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TESTS OF PHOTOABSORPTION SUM RULES IN MESODYNAMIC MODELS

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Проверка правил сумм для сечений фотопоглощения в моделях мезодинамики

Проведена проверка правил сумм, которые являются следствиями дисперсионных соотношений, алгебры токов и составных моделей и выражают интегралы от полных сечений взаимодействия фотонов с нуклонами и пионами через параметры электромагнитной структуры частиц (заряд, магнитный момент и среднеквадратичный радиус). Расчеты выполнены в низшем порядке теории возмущений. Рассмотрена модель псевдоскалярной связи мезонов с полем нуклонов и "сверхперенормируемые" модели юкавской связи полей с нулевым спином.

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Tests of Photoabsorption Sum Rules in Mesodynamic Models

Tests of sum rules following from disperision relations, current algebra and composite models and relating integrals of total photon-nucleon and photon-pion interaction cross sections to the particle electromagnetic characteristics (charge, magnetic moment and charge radius) are carried out. Calculations are performed in the lowest order perturbation theory. The pseudoscalar pion-nucleon coupling and the "superrenormalizable" Yukawa coupling of the zero-spin fields are considered.

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1. Introduotion and formulation of the problem

Present work carries on previously performed [1] tests of photoabsorption sum rules within the framework of perturbation theory and is devoted to sum rules for photon-nucleon (-pion) interaotion cross sections.

All the calculations are carried out on the basis of mesodynamic renormalized field theory models in the lowest order of perturbation theory. Our investigations are motivated by the following considerations.

At the present time a great number of sum rules are known, which connect energy integrals of real or virtual photon-hadron interaction amplitudes to the static characteristics (eleotrio charge, mass, etc.) and the form-factors of the partioles.

The methods most widely used for deriving sum rules are based on:

1. Dispersion relations and low energy theorems for Compton scattering amplitudes.

2. Equal time algebra of current commutators and infinite momentum techniques.

3. **Light cone algebra of current commutators and anticommutators.**

4. Composite models and infinite momentum techniques. Testing the sum rules is equivalent to verify the basic assumptions made in their derivation. Note that any sum rule derivation procedure usually requires several assumptions: some of them are fundamental hypotheses, while the others are more peculiar ones. One may ascribe to the first group of hypotheses such fundamental statements of the theory of dynamical symmetries as the assumptions about the algebraic structure of currents and of their commutators.

In the second group we would mention a number of rather general and experimentally well-verified assumptions related to the phenomenology of the high-energy hadronic interactions (for example, the use of an amplitude parametrization in terms of leading factorizable singularities in the complex J - plane).

Finally, the sum rule derivation procedure supposes the absence of specific non-Regge singularities (fixed poles in the J-plane) which contrary to purely hadronic processes may occur in the current-hadron interaction amplitudes and yield unknown additional contributions to the sum rules. This fact introduces some uncertainty in the interpretation of the experimental tests of the sum rules which simultaneously check all the assumptions on which their derivation is based.

The renormalized field theory model determines the current commutation relations and the Compton scattering amplitudes in the form of a perturbation expansion and permits to verify the sum rules exactly to a given order of the coupling constant.

In what sense do the results of this perturbation theory verification answer the question of validity of the sum rules?

We believe those sum rules, which are valid in the lowest order perturbation theory, may provide a more unambiguous check of the basic statements of current algebra or the current operator structure in composite models when compared with experiment.

The confirmation of the sum rules within the framework of field theory models yields some evidence for the absence of fixed poles in the corresponding Compton scattering amplitudes. Moreover, the perturbation theory calculation of the interaction amplitudes is useful for comparing with the parton model results and for checking the applicability of the infinite momentum techniques.

In the present paper, we shall study the following sum rules:

1. Sum rules for the nucleon isovector radius and the pion charge radius [2]:

$$2\pi^2\alpha\left(\frac{1}{3}\langle r_1^2\rangle^V-\left(\frac{\varkappa^V}{2m}\right)^2\right)=\int_0^\infty\frac{d\nu}{\nu}\left[\sigma^{tot}_{\gamma^- p}(\nu)-\sigma^{tot}_{\gamma^+ p}(\nu)\right]\equiv \tag{1}$$

$$\equiv\sigma_{-1}(\gamma^- p)-\sigma_{-1}(\gamma^+ p)$$

$$\frac{4\pi^2\alpha}{3}\langle r_\pi^2\rangle=\sigma_{-1}(\gamma^-\pi^+)-\sigma_{-1}(\gamma^+\pi^+). \tag{2}$$

2. Sum rule for the nucleon anomalous magnetic moment [3]:

$$2\pi^2\alpha \frac{\varkappa_N^2}{m^2} = \int_0^\infty \frac{d\nu}{\nu}\left[\sigma^{P}_{\gamma N}(\nu) - \sigma^{A}_{\gamma N}(\nu)\right] \equiv \Delta\sigma_{-1}(\gamma N). \qquad (3)$$

3. Linear sum rule for the nucleon isovector magnetic moment [4]:

$$4\pi^2\alpha\left(\frac{\mu^V}{2m}\right) = \int_0^\infty d\nu \left\{\left[\sigma^{P}_{\gamma^- p}(\nu) - \sigma^{P}_{\gamma^+ p}(\nu)\right] - \left[\sigma^{A}_{\gamma^- p}(\nu) - \sigma^{A}_{\gamma^+ p}(\nu)\right]\right\} \equiv \qquad (4)$$

$$\equiv \Delta\sigma_0(\gamma^- p) - \Delta\sigma_0(\gamma^+ p).$$

4. Sum rules for the proton [5] and pion [6] dipole moment fluctuation in the framework of parton models:

$$4\pi^2\alpha\left(\frac{1}{3}\langle r_1^2\rangle^P - \left(\frac{\varkappa_P}{2m}\right)^2\right) = \sigma_{-1}(\gamma p) \qquad (5)$$

$$\frac{4\pi^2\alpha}{3}\langle r_\pi^2\rangle = \begin{cases} \sigma_{-1}(\gamma\pi^+) & \textbf{(Fermi-Yang model)} \quad (6a) \\ \sigma_{-1}(\gamma\pi^+) + \frac{4}{5}\sigma_{-1}(\gamma\pi^0). & \textbf{(quark model)} \quad (6b) \end{cases}$$

5. Superconvergence dispersion sum rule for photon-pion interaction [7]:

$$\frac{2\pi^2\alpha}{\mu} = \int_0^\infty d\nu \left[\sigma^{tot}_{\gamma\pi^0}(\nu) - \sigma^{tot}_{\gamma\pi^+}(\nu)\right] \equiv \sigma_0(\gamma\pi^0) - \sigma_0(\gamma\pi^+). \qquad (7)$$

The photon-nucleon (-pion) interaction cross sections and the electromagnetic structure parameters of the particles were calculated to the lowest order of the constant, coupling the

pions to the nucleon field (or, as in the case of Eq. (6b), to the quark field):

$$\mathcal{L}_{int}(x) = g\,\bar\psi(x)\gamma_5\tau_i\psi(x)\pi_i(x). \qquad (8)$$

In order to estimate the influence of the particle spins on the fulfilment of the sum rules (1), (2), (5)-(7) we have also examined the model of a (pseudo) scalar "nucleon" isodoublet coupled to a scalar "pion" isotriplet and the "σ- model" of pions interacting with a scalar isosinglet:

$$\mathcal{L}_{int}(x) = \lambda_1 \phi^*(x)\tau_i\phi(x)\pi_i(x), \qquad (9)$$

$$\mathcal{L}_{int}(x) = \lambda_2 \pi_i^*(x)\pi_i(x)\sigma(x). \qquad (10)$$

The Feynman graphs for the lowest-order perturbation calculation of the cross sections and particle radii are drawn in Figs. 1-3.

2. Results and discussion

In the approximation considered, the right-hand sides of all the sum rules are of the order of g^2 (or λ^2), where g (or λ) is the meson coupling constant. But the left-hand sides of Eqs. (4) and (7) contain zeroth-order terms in $g(\lambda)$. One would therefore expect, according to the traditional perturbation approach to renormalizable field theories, that these relations cannot be valid and should be modified.

The simplest way to do this is to assume as was previously proposed [8,9] that the subtraotion oonstants in the dispersion approach are fixed at infinite energy by the Born-term contributions with "switched-off" meson interactions. Then, instead of Eqs. (4) and (7) one gets the modified relations:

$$4\pi^2\alpha\left(\frac{\mathcal{K}^V}{2m}\right) = \Delta\sigma_0(\gamma^- p) - \Delta\sigma_0(\gamma^+ p), \tag{11}$$

$$0 = \sigma_0(\gamma\pi^0) - \sigma_0(\gamma\pi^+), \tag{12}$$

whioh are relevant within the framework of weak-ooupling field -theory models. The phenomenologioal consequenoes of suoh a modification were discussed in Refs. [8,9].

Since the anomalous magnetio moment $\mathcal{K}$ is a quantity of the order $O(g^2)$ one has to negleot $\mathcal{K}^2 \sim O(g^4)$ in Eqs. (1),(3) and (5) for comparing quantities of the same order in the meson coupling constant. The results of our sum rule verification are summarized in Table I.

The explicit expressions for cross seotions and partiole radii are rather lengthy and oumbersome. We list them in the Appendix.

Summing up, we note that the sum rules (1)-(3) whose validity was confirmed in all the investigated models have passed also rather successfully through the experimental verification on the basis of meson photoproduction data [10-12].

The validity of Eqs. (5) and (6) speaks in favour of the infinite momentum method and of the use of operator relations between the currents and their moments within the framework of composite parton models.

The divergence of the integrals shows the inapplicability of unsubtracted dispersion relations to derive the sum rules (4) and (7) in the model with pseudoscalar pion-nucleon ooupling (8). This agrees with the results of other authors [10,13] showing that the sum rule (4) most probably breaks down.

The calculations, performed within the superrenormalizable and superoonvergent scalar models (9) and (10), verify and confirm in the λ^2- approximation the relativistic version of the generalized Thomas-Reiche-Kuhn sum rule [14]:

$$-\frac{Q_\pi^2}{4\pi\mu} = \frac{1}{2\pi^2}\,\sigma_0(\gamma\pi) + T_2^{\gamma\pi}(0), \tag{13}$$

where $Q_\pi(\mu)$ is the "pion" charge (mass) and $T_2^{\gamma\pi}(0)$ represents the zero-frequency contribution of the "seagull" graphs of Fig.4.

Combining the relation (13) for the $\gamma\pi^0$ and $\gamma\pi^+$ amplitudes yields the new sum rule:

$$\frac{2\pi^2\alpha}{\mu}(1-Z) = \sigma_0(\gamma\pi^0) - \sigma_0(\gamma\pi^+), \tag{14}$$

where Z is the charged-pion wave renormalization constant. In the considered superrenormalizable scalar model (9) the constant Z is finite. Its value to the lowest-order of perturbation theory is given in the Appendix (see Eq. A(20)).

The question to know whether the sum rule (14) is not only a specific model-dependent relation but is actually relevant to the real pions, remains of course open. Anyway Eq. (14) looks more general than Eqs. (7) and (12) and includes them as particular cases.

The original form of the sum rule (7) presupposes that Z=0, which is one of the possible "compositeness" criteria of particles (extensively discussed, e.g. in Ref. [15]) in the framework of quantum field theory. In this connection we have shown the sum rule (7) appears to be verified, when $\mu = 2m - \epsilon$, $\frac{\epsilon}{m} \to 0$, i.e. when the scalar "pion" is viewed as a weakly-bound nonrelativistic system of two scalar constituents of mass m, interacting through a zero-range potential.

Eq. (13) can be reduced in this case to the more familiar nonrelativistic form:

$$2\pi^2 \langle \psi_0 | [D_x, [H_0, D_x]] | \psi_0 \rangle = \sigma_0 \equiv \int_0^\infty d\omega \, \sigma_{tot}(\omega), \qquad (15)$$

where D_x is the x-component of the electric dipole moment operator, H_0 the nonrelativistic Hamiltonian and ψ_0 the ground state wave function.

The generalized nonrelativistic Thomas-Reiche-Kuhn sum rule (15) should be valid when one neglects the terms of the order of p^2/m^2, where p is the internal motion momentum of the constituents.

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APPENDIX

In this appendix we list the explicit expressions of the photon-nucleon and photon-pion cross sections and of the particle radii calculated to the lowest order of perturbation theory within the framework of the models (8), (9) and (10).

1. The cross sections for the processes $\gamma N \to \pi N$ and $\gamma \pi \to N\bar{N}$ within the pseudoscalar model (8) read:

$$\begin{aligned}
\sigma^{tot}_{\gamma N \to \pi N}(s) = \frac{\alpha g^2 \Delta}{8s(s-m^2)} \Big\{ & 2g_s^2 + 2g_u^2 - 7 g_s g_u - g_s g_t + 2 g_u g_t \\
& + g_s (g_u + g_t) \frac{m^2-\mu^2}{s} - 6 g_s (g_s - g_u - g_t) \frac{\mu^2}{s-m^2} + 4 (g_s^2 + g_u^2 + g_t^2) \frac{\mu^2 s}{(s-m^2)^2} \\
& + 2 g_u \frac{s}{\Delta} \Big[g_s - g_t - (3g_s - g_u - 3g_t) \frac{\mu^2}{s-m^2} - \\
& \qquad - 2 \left(2 g_s m^2 - (g_s - g_t) \mu^2 \right) \frac{\mu^2}{(s-m^2)^2} \Big] \ln \frac{s+m^2-\mu^2+\Delta}{s+m^2-\mu^2-\Delta} \\
& - 4 g_t \frac{s\mu^2}{\Delta (s-m^2)} \Big[g_t + (g_s + g_u) \frac{\mu^2}{s-m^2} \Big] \ln \frac{s-m^2+\mu^2+\Delta}{s-m^2+\mu^2-\Delta} \Big\},
\end{aligned} \qquad (A.1)$$

$$
\left(\sigma^{P}-\sigma^{A}\right)_{\gamma N\to\pi N} = \frac{\alpha g^2\Delta}{4s(s-m^2)}\Big\{-g_s^2-8g_u^2+2g_sg_u+2g_sg_t-4g_ug_t
$$

$$
-g_s^2\frac{m^2-\mu^2}{s}+2\left[g_s(g_u+g_t)\mu^2-2g_u(g_s+2g_u+g_t)m^2\right]\frac{1}{s-m^2}
$$

$$
+2g_u\frac{s}{\Delta}\left[g_u+2\left((g_s+2g_u+g_t)m^2-g_u\mu^2\right)\frac{1}{s-m^2}\right]\ln\frac{s+m^2-\mu^2+\Delta}{s+m^2-\mu^2-\Delta} \qquad (A.2)
$$

$$
-4g_t(g_s-g_u)\frac{s\mu^2}{\Delta(s-m^2)}\ \ln\frac{s-m^2+\mu^2+\Delta}{s-m^2+\mu^2-\Delta}\Big\},
$$

$$
\sigma^{tot}_{\gamma\pi\to N\bar N}(s)=\frac{\alpha g^2\Delta'}{4s(s-\mu^2)}\Big\{2(g_u-g_t)^2+4\left(g_s^2+g_u^2+g_t^2\right)\frac{\mu^2 s}{(s-\mu^2)^2}
$$

$$
+2\frac{s}{\Delta'}\Big[g_s(g_u-g_t)-2g_ug_t+\left(g_u^2+g_t^2+3g_sg_u-3g_sg_t-6g_ug_t\right)\frac{\mu^2}{s-\mu^2} \qquad (A.3)
$$

$$
+2\left(4g_ug_tm^2+(g_sg_u-g_sg_t-2g_ug_t)\mu^2\right)\frac{\mu^2}{(s-\mu^2)^2}\Big]\ln\frac{s+\Delta'}{s-\Delta'}\Big\},
$$

where $\Delta^2=\left[s-(m+\mu)^2\right]\left[s-(m-\mu)^2\right]$, (A.4)

$$
\Delta'^2=s(s-4m^2), \qquad (A.5)
$$

$\sqrt{s}$ is the total energy in the centre of mass system, $m\,(\mu)$ is the mass of the nucleon (pion), α is the fine structure constant, $g\equiv g_{pp\pi^0}$.
The numerical values of the coefficients g_s, g_u and g_t for the different channels of the reactions are given in Table II.

2. The cross sections for the processes $\gamma N\to\pi N$, $\gamma\pi\to N\bar N$ and $\gamma\pi\to\pi\sigma$ in the scalar models (9) and (10) are:

$$
\sigma^{tot}_{\gamma N\to\pi N}(s)=\frac{\alpha\lambda_1^2\Delta}{4s(s-m^2)^3}\Big\{-4\left(g_u^2+g_t^2+g_ug_t\right)s
$$

$$
+2g_u\frac{s}{\Delta}\left[g_u(s+m^2-\mu^2)+2g_tm^2\right]\ln\frac{s+m^2-\mu^2+\Delta}{s+m^2-\mu^2-\Delta} \qquad (A.6)
$$

$$
+2g_t\frac{s}{\Delta}\left[g_t(s-m^2+\mu^2)+2g_u\mu^2\right]\ln\frac{s-m^2+\mu^2+\Delta}{s-m^2+\mu^2-\Delta}\Big\},
$$

$$
\sigma^{tot}_{\gamma\pi\to N\bar N}(s)=\frac{\alpha\lambda_1^2\Delta'}{4s(s-\mu^2)^3}\Big\{-4\left(g_u^2+g_t^2-g_ug_t\right)s \qquad (A.7)
$$

$$
+2\frac{s}{\Delta'}\left[(g_u^2+g_t^2)s-4g_ug_tm^2\right]\ln\frac{s+\Delta'}{s-\Delta'}\Big\},
$$

$$
\sigma^{tot}_{\gamma\pi\to\pi\sigma}(s)=\frac{\alpha\lambda_2^2\Delta''g_u^2}{4s(s-\mu^2)^3}\Big\{-4s+2\frac{s}{\Delta''}\left(s-M_\sigma^2+\mu^2\right)\cdot \qquad (A.8)
$$

$$
\cdot\ln\frac{s-M_\sigma^2+\mu^2+\Delta''}{s-M_\sigma^2+\mu^2-\Delta''}\Big\},
$$

where $\Delta''^2=\left[s-(M_\sigma+\mu)^2\right]\left[s-(M_\sigma-\mu)^2\right]$, (A.9)

M_σ is the mass of the σ- meson, Δ and Δ' have been defined by the Eqs. (A.4) and (A.5) . The values of the coefficients g_s, g_u, g_t are listed in table II.

3. We provide hereafter the notations and formulae defining the nucleon and pion electromagnetic vertices. For spin 1/2 particles the matrix element of the current is parametrized as usually:

$$\langle p_2 | j^a_\mu(0) | p_1 \rangle = e\,\bar{u}(p_2) M^a \left[\gamma_\mu F^a_1(k^2) + i \frac{\sigma_{\mu\nu}}{2m} k^\nu F^a_2(k^2) \right] u(p_1) , \tag{A.10}$$

$$\langle r_1^2 \rangle^a = 6\, F_1^{a\prime}(0)$$

with $a = S, V$; $M^S = \frac{1}{2}$, $M^V = \frac{1}{2}\tau_3$.

The current is normalized so that $F_1^V(0) = F_1^S(0) = 1$ and $F_2^{S(V)}(0) = \chi_p \pm \chi_n$.

For spin 0 particles with isotopic spins 1/2 and 1 we have, respectively :

$$\langle p_2 | j^a_\mu(0) | p_1 \rangle = e\,(p_1 + p_2)_\mu F^a(k^2)\, \xi_2^{\dagger} M^a \xi_1 , \tag{A.11}$$

$$\langle p_2 | j^V_\mu(0) | p_1 \rangle = ie\,(p_1 + p_2)_\mu\, \epsilon^{3ab} F_\pi(k^2) , \tag{A.12}$$

where $F^S(0) = F^V(0) = F_\pi(0) = 1$. ξ_1 and ξ_2 are the isotopic spinors describing the (pseudo) scalar particle with isospin 1/2.

Within the pseudoscalar model (8) the nucleon and pion mean square radii are given by the following equations:

$$\langle r_1^2 \rangle^V = \frac{g^2}{32\pi^2 m^2} \left\{ -\frac{172 - 163\eta + 30\eta^2}{4-\eta} - (12 - 44\eta + 15\eta^2) \ln\eta + 2\eta^{1/2} \frac{280 - 366\eta + 134\eta^2 - 15\eta^3}{(4-\eta)^{3/2}}\, \mathrm{tg}^{-1} \left(\frac{4-\eta}{\eta} \right)^{1/2} \right\} \tag{A.13}$$

$$\langle r_1^2 \rangle^S = \frac{3g^2}{32\pi^2 m^2} \left\{ \frac{12 + 33\eta - 10\eta^2}{4-\eta} + (4 - 5\eta)\eta \ln\eta - 2\eta^{3/2} \frac{54 - 34\eta + 5\eta^2}{(4-\eta)^{3/2}}\, \mathrm{tg}^{-1} \left(\frac{4-\eta}{\eta} \right)^{1/2} \right\} , \tag{A.14}$$

$$\langle r_\pi^2 \rangle = \frac{g^2}{4\pi^2 m^2} \left\{ -\frac{6-\eta}{\eta(4-\eta)} + 4\, \frac{6 - 6\eta + \eta^2}{\eta^{3/2}(4-\eta)^{3/2}}\, \mathrm{tg}^{-1} \left(\frac{\eta}{4-\eta} \right)^{1/2} \right\} , \tag{A.15}$$

where $\eta = \frac{\mu^2}{m^2}$.

Within the scalar model (9) one obtains:

$$\langle r_1^2 \rangle^V = \frac{\lambda_1^2}{32\pi^2 m^4} \left\{ 4\, \frac{4 - 13\eta + 3\eta^2}{\eta(4-\eta)} - (11 - 6\eta) \ln\eta - 2\, \frac{40 - 102\eta + 47\eta^2 - 6\eta^3}{\eta^{1/2}(4-\eta)^{3/2}}\, \mathrm{tg}^{-1} \left(\frac{4-\eta}{\eta} \right)^{1/2} \right\} , \tag{A.16}$$

$$\langle r_1^2 \rangle^S = \frac{3\lambda_1^2}{32\pi^2 m^4} \left\{ -4\, \frac{3-\eta}{4-\eta} - (1 - 2\eta) \ln\eta + 2\eta^{1/2} \frac{18 - 13\eta + 2\eta^2}{(4-\eta)^{3/2}}\, \mathrm{tg}^{-1} \left(\frac{4-\eta}{\eta} \right)^{1/2} \right\} , \tag{A.17}$$

$$\langle r_\pi^2 \rangle = \frac{\lambda_1^2}{4\pi^2 m^4} \left\{ -\frac{3-\eta}{\eta^2(4-\eta)} + \frac{12 - 6\eta + \eta^2}{\eta^{5/2}(4-\eta)^{3/2}}\, \mathrm{tg}^{-1} \left(\frac{\eta}{4-\eta} \right)^{1/2} \right\} \tag{A.18}$$

and within the "σ- model"(10) the pion radius reads:

$$\langle \lambda_\pi^2 \rangle = \frac{\lambda_2^2}{32\pi^2 M_\sigma^4} \left\{ -4\,\frac{1-3\eta'}{\eta'^2(1-4\eta')} - \frac{2-\eta'}{\eta'^3}\ln\eta' + \frac{2-13\eta'+18\eta'^2}{\eta'^3(1-4\eta')^{3/2}}\ln\xi \right\}, \tag{A.19}$$

where $\eta' = \frac{\mu^2}{M_\sigma^2}$ and $\xi = \frac{2\eta'}{1-2\eta'+(1-4\eta')^{1/2}}$.

4. The pion wave renormalization constant in the λ^2-approximation of the scalar model (9) reads:

$$Z = 1 + \frac{\lambda_1^2}{8\pi^2 m^2}\left[\frac{1}{\eta} - \frac{4}{\eta^{3/2}(4-\eta)^{1/2}}\,\mathrm{tg}^{-1}\left(\frac{\eta}{4-\eta}\right)^{1/2}\right]. \tag{A.20}$$

Table 1

Sum rules	Convergence of the integral	Lowest-order perturbation verification
(1)	yes	yes
(2)	yes	yes
(3)	yes	yes
(4), (11)	no $(\sim \ln^2 \nu,\ \nu\to\infty)$	no
(5)	yes	yes
(6)	yes	yes
(7), (12)	(a) no $(\sim \ln\nu,\ \nu\to\infty)$	(a) no
	(b) yes	(b) no
(13), (14)	(b) yes	(b) yes

(a) Within the framework of the pseudoscalar model (8).

(b) Within the framework of the scalar models (9) and (10).

Table II

Reaction	g_s	g_u	g_t
$\gamma p \to \pi^+ n$	$\sqrt{2}$	0	$\sqrt{2}$
$\gamma p \to \pi^0 p$	1	1	0
$\gamma n \to \pi^- p$	0	$\sqrt{2}$	$-\sqrt{2}$
$\gamma n \to \pi^0 n$	0	0	0
$\gamma^+ p \to \pi^+ p$	0	1	-1
$\gamma^- p \to \pi^- p$	1	0	1
$\gamma^- p \to \pi^0 n$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$-\sqrt{2}$
$\gamma \pi^+ \to p \bar{n}$	$\sqrt{2}$	0	$\sqrt{2}$
$\gamma \pi^0 \to p \bar{p}$	0	1	1
$\gamma \pi^0 \to n \bar{n}$	0	0	0
$\gamma^- \pi^+ \to p \bar{p}$	-1	1	0
$\gamma^- \pi^+ \to n \bar{n}$	1	0	1
$\gamma \pi^+ \to \pi^+ \sigma$	1	1	0
$\gamma \pi^0 \to \pi^0 \sigma$	0	0	0
$\bar{\gamma} \pi^+ \to \pi^0 \sigma$	-1	-1	0

References:

1. S.B.Gerasimov, J.Moulin. JINR,E2-6722, Dubna (1972).
2. N.Cabibbo, L.Radicati. Phys.Lett., 19, 697 (1966).
3. С.Б.Герасимов. Ядерная физика,2, 598 (1965).
S.D.Drell, A.C.Hearn. Phys.Rev.Lett., 16, 908 (1966).
4. M.A.Beg. Phys.Rev., 150, 1276 (1966).
K.Kawarabayashi, M.Suzuki. Phys.Rev., 150, 1181 (1966).
5. K.Gottfried. Phys.Rev.Lett., 18, 1174 (1967).
6. S.B.Gerasimov. JINR,E2-4295, Dubna (1969).
7. В.А.Матвеев, Л.Д.Соловьев и др. ОИЯИ,Р2-3II8, Дубна (1967).
H.Pagels. Phys.Rev.Lett., 18, 316 (1967);
H.Harari, ibid., 18, 319 (1967).
8. W.W.Wada. Nuovo Cimento, 53A, 833 (1968).
9. L.D.Soloviev, in High Energy Physics, Proceedings of the XVth International Conference on High Energy Physics, Naukova Dumka Publ., Kiev, 1972, p. 534.
10. G.C.Fox, D.Z.Freedman. Phys.Rev., 182, 1628 (1969).
11. G.J.Gounaris. Phys.Lett., 41B, 329 (1972).
G.J.Aubrecht , W.W.Wada. Ann.Phys., 78, 376 (1973).
12. I.Carliner. SLAC-PUB-1179 (1973).
13. D.A.Dicus, R.Jackiw, V.L.Teplitz. Phys.Rev., D4, 1733 (1971).
14. S.B.Gerasimov. Phys.Lett., 13, 240 (1964).
15. K.Hayashi, M.Hirayama et al. Fortsch.Phys., 15, 625 (1967).
16. J.S.O'Connell. Electromagnetic Sum Rules, Preprint National Bureau of Standards (1973).
17. T.Matsuura, K.Yazaki. A Comment on the Electric Dipole Sum Rule for Nuclear Photoeffect, Preprint Univ. of Tokyo, (1973).

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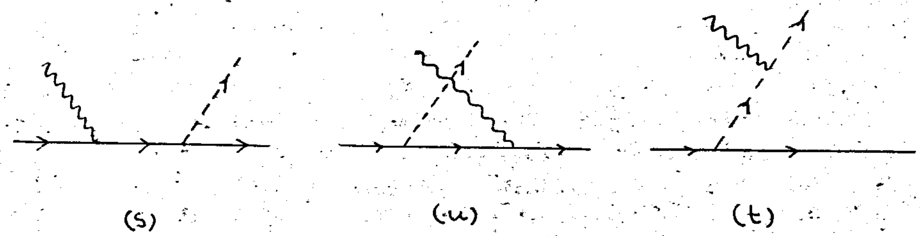
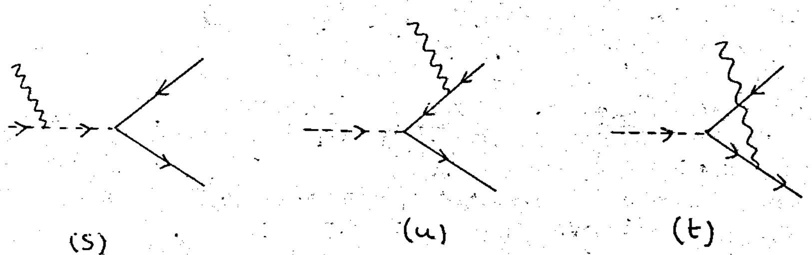
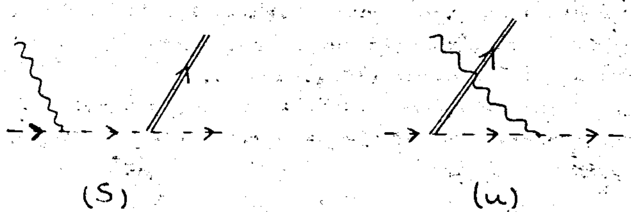
on October 12, 1973.



1a

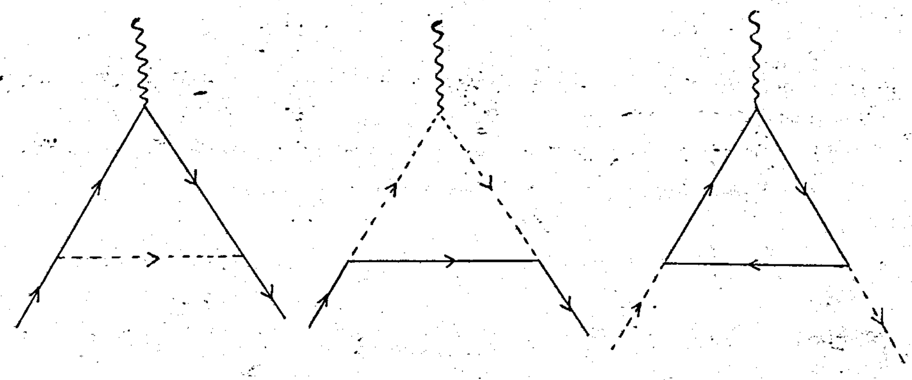


1b



1c

Fig.1. Lowest-order Feynman graphs for the processes:
$\gamma N \to \pi N$ (s-, u- and t- channels, Fig.1a)
$\gamma \pi \to N\bar{N}$ (s-, u- and t- channels, Fig.1b)
and $\gamma \pi \to \pi \sigma$ (s- and u- channels, Fig. 1c).



2a   2b

Fig.2. Lowest-order Feynman graphs for the electromagnetic vertices of the nucleon (Fig.2a) and of the pion (Fig.2b).

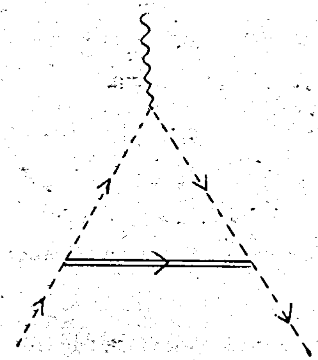


Fig.3. Lowest-order Feynman graph for the electromagnetic vertex of the pion in the "σ- model" of the Lagrangian (10).

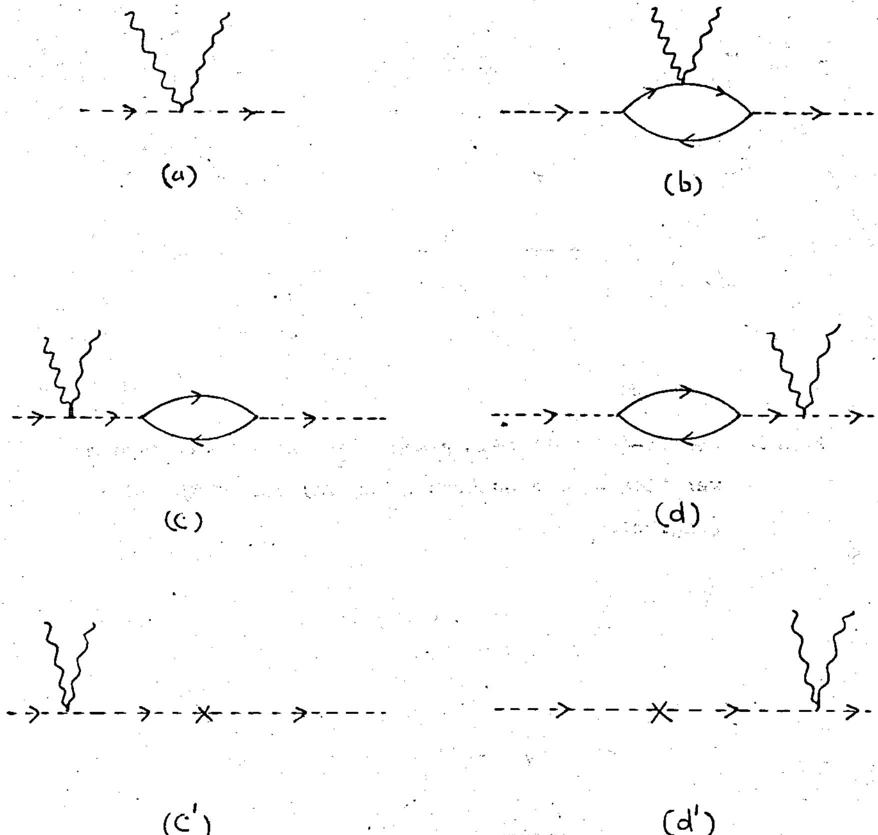


Fig.4. "Seagull" graphs contributing to $T_2^{\gamma\pi}(0)$ in the scalar model of the Lagrangian (9), including the "catastrophic" photon-meson interaction term (Fig.4a) and the mass renormalization diagrams (Fig.4c' and Fig.4 d'). Similar diagrams occur in the " σ- model" (10).

The solid lines represent the nucleon, the wave lines-the photon, the dotted lines-the pion and the double solid lines – the σ- meson.