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RESTRICTIONS ON THE ANOMALOUS DIMENSIONS FROM ANALYSIS OF DEEP INELASTIC e-p SCATTERING DATA

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RESTRICTIONS ON THE ANOMALOUS DIMENSIONS FROM ANALYSIS OF DEEP INELASTIC e-p SCATTERING DATA

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Об ограничениях на аномальные размерности из анализа данных по глубоконеупругому е-р рассеянию

Анализируются данные SLAC-MIT по глубоконеупругому е-р рассеянию. При этом к "скэйлинговому" члену в выражении для структурной функции νW_2 добавляется член, обусловленный аномальными размерностями. Показано, что данные хорошо описываются при разных выражениях для $R \equiv \sigma_L/\sigma_T$. Показано также, что эти данные не исключают аномальных размерностей. Величина соответствующего параметра существенно зависит от параметризации R.

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Bilenkaya S.I., Hristova E.H., Stamenov D.B. E2 - 7509

Restrictions on the Anomalous Dimensions from Analysis of Deep Inelastic e-p Scattering Data

The SLAC-MIT data on deep inelastic e-p scattering are analyzed. In the expression for the structure function νW_2 a scaling breaking term, due to possible anomalous dimensions, is added. It is shown that the data are fitted well by different parametrizations of $R \equiv \sigma_L/\sigma_T$. It is shown also that the presently available SLAC-MIT data do not exclude anomalous dimensions. The value of the anomalous dimensions depends strongly on the parametrization of R.

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Introduction

The idea of scale invariance at short distances has received a recent development due to the experimental indications of scaling in deep inelastic lepton-nucleon processes. In references /1-3/ it has been shown that the hypothesis of scale invariance does not imply Bjorken's scaling /4/ for the structure functions of the deep inelastic e-N scattering ($MW_1(\nu, q^2) \to F_1(\omega)$, $\nu W_2(\nu, q^2) \to F_2(\omega)$) in the limit $q^2 \to \infty$, $\nu \to \infty$ at $\omega = \frac{2M\nu}{q^2}$ fixed). In order to obtain Bjorken's scaling one has to assume in addition that the dimensions of the operators involved in Wilson's expansion /5/ of the product of two currents near the light cone should be canonical. However, in the framework of quantum field theory it is necessary to assume anomalous dimensions (at least of some basic fields) if we want to construct a theory, which should not be free in the asymptotic (i.e. not only high energies and high momentum transfer, but also high external momentum squared) region /6/.

So it is important to analyse the present experimental data in order to check the possible existence of anomalous dimensions. Basing on the results of a field-theory model with anomalous dimensions Parisi /7/ has derived a logarithmic in q^2 term in addition to the scaling one of the function νW_2. Using the data on deep inelastic e-p scattering the author concludes that the present data are not in contradiction with the possible existence of anomalous dimensions and so they do not exclude a violation of Bjorken's scaling for the function νW_2.

The present paper gives the results of a more detailed analysis of the SLAC-MIT /8,9/ data on deep inelastic e–p scattering. As in ref. /7/, a term, caused by anomalous dimensions is added to the scaling term in νW_2. Various expressions for the ratio R are considered.

Our analysis shows that the SLAC-MIT data on deep inelastic e–p scattering do not exclude anomalous dimensions. However, the value of the parameter, associated with them, depends strongly on the parametrization of R.

In our analysis we have used a method which has been applied earlier when analysing the data on elastic /10/ and deep inelastic /11/ e–p scattering. It was considered in more details in ref. /10/. This method of determining the functions νW_2 and R implies that they should be parametrized at the very beginning. The appropriate parameters are determined directly from the experimental data by minimizing the functional χ^2.

Parametrization of the Structure Function νW_2

The cross section of the process $e+p \to e+$ hadrons for unpolarized initial particles in the one-photon exchange approximation has the following form:

$$\frac{d^2\sigma}{d\Omega dE'} = \left(\frac{d\sigma}{d\Omega}\right)_M \left[W_2(\nu,q^2) + 2\,\mathrm{tg}^2\frac{\theta}{2} W_1(\nu,q^2)\right], \quad (1)$$

where

$$\left(\frac{d\sigma}{d\Omega}\right)_M = \frac{\alpha^2 \cos^2\frac{\theta}{2}}{4E^2 \sin^4\frac{\theta}{2}},$$

$$\alpha = \frac{e^2}{4\pi}, \quad \nu = -\frac{pq}{M} = E - E', \quad q^2 = 4EE'\sin^2\frac{\theta}{2} \quad (\nu > 0, q^2 > 0).$$

Here E,E′ and θ are the electron initial energy, final energy and scattering angle, respectively, and M is the proton mass.

The transverse and longitudinal absorption cross-sections of the virtual photon σ_T and σ_L are related to the structure functions W_1 and W_2 by:

$$\sigma_T = (2\pi)^2 \alpha \frac{1}{k} W_1, \quad (2a)$$

$$\sigma_L = (2\pi)^2 \alpha \frac{1}{k}\left(-W_1 + \frac{q^2+\nu^2}{q^2} W_2\right), \quad (2b)$$

$$R \equiv \frac{\sigma_L}{\sigma_T} = \frac{-W_1 + \frac{q^2+\nu^2}{q^2} W_2}{W_1}, \quad (2c)$$

where

$$k = \nu - \frac{q^2}{2M}.$$

As was already pointed out, in our analysis we shall use a parametrization of the function νW_2 taking into account anomalous dimensions. We shall fix the main assumptions used in deriving the considered expression for νW_2 (see eq. (7), (11) below).

Basing on Wilson's theory /5/ of broken scale invariance the following sum rules for $\nu W_2(\nu,q^2) \equiv F_2(\omega,q^2)$ can be obtained /1,2/ (in the limit $q^2 \to \infty$):

$$\int_1^\infty d\omega\, \omega^{-n} F_2(\omega,q^2) = C_n \left(\frac{M^2}{q^2}\right)^{1/2\,\delta_n} \quad n=2,4,\ldots, \quad (3)$$

where

$$\omega = \frac{2M\nu}{q^2}, \quad (4)$$

$$\delta_n = d_n - n - 2.$$

In (4) n and d_n are the spin and the dimension (in general anomalous) of the symmetric trassless tensor operators involved in Wilson's expansion of two currents near the light cone, respectively. Canonical dimensions imply $d_n = n+2$. The quantity δ_n is the anomalous part of the dimension of the spin n tensor operator.

Equation (3) implies that Bjorken's scaling for νW_2 can be true only if we deal with canonical dimensions ($\delta_n = 0$ for all n).

We shall remind that conservation of the energy momentum tensor requires that $\delta_2 = 0$ and positively of W_2 leads to $\delta_{n+2} \geq \delta_n$ (see ref. /1/).

Further on the following parametrization of δ_n is adopted:

$$\delta_n = A\left(1 - \frac{12}{(n+1)(n+2)}\right). \tag{5}$$

It is easy to verify that the conditions $\delta_2 = 0$ and $\delta_{n+2} \geq \delta_n$ are fulfilled. Such an expression has been obtained in the framework of a field-theory model /1/.

The value of the parameter A, which characterizes anomalous dimensions, will be determined from the experimental data.

It is convenient to rewrite the sum rules (3) as:

$$\int_1^\infty d\omega\, \omega^{-n} F_2(\omega, q^2) = C_n \left(\frac{M^2}{K^2}\right)^{\frac{\delta_n}{2}} e^{-\frac{1}{2}\delta_n \ln\left(\frac{q^2}{K^2}\right)}, \tag{6}$$

where K^2 is any fixed value of q^2.

Expanding the RHS of eq. (6) in powers of $\delta_n \ln\left(\frac{q^2}{K^2}\right)$ and using the Mellin transform we obtain for F_2 finally:

$$F_2(\omega, q^2) = F_2(\omega, K^2)\Big[1 - \Delta_1(\omega, K^2)\frac{A}{2}\ln\left(\frac{q^2}{K^2}\right) + \frac{1}{2}\Delta_2(\omega, K^2)\left(\frac{A}{2}\right)^2 \ln^2\left(\frac{q^2}{K^2}\right) + \dots\Big], \tag{7}$$

where

$$\Delta_1(\omega, q^2) = 1 - 12\int_{1/\omega}^{1} dx\,(x - x^2)\frac{F_2(\omega x, q^2)}{F_2(\omega, q^2)}, \tag{8}$$

$$\Delta_2(\omega, q^2) = 1 - 24\int_{1/\omega}^{1} dx\,(x - x^2)\frac{F_2(\omega x, q^2)}{F_2(\omega, q^2)} + 144\int_{1/\omega}^{1} dx \int_{1/x\omega}^{1} dy\,(x - x^2)(y - y^2)\frac{F_2(\omega x y, q^2)}{F_2(\omega, q^2)}. \tag{9}$$

The term linear in $\ln(q^2/K^2)$ in eq. (7) was obtained in ref. /7/ and was used by the author to receive some restrictions on the value of the anomalous dimensions *.

The functions Δ_1 and Δ_2 satisfy the following asymptotic conditions:

$$\Delta_1(\omega, q^2) \xrightarrow[\omega \to 1]{} 1, \qquad \Delta_1(\omega, q^2) \xrightarrow[\omega \to \infty]{} -1;$$

$$\Delta_2(\omega, q^2) \xrightarrow[\omega \to 1]{} 1, \qquad \Delta_2(\omega, q^2) \xrightarrow[\omega \to \infty]{} 1. \tag{10}$$

Our analysis is based on formula (7), provided the function $F_2(\omega, K^2)$ is parametrized as follows /9/ (K^2 - fixed):

$$F_2(\omega, K^2) = \sum_{i=0} a_i \left(1 - \frac{1}{\omega}\right)^{i+3}, \tag{11}$$

where a_i are free parameters.

Eq. (11) is consistent with the usual Regge asymptotic behaviour /12/

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* In this paper there is a misprint in the expression for Δ_1.

$$F_2(\omega) \underset{\omega\to\infty}{\to} \text{const} \quad (12)$$

and with the threshold behaviour [13,14]

$$F_2(\omega) \underset{\omega\to 1}{\to} \left(1-\frac{1}{\omega}\right)^3. \quad (13)$$

The value of K^2 is fixed in such a way as to ignore the quadratic in $\ln(\frac{q^2}{K^2})$ term in eq. (7) in the considered interval of q^2. The analysis of the data gives the parameter A to be rather small, so we can extend the interval of q^2 in which the linear term is a good approximation. We notice that estimates for the quadratic in $\ln(\frac{q^2}{K^2})$ term in the region of ω, where $\Delta_i \to 0$ indicate that it is small and experimental precision is not enough to account it. So we can neglect the quadratic in $\ln(\frac{q^2}{K^2})$ term in this region of ω too. We emphasize that in the derivation of these estimates we make use of eq. (9) for Δ_2 and the smallness of parameter A.

As the hypothesis of scale invariance does not allow to make any conclusions about R at present[1], various expressions for R are considered. (Some of them are derived in the framework of theoretical models.) All free parameters associated with the functions F_2 and R are determined from the experimental data by minimizing the functional

$$\chi^2 = \sum_i \frac{1}{\epsilon_i^2}(\sigma_i^{exp.} - \sigma_i^{theor.})^2. \quad (14)$$

Here $\sigma_i^{exp.}$ is the differential cross section of the process at the i-th point, ϵ_i is the error of $\sigma_i^{exp.}$, $\sigma_i^{theor.}$ is the cross section at the i-th point calculated by formula (1). Minimization of χ^2 is achieved through the linearization method [15].

Results of Analysis

In this section we give the results of analysis of the SLAC-MIT data [8,9]. The background of our analysis - eq. (7) is an asymptotic equation with respect to the variable q^2. Therefore exploiting the present experimental data at q^2 $(0.25 \le q^2 \le 19.2\ (\text{GeV/c})^2)$ we make the following restrictions on q^2_{min}:

$$q^2_{min} = 1{,}5\left(\frac{\text{GeV}}{c}\right)^2, \quad q^2_{min} = 3\left(\frac{\text{GeV}}{c}\right)^2, \quad q^2_{min} = 5\left(\frac{\text{GeV}}{c}\right)^2.$$

Various regions for W (W is the mass of the final hadrons) have been considered, too:

$W \ge 1{,}8$ GeV, $W \ge 2$ GeV, $W \ge 2{,}3$ GeV.

Our analysis indicates that formula (7) fits the data in all considered intervals of q^2 provided $W \ge 2.3$ GeV. Throughout the analysis various expressions for R have been considered. We have examined the data at different fixed values of the variable K^2. Within the errors, the fit of the data at fixed q^2_{min} does not depend on K^2. We shall discuss further the results of the analysis at $K^2 = 8\ (\text{GeV/c})^2$. As in ref. [11] only the parameters a_0 and a_2 in the expansion (11) are assumed to be nonzero.

1. Suppose that

$$R = \frac{q^2}{\nu^2}. \quad (15)$$

Such a parametrization of R leads to the Callan-Gross relation [16] between the structure functions W_1 and νW_2. The results of analysis of the data in region $W \ge 2.3$ GeV are listed in Table I. One can see that the experimental data can be fitted well by means of (7) and (15) in all considered regions of q^2. Parameter A is nonzero and as q^2_{min} increases its mean value grows. Note that as was shown in ref. [11] no satisfactory description of the available data can be obtained if one assumes for R and F_2 the relations (15) and (11) respectively in the region $W \ge 2.3$ GeV.

Table I

The Results of Analysis of SLAC-MIT data ($R=\frac{q^2}{\nu^2}$, $K^2=8\left(\frac{GeV}{c}\right)^2$, $W\geq 2.3\,GeV$)

$q^2_{min}\left(\frac{GeV}{c}\right)^2$	a_0	a_2	A	$\chi^2/\bar{\chi}^2$
1.5	1.44±0.01	-1.19±0.02	0.21±0.03	138/133
3	1.48±0.02	-1.34±0.04	0.36±0.04	92/116
5	1.56±0.02	-1.61±0.07	0.47±0.06	61/74

A disagreement of the scaling of F_2 with eq. (15) has been obtained also from the ρ-electroproduction data in ref. /17/.

Table I illustrates that the logarithmic term in eq. (7) allows us to fit the data by means of parametrization (15) of R.

2. We analyzed also the data using a more general than (15), parametrization /18/ of R:

$$R=\frac{q^2}{\nu^2}f(\omega),$$

where

$$1)\ f(\omega)=1+a_1\omega^2$$

$$2)\ f(\omega)=a_2.$$

Within errors the results of analysis coincide with those obtained when eq. (15) has been supposed. We find the following values of the parameters a_1 and a_2 :

$$a_1=0{,}04\pm 0{,}02,$$

$$a_2=1{,}15\pm 0{,}34.$$

3. We have further analysed the data considering the expressions:

$$R=\text{const}, \tag{16}$$

$$R=a_3\frac{1}{\omega}. \tag{17}$$

The results are presented in Table II. The data in all considered intervals of q^2 turned out to be fitted quite well by formula (7) and the parametrizations (16) and (17). The parameter A is nonzero and as q^2_{min} increases its mean value grows as in the case of eq. (15). We point out that our analysis with R = const and with F_2 according to eq. (7) gives an essential improvement of the description of the SLAC-MIT data in comparison with the one that makes use of parametrization (11) of F_2 as in ref. /11/. At $R=a_3\frac{1}{\omega}$ a reasonable description has also been obtained assuming scaling for F_2 /11/.

Table II
The Results of Analysis of SLAC-MIT data ($K^2 = 8\left(\frac{GeV}{c}\right)^2$, $W \geq 2.3$ GeV)

	$q^2_{min}\left(\frac{GeV}{c}\right)^2$	a_0	a_2	A		$\chi^2/\bar{\chi}^2$
$R = const$	1.5	1.48±0.02	-1.23±0.03	0.14±0.03	R =0.21±0.04	100/132
	3.0	1.53±0.03	-1.32±0.04	0.25±0.05	R =0.26±0.05	68/115
	5.0	1.56±0.03	-1.46±0.08	0.34±0.07	R =0.22±0.07	55/73
$R=\alpha_3 \frac{1}{\omega}$	1.5	1.56±0.03	-1.38±0.05	0.066±0.036	α_3=0.65±0.11	100/132
	3.0	1.60±0.03	-1.47±0.05	0.19±0.05	α_3=0.65±0.13	71/115
	5.0	1.61±0.04	-1.62±0.08	0.31±0.07	α_3=0.48±0.15	54/73

4. Tables I and II show that if we assume expressions (15), (16) and (17) for R the analysis of the data leads to nonzero values for the parameter associated with anomalous dimensions. A further analysis shows that any conclusions on the existence of anomalous dimensions essentially depend on the parametrization for R. Table III presents the values of the corresponding parameters obtained in the analysis of the data, when

$$R = \alpha_4 \frac{q^2}{M^2}, \tag{18}$$

$$R = \alpha_5 \frac{q^2}{W^2}. \tag{19}$$

As one can see from this Table parameter A equals zero within two errors. Addition of the term logarithmic in q^2 appears to be inessential in this case.

Summary

The SLAC-MIT data on deep inelastic e–p scattering have been examined assuming the existence of anomalous dimensions. Various parametrizations of R have been considered. It is shown that:

1. The present experimental data do not exclude anomalous dimensions.

2. Supposing the existence of anomalous dimensions when we have eq. (7) and supposing $R = \frac{q^2}{\nu^2}$ we obtain a good description of the experimental data. We shall note here, that if we suppose eq. (11) for F_2 and eq. (15) for R a satisfactory description of the data is impossible in the region $W \geq 2.3$ GeV.

3. The value of the parameter A characterizing anomalous dimensions depends strongly on the parametrization of R.

Table III
The Results of Analysis of SLAC-MIT data $\left(q^2 \geq 1 \left(\frac{GeV}{c}\right)^2, \ W \geq 2.3\ GeV \right)$

	a_0	a_2	A	α_i	$\chi^2/\bar{\chi}^2$
$R = \alpha_4 \frac{q^2}{M^2}$	1.58 ± 0.03	-1.44 ± 0.05	0.06 ± 0.06	0031 ± 0.006	64/115
$R = \alpha_5 \frac{q^2}{W^2}$	1.62 ± 0.04	-1.54 ± 0.06	0.10 ± 0.06	0.37 ± 0.08	79/115

More definite conclusions on the existence and on the value of the anomalous dimensions require further data on the cross-sections of deep inelastic $e-p$ scattering in such kinematical regions that would allow of more complete determination of R.

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