F_\pi^4}\Big[\frac{s^2}{2}\ln\left(\frac{-s-i\varepsilon}{4\pi^2F_\pi^2}\right) + \qquad \text{(D.7)}$$

$$+\frac{t(t-u)}{6}\ln\left(\frac{-t-i\varepsilon}{4\pi^2F_\pi^2}\right) + \frac{u(u-t)}{6}\ln\left(\frac{-u-i\varepsilon}{4\pi^2F_\pi^2}\right) +$$

$$+\left(c-\frac{13}{15}\right)(s^2+t^2+u^2) + \frac{s^2}{3}\Big].$$

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