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Introduction

A conformal covariant operator expansion for the product of two scalar fields was obtained by Ferrara, Gatto and Grillo[1] and Bonora, Sartori and Tonin[2]. It was interesting to derive such an expansion for the product of two Dirac fields. It turns out that the basis of irreducible Lorentz tensors of the type $\left(\frac{n}{2},\frac{n}{2}\right)$ used in papers[1,2] is not sufficient here. We have found out the basis needed here and constructed the operator product expansion. We use the manifest conformal covariant formulation of local field theory based on the isomorphism between the conformal group and the group $SO(4,2)/Z_2$ of pseudorotations in 6-dimensional space[3].

The paper is organized as follows:

In sections 1.1 and 1.2 we summarize some background material contained in ref.[3]. Section 1.3 deals with symmetric tensors. Section 1.4 describes the antisymmetric tensors needed for the expansion. Some calculations concerning these tensors are collected in Appendix A. Chapter 2 contains the derivation of the expansion. The final results are given in Chapter 3. Some of the two - and three - point functions needed in the text are given in Appendix B. In Appendix C the relation between the OPE and the three - point functions is illustrated in the case n=1.

1 Manifest Conformal Covariant Formalism

1.1 Relation between the manifestly conformal covariant fields and the fields in x-space

A Poincare covariant quantized field $\varphi(x)$ (or shorter x-field) is called conformal covariant if it has a definite scale dimension d_φ (in mass units) and in addition is covariant under infinitesimal special conformal transformations. We can obtain our x-fields starting from manifest conformal covariant fields $\Phi(\eta)$. The fields $\Phi(\eta)$ are multispinors defined on the subset:

$$\eta_5 + \eta_6 = \varkappa > 0$$

of the light cone in 6-dimensions:

$$C_{4,2} = \left\{ \eta = (\eta_0, \eta_1, \eta_2, \eta_3, \eta_5, \eta_6) \mid \eta^2 = g^{AB}\eta_A\eta_B = 0 \right\} \qquad (1.1)$$

(g^{AB} is diagonal and $g^{AA} = (1, -1, -1, -1, -1, 1)$), they are homogeneous functions of degree $-d_\Phi$ on the cone $C_{4,2}$ and transform under the action of the $O(4,2)$ generators J_{AB} according to:

$$\delta\Phi(\eta) = -i\varepsilon^{AB} J_{AB}\Phi(\eta) = -i\varepsilon^{AB}(L_{AB} + S_{AB})\Phi(\eta) , \qquad (1.2)$$

where

$$L_{AB} = i(\eta_A\partial_B - \eta_B\partial_A) = \frac{i}{\varkappa}\left\{\eta_A\left[g^\mu_B\partial_\mu + (g_{B5}+g_{B6})\left(\varkappa\frac{\partial}{\partial\varkappa} - x.\partial\right)\right] - \eta_B\left[g^\mu_A\partial_\mu + (g_{A5}+g_{A6})\left(\varkappa\frac{\partial}{\partial\varkappa} - x.\partial\right)\right]\right\}$$

and S_{AB} is some finite dimensional representation of the $O(4,2)$ acting on the indices of $\Phi(\eta)$ only. Then the formula

$$\varphi(x) = \varkappa^{d_\phi} T(x)\,\Phi(\eta) \quad , \quad T(x) = \exp\left[-i(S_{6\mu} + S_{5\mu})\, x^\mu\right] , \tag{1.3}$$

where $\eta_\mu = \varkappa x_\mu$, $\eta_5 + \eta_6 = \varkappa$, $\eta_5 - \eta_6 = \varkappa x^2$,
gives us an x-field in our sense if $d_\phi = d_\varphi - \tilde{d}$, where $\tilde{d}$ is a suitably chosen eigenvalue of $i S_{65}$ /3/.

In all cases (except scalar fields) we must in addition to the above procedure, impose on the η -fields $\Phi(\eta)$ some subsidiary conditions (to be specified in each case) in order to exclude unwanted (we also call them unphysical) components of the x-fields.

In the following sections of this chapter we are going to consider the fields needed for our expansion.

1.2 Spin $1/2$ field

The "Dirac" η -fields are 8 - dimensional. Here

$$S_{AB} = S^{1/2}_{AB} = \frac{i}{4}\left[\beta_A, \beta_B\right] , \tag{1.4}$$

where β_A are 8x8 matrices which transform as a 6-vector. The β_A can be defined as the lowest order (irreducible) representation of the Clifford algebra

$$\left\{ \beta_A , \beta_B \right\} = 2 g_{AB} \mathbf{1}_8 \quad .$$

We shall use the following direct-product realization of the β -matrices[3]

$$\beta_\mu = \tau_3 \gamma_\mu \ , \quad \beta_5 = i \tau_1 \mathbf{1}_4 \ , \quad \beta_6 = \tau_2 \mathbf{1}_4 \tag{1.5}$$

or $\beta_A = g_A^{\mu} \tau_3 \gamma_\mu - i(g_{A5} + g_{A6}) \tau_+ + i(g_{A6} - g_{A5}) \tau_- \ , \quad \tau_\pm = \frac{1}{2}(\tau_1 \pm \tau_2)$

(τ_i are the Pauli matrices).

In this basis the generators S_{AB} assume the form

$$S_{\mu\nu}^{(1/2)} = \mathbf{1}_2 \frac{i}{4}[\gamma_\mu, \gamma_\nu] \ , \quad S_{\mu 5}^{(1/2)} = -\frac{i}{2} \tau_2 \gamma_\mu \ , \quad S_{\mu 6}^{(1/2)} = \frac{1}{2} \tau_1 \gamma_\mu \ , \quad S_{56}^{(1/2)} = -\frac{i}{2} \tau_3 \mathbf{1}_4 \ . \tag{1.6}$$

There exists also a conformal pseudoscalar

$$\beta_7 = -\beta_0 \beta_1 \beta_2 \beta_3 \beta_5 \beta_6 = \tau_3 \gamma_5 \quad , \qquad \gamma_5 = \gamma_0 \gamma_1 \gamma_2 \gamma_3 \ , \tag{1.7}$$

$$\beta_7^2 = -1 \quad , \quad \{ \beta_7, \beta_A \} = 0 \ .$$

The 8-component η -field $\chi(\eta)$ is homogeneous of degree $-d_\chi$ and satisfies the following subsidiary condition

$$(\beta \eta) \chi(\eta) = 0 \ , \tag{1.8}$$

where $(\beta\eta) = \beta^A \eta_A \quad , \quad -d_\chi = -d_\psi + \frac{1}{2} \quad \left(\tilde{d} = \frac{1}{2}\right)$.

The Dirac conjugation of the field χ is defined in the above basis for the β -matrices by

$$\tilde{\chi} = \chi^* \gamma_0 \tau_1$$

The relation between the x-field $\psi(x)$ and $\chi(\eta)$ is given by (1.3), where in the basis (1.5)

$$T(x) = T^{1/2}(x) = \exp\left[-i\left(S^{(1/2)}_{6\mu} + S^{(1/2)}_{5\mu}\right)x^{\mu}\right] = \mathbb{1}_8 + i\tau_- x\!\!\!/ . \tag{1.9}$$

The subsidiary condition (1.8) is equivalent to

$$\mathbb{1}_4 \tau_+ \Psi(x) = 0$$

which leaves us with an exactly 4-component spin $1/2$ field.

1.3 Symmetric tensors

The x-fields $O_{\mu_1 \dots \mu_n}(x)$ we consider here are irreducible Lorentz tensors of type $\left(\frac{n}{2}, \frac{n}{2}\right)$ (symmetric and traceless). The corresponding η -fields $\theta_{c_1 \dots c_n}(\eta)$ are symmetric and traceless, homogeneous of degree $-d_n$ $(\tilde{d} = 0)$ and satisfy two subsidiary conditions (for $n \neq 0$)[3]:

$$\eta^{c_1} \, \theta_{c_1 \dots c_n}(\eta) = 0, \tag{1.10}$$

$$\left(L_c{}^{c_1} - i g_c^{c_1}\right) \theta_{c_1 \dots c_n}(\eta) = 0. \tag{1.11}$$

Here the generators S_{AB} are given by[1]:

$$\left(S^{(n)}_{AB}\right)^{D_1 \dots D_n}_{C_1 \dots C_n} = i \sum_{j=1}^{n} \left(g_{AC_j} g_B^{D_j} - g_{BC_j} g_A^{D_j}\right) g^{D_1}_{C_1} \cdots g^{D_{j-1}}_{C_{j-1}} g^{D_{j+1}}_{C_{j+1}} \cdots g^{D_n}_{C_n} \tag{1.12}$$

$$S^{(0)}_{AB} = 0 .$$

In the case n=1 the operator $T(x)$ takes the form[4]:

$$T(x)=T^{(1)}(x)=\exp\left[-i\left(S^{(1)}_{5\mu}+S^{(1)}_{6\mu}\right)x^{\mu}\right]=\mathbb{1}_6+\omega_x+\frac{1}{2}\omega_x^2 , \qquad (1.13)$$

where

$$(\omega_x)^A_B = x^{\mu}\left(g^A_5+g^A_6\right)g_{\mu B}-g^A_{\mu}\left(g_{5B}+g_{6B}\right)$$

$$(\omega_x^2)^A_B = -x^2\left(g^A_5+g^A_6\right)\left(g_{5B}+g_{6B}\right) .$$

In the general case of a tensor of rank n $T(x)=T^{(n)}(x)$, where

$$T^{(n)}(x)=\bigotimes_{i=1}^{n} T^{(1)}_i(x) \qquad \text{for } n\neq 0, \quad T^{(0)}(x)=1 \qquad (1.14)$$

and $T^{(1)}_i(x)$ is $T^{(1)}(x)$ acting on the i-th index of $\theta_{c_1\ldots c_n}(\eta)$.

As a consequence of the first subsidiary condition (1.10) one can obtain[1] for the fields $O_{c_1\ldots c_n}(x)$:

$$O_{\mu_1\ldots\mu_k 5\ldots 5 6\ldots 6}(x) = O_{\mu_1\ldots\mu_k 5\ldots 5}(x) . \qquad (1.15)$$

From the second condition (1.11) using (1.15) we get[1]

$$O_{\mu_1\ldots\mu_{n-k}\underbrace{5\ldots 5}_{k}}(x) = \frac{1}{\lambda^k}\frac{\Gamma(d_n-2-n)}{\Gamma(d_n-2-n+k)}\partial^{\mu_{n-k+1}}\ldots\partial^{\mu_n}O_{\mu_1\ldots\mu_n}(x) . \qquad (1.16)$$

Equation (1.16) becomes meaningless for canonical+) dimensions $d=2+n$, $n\neq 0$. The reason is that for tensors

+) If A and B were free zero mass fields, satisfying canonical commutation relation so that $d_A=d_B=1$, then the irreducible basis tensors $O(x)=:A(x)B(x):$, $O_{\mu}(x)=:A(x)\overleftrightarrow{\partial_{\mu}}B(x):$,
$O_{\mu_1\mu_2}(x)=\frac{1}{2}:\partial_{\mu_1}A(x)\partial_{\mu_2}B(x)+\partial_{\mu_2}A(x)\partial_{\mu_1}B(x):-\frac{1}{2}g_{\mu_1\mu_2}:\partial^{\lambda}A(x)\partial_{\lambda}B(x):+\frac{1}{6}\left(g_{\mu_1\mu_2}\Box-\partial_{\mu_1}\partial_{\mu_2}\right):A(x)B(x):$, etc.
would have canonical dimensions 2,3,4, etc.

with such dimensions the second subsidiary condition (1.11) can not be imposed, unless the tensors are conserved $\left(\partial^{\mu_1} O_{\mu_1 \dots \mu_n}(x) = 0\right)$ but in this case it is just an identity.

1.4 Antisymmetric tensors

The x-fields $F_{\alpha\beta\mu_1 \dots \mu_n}(x)$ we are going to consider in this section are irreducible Lorentz tensors of type $\left(\frac{n+2}{2}, \frac{n}{2}\right) \oplus \oplus \left(\frac{n}{2}, \frac{n+2}{2}\right)$, i.e. they are antisymmetric in the first two indices, symmetric in the rest, traceless in every pair of indices and satisfy

$$\varepsilon^{\gamma\alpha\beta\mu_1} F_{\alpha\beta\mu_1 \dots \mu_n}(x) = 0 \qquad (1.17)$$

where $\varepsilon^{\gamma\alpha\beta\mu_1}$ is the totally antisymmetric tensor $\left(\varepsilon^{0123} = 1\right)$.

The corresponding η -fields $\mathcal{F}_{ABC_1 \dots C_n}(\eta)$ are antisymmetric in A,B, symmetric in C, traceless in every pair of indices and satisfy $\varepsilon^{ABC_1 DD'D''} \mathcal{F}_{ABC_1 \dots C_n}(\eta) = 0$. In this case the generators S_{AB} and the operator $T(x)$ are given by (1.12) and (1.14) as for the symmetric tensors $(n \to n+2)$.

The tensors $\mathcal{F}_{ABC_1 \dots C_n}(\eta)$ are homogeneous of degree $-d_n^a$ $(\tilde{d} = 0)$ and satisfy two subsidiary conditions analogous to (1.10) and (1.11) on both sets of indices

$$\eta^B \mathcal{F}_{ABC_1 \dots C_n}(\eta) = 0, \qquad (1.18a)$$

$$\eta^{C_1}\mathcal{F}_{ABC_1\ldots C_n}(\eta)=0\ ,\quad n\neq 0\ , \tag{1.18b}$$

$$\left(L_{B'}^{B}-ig_{B'}^{B}\right)\mathcal{F}_{ABC_1\ldots C_n}(\eta)=0\ , \tag{1.19a}$$

$$\left(L_{C}^{C_1}-ig_{C}^{C_1}\right)\mathcal{F}_{ABC_1\ldots C_n}(\eta)=0\ ,\quad n\neq 0\ . \tag{1.19b}$$

We get the following results for the X – tensors $F_{ABC_1\ldots C_n}(x)$.

From (1.18a) we obtain:

$$F_{\alpha 5C_1\ldots C_n}(x)=F_{\alpha 6C_1\ldots C_n}(x)\ ,\qquad F_{56C_1\ldots C_n}(x)=0\ . \tag{1.20}$$

From (1.18b),

$$F_{AB\mu_1\ldots\mu_n 5\ldots 56\ldots 6}(x)=F_{AB\mu_1\ldots\mu_k 5\ldots 5}(x)\ . \tag{1.21}$$

From (1.19) using (1.20) and (1.21) we get:

$$F_{\alpha 5\mu_1\ldots\mu_{n-k}\underbrace{5\ldots 5}_{k}}=\frac{1}{2^{k+1}}\frac{\Gamma(d_n^a-n-3)}{\Gamma(d_n^a-n-2+k)(d_n^a-1)(d_n^a-2)}\partial^\lambda\partial^{\mu_{n-k+1}}\ldots\partial^{\mu_n} \tag{1.22}$$

$$\cdot\left\{\left[(d_n^a-n-2)(d_n^a-2)-1\right]F_{\alpha\lambda\mu_1\ldots\mu_n}+(d_n^a-2)\sum_{i=1}^{n}F_{\alpha\mu_i\lambda\mu_1\ldots\hat{\mu}_i\ldots\mu_n}-\sum_{i=1}^{n}F_{\mu_i\lambda\alpha\mu_1\ldots\hat{\mu}_i\ldots\mu_n}\right\}=$$

$$=\frac{1}{2^{k+1}}\frac{\Gamma(d_n^a-n-3)}{\Gamma(d_n^a-n-2+k)(d_n^a-1)(d_n^a-2)}\partial^\lambda\partial^{\mu_{n-k+1}}\ldots\partial^{\mu_n}\,\tilde{\tilde{F}}_{\alpha\lambda\mu_1\ldots\mu_n}\ ,\quad n\neq 0$$

$$F_{\alpha\beta\mu_1\ldots\mu_{n-\kappa}\underbrace{5\ldots5}_{\kappa}} = \frac{1}{2^{\kappa}} \frac{\Gamma(d_n^a - n - 3)}{\Gamma(d_n^a - n - 2 + \kappa)(d_n^a - 1)} \partial^{\mu_{n-\kappa+1}} \ldots \partial^{\mu_n} \cdot$$

$$\cdot \Big\{ \big[(d_n^a - n - 3)(d_n^a - 1) + 2\kappa \big] F_{\alpha\beta\mu_1\ldots\mu_n} + \kappa (d_n^a - n - 2) \big(F_{\alpha\mu_n\mu_1\ldots\mu_{n-1}\beta} + F_{\mu_n\beta\mu_1\ldots\mu_{n-1}\alpha} \big) + \tag{1.23}$$

$$+ \kappa \sum_{i=1}^{n-1} \big(F_{\alpha\mu_i\mu_1\ldots\hat{\mu}_i\ldots\mu_n\beta} + F_{\mu_i\beta\mu_1\ldots\hat{\mu}_i\ldots\mu_n\alpha} \big) \Big\} \equiv \frac{1}{2^{\kappa}} \frac{\Gamma(d_n^a - n - 3)}{\Gamma(d_n^a - n - 2 + \kappa)(d_n^a - 1)} \partial^{\mu_{n-\kappa+1}} \ldots \partial^{\mu_n} \tilde{F}_{\kappa\,\alpha\beta\mu_1\ldots\mu_n}$$

for $n = 0$

$$F_{\alpha 5} = \frac{1}{2(d_0^a - 2)} \partial^{\lambda} F_{\alpha\lambda} . \tag{1.24}$$

The derivation of (1.22), (1.23) and (1.24) is given in Appendix A. (1.22) is not valid for $d_n^a = 2$ and $d_n^a = n+3$, $n \geq 1$ and (1.23) is not valid for $d_n^a = n+3$ ($d_n = 1$ gives no trouble because positivity requirements yield $d_n^a \geq n+3$ $(n \geq 1)$, $d_0^a \geq 2$). So we call $d_0^a = 2$ and $d_n^a = n+3$, $n \geq 1$ canonical dimensions. For such values we cannot impose (1.19) unless the tensors are conserved in both kinds of indices, but in such a case we cannot derive (1.22), (1.23) and (1.24) in the same way as in the symmetric case (1.16).

2. The OPE of Two Dirac Fields in 6-Dimensional Space and the Transition to Minkowski Space

2.1 Contributions to the expansion

The product of two scalar fields is expanded in the basis of the symmetric tensors described in section 1.3 (see ref. /1/). Such a set is not sufficient for the expansion of the product of two Dirac fields. We must add the anti-symmetric tensors described in 1.4. We have to distinguish tensors $O_{\mu_1 \ldots \mu_n}$ and pseudotensors $O^{P}_{\mu_1 \ldots, \mu_n}$ in contrast to the scalar expansion which can include only one of the two kinds depending on whether the two scalar fields have identical or different P-parity. Finally if we assume γ_5-invariance, i.e., invariance under which the spinor fields transform as

$$\Psi \to \gamma_5 \Psi \, , \quad \tilde{\Psi} \to \tilde{\Psi} \gamma_5 \tag{2.1}$$

we must include in the expansion both γ_5-"even" and "odd" tensors, i.e. tensors transforming as

$$A_{\mu_1 \ldots \mu_n} \to A_{\mu_1 \ldots \mu_n} \quad \text{and} \quad A^{5}_{\mu_1 \ldots \mu_n} \to -A^{5}_{\mu_1 \ldots \mu_n},$$

respectively. ($A_{\mu_1 \ldots \mu_n}$ stands here for both symmetric and antisymmetric tensors). We must note again that the OPE for two scalar fields excludes one of these types of tensors depending on the relative γ_5-"parity" of the two fields as above for P-parity. Obviously the results obtained in sect.1.3 and 1.4 are valid for all six kinds of tensors:

$O_{\mu_1\cdots\mu_n}$, $O^{(s)}_{\mu_1\cdots\mu_n}$, $O^{p}_{\mu_1\cdots\mu_n}$, $O^{p(s)}_{\mu_1\cdots\mu_n}$, $F_{\alpha\beta\mu_1\cdots\mu_n}$, $F^{(s)}_{\alpha\beta\mu_1\cdots\mu_n}$.

It is important to note that a conformal covariant field contributes to the expansion of two Dirac fields if and only if the three-point function of the two Dirac fields with the field in consideration is not zero. From the point of view of conformal covariance the question is whether we can build a conformal covariant structure in the expansion corresponding to the field in consideration.

2.2 OPE in 6-dimensional space

We are going to build a manifest conformal covariant OPE for the product of two Dirac fields $X(\eta_1)$ and $\tilde{X}(\eta_2)$ as described in 1.2 (we denote $d_{\psi_1}=d_1'$ and $d_{\psi_2}=d_2'$).
The most general form of the expansion of these fields is then

$$X(\eta_1)\tilde{X}(\eta_2)=\sum_{n=0}^{\infty}\left[\mathcal{D}_s^{(n)C_1\dots C_n}(\eta_1,\eta_2)\,\Theta_{C_1\dots C_n}(\eta_2)+\mathcal{D}_a^{(n)ABC_1\dots C_n}(\eta_1,\eta_2)\,\mathcal{F}_{ABC_1\dots C_n}(\eta_2)\right]+ \tag{2.2}$$

+ contributions from Θ^{p}, $\Theta^{(s)}$, $\Theta^{p(s)}$, $\mathcal{F}^{(s)}$.

The general structure of $\mathcal{D}_s^{(n)}$ $\left(\mathcal{D}_s^{(n)}\equiv\mathcal{D}_s^{(n)}, \text{ or } \mathcal{D}_s^{(n)}\equiv\mathcal{D}_s^{5(n)}\right)$ is

$$\mathcal{D}_s^{(n)C_1\dots C_n}(\eta_1,\eta_2)=\sum_{l}c_l^{(n)}(\eta_1\eta_2)^{d_l}(\beta\eta_1)\beta^{C_1}\dots\beta^{C_l}\eta_1^{C_{l+1}}\dots\eta_1^{C_n}(\beta\eta_2)\tilde{D}_{d_n}^{h_l}(\eta_1,\eta_2)+ \tag{2.3}$$

$$+\sum_{m}c_m'^{(n)}(\eta_1\eta_2)^{d_m'}(\beta\eta_1)(\beta\partial_2)\beta^{C_1}\dots\beta^{C_m}\eta_1^{C_{m+1}}\dots\eta_1^{C_n}(\beta\eta_2)\tilde{D}_{d_n}^{h_m'}(\eta_1,\eta_2)$$

here $(\beta\eta_1)(\beta\partial_1) = i(\eta_1\eta_2)^{-1}(\beta\eta_1)\beta^A\eta_2^B L_{1AB}$ is well defined on the cone $\eta_1^2 = 0$.

In the above sums $0 \le \ell, m \le 1$ because of $\theta_{c_1 \dots c_n}$ being symmetric and traceless and ℓ takes even values and m takes odd values for γ_5 "even" tensors and vice-versa for γ_5 – "odd" tensors. The numbers α_k, h_ℓ, α'_m, h'_m are easily found from considerations of homogeneity.

Six vectors η_2^A are not included in the tensor structure of $\mathcal{D}$ because their contribution is proportional to the structure written above (see ref. /1/). $\tilde{D}_d^h$ must be a differential operator defined on the cones $\eta_1^2 = \eta_2^2 = 0$, finite for $(\eta_1\eta_2) = 0$. In fact it turns out that this operator is essentially the one used in ref. /1/, i.e.

$$\tilde{D}_d^h = \left[(\eta_1\eta_2)^{-1}\eta_1^A\eta_1^C g^{BD} L_{2AB} L_{2CB}\right]^h =$$
$$= \left(\frac{x_1}{x_2}\right)^h \frac{(-1)^h}{\Gamma(-h)} \int_0^1 du\, u^{-h-1}(1-u)^{d+h-1} e^{u(12)\partial_2} {}_0F_1\left(d-1; \frac{x_{12}^2}{4} u(1-u)\Box_2\right) \equiv \left(\frac{x_1}{x_2}\right)^h D_d^h \tag{2.4}$$

here

$$x_{12}^2 = i0(x_{10} - x_{20}) - (x_1 - x_2)^2, \quad (12)_\mu = (x_1 - x_2)_\mu .$$

It can be easily checked that any other covariant combination correctly defined on the cones $\eta_1^2 = 0$ and $\eta_2^2 = 0$ reduces to (2.3). The operator $\mathcal{D}_a^{(n|ABc_1\dots c_n}(\eta_1, \eta_2)$ is analogous to $\mathcal{D}_s^{(n|c_1\dots c_n}(\eta_1, \eta_2)$ and is written in detail below in (2.7) and (2.8).

Now we are going to write down the explicit form of each term of the expansion.

1. Symmetric γ_5-"even" tensor part (S.P.) of the expansion

$$\text{S.P. } X(\eta_1)\tilde{X}(\eta_2)=\sum_{n=1}^{\infty} s_n(\eta_1\eta_2)^{d_n+\frac{1}{2}}(\beta\eta_1)(\beta\partial_1)\beta^{A_1}\eta_1^{A_2}\cdots\eta_1^{A_n}(\beta\eta_1)\tilde{D}_{d_n}^{h+1}\Theta_{A_1\ldots A_n}(\eta_2)+$$
$$+\sum_{n=0}^{\infty} s_n'(\eta_1\eta_2)^{d_n-\frac{1}{2}}(\beta\eta_1)\eta_1^{A_1}\cdots\eta_1^{A_n}(\beta\eta_2)\tilde{D}_{d_n}^{h}\,\Theta_{A_1\ldots A_n}(\eta_2) \qquad (2.5)$$

$$d_n=-\frac{1}{2}\left(d_1'+d_2'-d_n+n\right)$$

$$h_n=-\frac{1}{2}\left(d_1'-d_2'+d_n+n\right)$$

$$h=h_n$$

2. Symmetric γ_5 - "odd" tensor part ($\overset{(5)}{\text{S}}$.P)

$$\text{S}^{(5)}\text{P. } X(\eta_1)\tilde{X}(\eta_2)=$$

$$=\sum_{n=1}^{\infty} s_n^{(5)}(\eta_1\eta_2)^{d_n^{(5)}}(\beta\eta_1)(\beta\partial_1)\eta_1^{A_1}\cdots\eta_1^{A_n}(\beta\eta_2)\tilde{D}_{d_n}^{h^{(5)}+\frac{1}{2}}\Theta^{(5)}_{A_1\ldots A_n}(\eta_2)+ \qquad (2.6)$$
$$+\sum_{n=0}^{\infty} s_n'^{(5)}(\eta_1\eta_2)^{d_n^{(5)}}(\beta\eta_1)\beta^{A_1}\eta_1^{A_2}\cdots\eta_1^{A_n}(\beta\eta_2)\tilde{D}_{d_n}^{h^{(5)}+\frac{1}{2}}\Theta^{(5)}_{A_1\ldots A_n}(\eta_2)$$

here $d_n^{(5)}$ and $h^{(5)}$ are defined as d_n and h in (2.5) changing $d_n \to d_n^{(5)}$.

3. Antisymmetric γ_5 -"even" tensor part (A.P.)

$$\text{A.P. } X(\eta_1)\tilde{X}(\eta_2)=$$

$$=\sum_{n=1}^{\infty} a_n(\eta_1\eta_2)^{d_n^a+\frac{1}{2}}(\beta\eta_1)(\beta\partial_1)\beta^A\beta^B\beta^{C_1}\eta_1^{C_2}\cdots\eta_1^{C_n}(\beta\eta_2)\tilde{D}_{d_n^a}^{h^a+1}\,\mathcal{F}_{ABC_1\ldots C_n}(\eta_2)+$$
$$+\sum_{n=0}^{\infty} a_n'(\eta_1\eta_2)^{d_n^a-\frac{1}{2}}(\beta\eta_1)\beta^A\beta^B\eta_1^{C_1}\cdots\eta_1^{C_n}(\beta\eta_2)\,\tilde{D}_{d_n^a}^{h^a}\,\mathcal{F}_{ABC_1\ldots C_n}(\eta_2)\;+ \qquad (2.7)$$

$$+\sum_{n=1}^{\infty} b_n(\eta_1\eta_2)^{d_n^a-\frac{1}{2}}(\beta\eta_1)\beta^A\eta_1^B\beta^{C_1}\eta_1^{C_2}\cdots\eta_1^{C_n}(\beta\eta_2)\tilde{D}^{h^a}_{d_n^a}\mathcal{F}_{ABC_1\ldots C_n}(\eta_2)+$$

$$+\sum_{n=0}^{\infty} b'_n(\eta_1\eta_2)^{d_n^a-\frac{1}{2}}(\beta\eta_1)(\beta\partial_1)\beta^A\eta_1^B\eta_1^{C_1}\cdots\eta_1^{C_n}(\beta\eta_2)\tilde{D}^{h^a}_{d_n^a}\mathcal{F}_{ABC_1\ldots C_n}(\eta_2).$$

4. Antisymmetric γ_5-"odd" tensor part ($\overset{(5)}{A}$.P.)

$$\overset{(5)}{A}.P.\ \chi(\eta_1)\tilde{\chi}(\eta_2)=$$

$$=\sum_{n=1}^{\infty} a_n^{(5)}(\eta_1\eta_2)^{d_n^{a(5)}}(\beta\eta_1)\beta^A\beta^B\beta^{C_1}\eta_1^{C_2}\cdots\eta_1^{C_n}(\beta\eta_2)\tilde{D}^{h^{a(5)}+\frac{1}{2}}_{d_n^a}\mathcal{F}^{(5)}_{ABC_1\ldots C_n}(\eta_2)+$$

$$+\sum_{n=0}^{\infty} a_n^{\prime(5)}(\eta_1\eta_2)^{d_n^{a(5)}}(\beta\eta_1)(\beta\partial_1)\beta^A\beta^B\eta_1^{C_1}\cdots\eta_1^{C_n}(\beta\eta_2)\tilde{D}^{h^{a(5)}+\frac{1}{2}}_{d_n^a}\mathcal{F}^{(5)}_{ABC_1\ldots C_n}(\eta_2)\quad+\qquad(2.8)$$

$$+\sum_{n=1}^{\infty} b_n^{(5)}(\eta_1\eta_2)^{d_n^{a(5)}}(\beta\eta_1)(\beta\partial_1)\beta^A\eta_1^B\beta^{C_1}\eta_1^{C_2}\cdots\eta_1^{C_n}(\beta\eta_2)\tilde{D}^{h^{a(5)}+\frac{1}{2}}_{d_n^a}\mathcal{F}^{(5)}_{ABC_1\ldots C_n}(\eta_2)+$$

$$+\sum_{n=0}^{\infty} b_n^{\prime(5)}(\eta_1\eta_2)^{d_n^{a(5)}-1}(\beta\eta_1)\beta^A\eta_1^B\eta_1^{C_1}\cdots\eta_1^{C_n}(\beta\eta_2)\tilde{D}^{h^{a(5)}-\frac{1}{2}}_{d_n^a}\mathcal{F}^{(5)}_{ABC_1\ldots C_n}(\eta_2).$$

We must note that the four-constants in (2.7) correspond to three and in some cases two constants in the three-point functions $\langle\chi(\eta_1)\tilde{\chi}(\eta_2)\mathcal{F}_{ABC_1\ldots C_n}(\eta)\rangle_0$. The same is true for (2.8). (The relation between the three-point functions and the corresponding terms in the expansion is illustrated in Appendix C with the case n=1 for $O_{\mu_1\ldots\mu_n}$).

To account for pseudo-tensors we must add to S.P. and $\overset{(5)}{S}$.P. an expression that is obtained from them by multiplying by β_7 and changing the notations respectively.

2.3 Transition to Minkowski space

Let us denote $\binom{n}{k}(x_{12}^2)^k(12)^{\mu_1}\dots(12)^{\mu_{n-k}} = X_n^{\mu_1 \dots \mu_{n-k}}$.

Using the relation (1.3) between x-fields and η -fields we get from (2.5):

$$S.P.\ \psi(x_1)\tilde{\psi}(x_2) =$$

$$= -i\sum_{n=1}^{\infty} s_n\left(\frac{x_{12}^2}{2}\right)^{d_n+1/2}\left\{\left[\left(h'\gamma^\mu + \gamma_1\gamma^\mu \not x\right) D - 2\gamma_1 D(12)^\mu\right]\cdot\sum_{k=0}^{n-1} X_{n-1}^{\mu_1\dots\mu_{n-k-1}}\ O_{\mu\mu_1\dots\mu_{n-k-1}\underbrace{s\dots s}_{k}}(x_2) + \right.$$

$$\left. + 2\left[\gamma_1 \not x D + h'(\not x D - D\not x) - 2\gamma_1(12)^\nu D(12)_\nu - \gamma_1 D x_{12}^2\right]\cdot\sum_{k=0}^{n-1} X_{n-1}^{\mu_1\dots\mu_{n-k-1}}\ O_{\mu_1\dots\mu_{n-k-1}\underbrace{s\dots s}_{k+1}}(x_2)\right\} - \tag{2.9}$$

$$-i\sum_{n=0}^{\infty} s_n'\left(\frac{x_{12}^2}{2}\right)^{d_n-1/2}\not x D'\sum_{k=0}^{n} X_n^{\mu_1\dots\mu_{n-k}}\ O_{\mu_1\dots\mu_{n-k}\underbrace{s\dots s}_{k}}(x_2)$$

here $h' = d_2' - d_1' - d_n + n + 4$; $D = D_{d_n}^{h+1}$, $D' = D_{d_n}^{h}$.

Deriving (2.9) we have used the relation [1]:

$$\eta_1^{c_1}\dots\eta_1^{c_n}\ \Theta_{c_1\dots c_n}(\eta_2) = \sum_{k=0}^{n}\binom{n}{k}(12)^{\mu_1}\dots(12)^{\mu_{n-k}}(x_{12}^2)^k\ O_{\mu_1\dots\mu_{n-k}\underbrace{s\dots s}_{k}}(x_2) = \tag{2.10}$$

$$= \sum_{k=0}^{n} X_n^{\mu_1\dots\mu_{n-k}}\ O_{\mu_1\dots\mu_{n-k}\underbrace{s\dots s}_{k}}(x_2)$$

To get the final result it remains to use (1.16) for non-canonical dimensions. This is done in Chapter 3.

For the symmetric γ_5 - "odd" tensor part we get from (2.6):

$$
\begin{aligned}
& S^{(s)}_{\cdot} P_{,} \psi(x_1) \bar{\psi}(x_2) = \\
& = -i \sum_{n=0}^{\infty} s_n^{(s)} \left(\frac{x_{12}^2}{2}\right)^{h^{(s)}} \left(h^{\prime(s)} - \not{\partial}_1 \not{x}_{12} \right) D \cdot \sum_{k=0}^{n} X_n^{\mu_1 \ldots \mu_{n-k}} O^{(s)}_{\mu_1 \ldots \mu_{n-k} \underbrace{s \ldots s}_{k}}(x_2) - \\
& - i \sum_{n=1}^{\infty} s_n^{\prime(s)} \left(\frac{x_{12}^2}{2}\right)^{d_n^{(s)}} \Big\{ 2 \left[D x_{12}^2 - \not{x}_{12} D \not{x}_{12} + 2 (x_{12})^{\nu} D (x_{12})_{\nu} \right] \sum_{k=0}^{n} X_{n+1}^{\mu_1 \ldots \mu_{n-k+1}} O^{(s)}_{\mu_1 \ldots \mu_{n-k+1} \underbrace{s \ldots s}_{k+1}}(x_2) + \\
& + \left[\not{x}_{12} \gamma^{\mu} D + 2 \left(D (x_{12})^{\mu} - (x_{12})^{\mu} D \right) \right] \sum_{k=0}^{n-1} X_{n-1}^{\mu_1 \ldots \mu_{n-1-k}} O^{(s)}_{\mu \mu_1 \ldots \mu_{n-k-1} \underbrace{s \ldots s}_{k}}(x_2) \Big\}
\end{aligned}
\tag{2.11}
$$

here $D = D^{h^{(s)}}_{d_n^{(s)}}$, $h^{\prime(s)} = d_2' - d_1' - d_n^{(s)} + n + 4$.

We will not reproduce here the corresponding formulae for the antisymmetric contributions.

3. Summary of Results

3.1 Noncanonical dimensions

Introduce the operators:

$$
\begin{aligned}
& Y_{n,k,r}^{\mu_1 \ldots \mu_p} = \frac{\Gamma(d_n - 2 - n)}{2^k \, \Gamma(d_n - 2 - n + k + r)} X_{p-r}^{\mu_1 \ldots \mu_{p-r-k}} \partial^{\mu_{p-r-k+1}} \ldots \partial^{\mu_p} , \\
& \tilde{Y}_{n,k,r}^{\mu_1 \ldots \mu_p} = \frac{\Gamma(d_n^a - n - 3)}{2^k \, \Gamma(d_n^a - 2 - n + k + r)(d_n^a - 1)} X^{\mu_1 \ldots \mu_{p-r-k}} \partial^{\mu_{p-r-k+1}} \ldots \partial^{\mu_p} , \\
& \tilde{\tilde{Y}}_{n,k,r}^{\lambda \mu_1 \ldots \mu_p} = \frac{1}{d_n^a - 2} \tilde{Y}_{n,k,r}^{\mu_1 \ldots \mu_p} \partial^{\lambda} .
\end{aligned}
\tag{3.1}
$$

Then we finally obtain for the S.P. from (2.9):

$$S.P.\ \Psi(x_1)\tilde{\Psi}(x_2) =$$

$$= -i\sum_{n=1}^{\infty} S_n\left(\frac{x_{12}^2}{2}\right)^{d_n+1/2}\Big\{\left[\left(h'\gamma^\mu + \mathcal{Z}_1\gamma^\mu \not{x}\right)D - 2\mathcal{Z}_1 D(12)^\mu\right]\sum_{k=0}^{n-1} Y_{n,k,0}^{\mu_1\cdots\mu_{n-1}} O_{\mu\mu_1\cdots\mu_{n-1}}(x_2) +$$

$$+\left[\mathcal{Z}_1\not{x}D\not{x} + h'(\not{x}D - D\not{x}) - 2\mathcal{Z}_1(12)^\nu D(12)_\nu - \mathcal{Z}_1 D x_{12}^2\right]\sum_{k=0}^{n-1} Y_{n,k,1}^{\mu_1\cdots\mu_n} O_{\mu_1\cdots\mu_n}(x_2)\Big\} - \tag{3.2}$$

$$- i\sum_{n=0}^{\infty} S'_n\left(\frac{x_{12}^2}{2}\right)^{d_n-1/2} \not{x}D\sum_{k=0}^{n} Y_{n,k,0}^{\mu_1\cdots\mu_n} O_{\mu_1\cdots\mu_n}(x_2).$$

Similarly for $\overset{(5)}{S}.P.$ we obtain from (2.11):

$$S^{(5)}P.\ \Psi(x_1)\tilde{\Psi}(x_2) =$$

$$= -i\sum_{n=0}^{\infty} S_n^{(5)}\left(\frac{x_{12}^2}{2}\right)^{d_n^{(5)}}\left(h^{\prime(5)} - \mathcal{Z}_1\not{x}\right)D.\sum_{k=0}^{n} Y_{n,k,0}^{\mu_1\cdots\mu_n} O^{(5)}_{\mu_1\cdots\mu_n}(x_2) - \tag{3.3}$$

$$- i\sum_{n=1}^{\infty} S_n^{\prime(5)}\left(\frac{x_{12}^2}{2}\right)^{d_n}\Big\{\left[D x_{12}^2 - \not{x}D\not{x} + 2(12)^\nu D(12)_\nu\right]\sum_{k=0}^{n-1} Y_{n,k,1}^{\mu_1\cdots\mu_n} O^{(5)}_{\mu_1\cdots\mu_n}(x_2) +$$

$$+\left[\not{x}\gamma^\mu D + 2\left(D(12)^\mu - (12)^\mu D\right)\right]\sum_{k=0}^{n-1} Y_{n,k,0}^{\mu_1\cdots\mu_{n-1}} O^{(5)}_{\mu\mu_1\cdots\mu_{n-1}}(x_2)\Big\}.$$

Now for A.P. we obtain from (2.7):

$$A.P.\ \Psi(x_1)\tilde{\Psi}(x_2) = -i\sum_{n=1}^{\infty} a_n\left(\frac{x_{12}^2}{2}\right)^{d_n^a+1/2}\Big\{\Big[\frac{1}{2}\mathcal{Z}_1[\gamma^\sigma,\gamma^\tau]\gamma^\mu\not{x}D +$$

$$+\frac{h_1^{\prime a}}{2}[\gamma^\sigma,\gamma^\tau]\gamma^\mu D + 4\mathcal{Z}_1\gamma^\sigma\gamma^\mu D(12)^\tau - \mathcal{Z}_1[\gamma^\sigma,\gamma^\tau]D(12)^\mu\Big]\sum_{k=0}^{n-1}\tilde{Y}_{n,k,0}^{\mu_1\cdots\mu_{n-1}}\ {}_k\tilde{F}_{\sigma\tau\mu\mu_1\cdots\mu_{n-1}}(x_2) + \tag{3.4}$$

$$+\Big[\frac{1}{2}\mathcal{Z}_1[\gamma^\sigma,\gamma^\tau]\not{x}D\not{x} - \mathcal{Z}_1[\gamma^\sigma,\gamma^\tau](12)^\nu D(12)_\nu + \frac{h_1^{\prime a}}{2}[\gamma^\sigma,\gamma^\tau](\not{x}D - D\not{x}) -$$

$$- \frac{\mathcal{Z}_1}{2}[\gamma^\sigma,\gamma^\tau]D x_{12}^2 + 4\mathcal{Z}_1\gamma^\sigma(\not{x}D - D\not{x})(12)^\tau\Big]\sum_{k=0}^{n-1}\tilde{Y}_{n,k,1}^{\mu_1\cdots\mu_n}\ {}_{k+1}\tilde{F}_{\sigma\tau\mu_1\cdots\mu_n}(x_2) +$$

$$+\Big[4\mathcal{Z}_1\gamma^\sigma(D\not{x} - \not{x}D)(12)^\mu + 2\mathcal{Z}_1\gamma^\sigma\gamma^\mu D x_{12}^2 - 4\mathcal{Z}_1\gamma^\mu(D\not{x} - \not{x}D)(12)^\sigma + 4\mathcal{Z}_1\gamma^\sigma\gamma^\mu(12)^\lambda D(12)_\lambda +$$

$$+2h_1'^a\gamma^\sigma\gamma^\mu(D\varkappa-\varkappa D)-2\gamma_1\gamma^\sigma\gamma^\mu\varkappa D\varkappa\Big]\sum_{K=0}^{n-1}\tilde{\tilde{Y}}^{\lambda\mu_1\cdots\mu_{n-1}}_{n,K,0}\tilde{\tilde{F}}_{\sigma\lambda\mu\mu_1\cdots\mu_{n-1}}(x_2)\Big\}-$$

$$-i\sum_{n=0}^{\infty}a_n'\left(\frac{x_{12}^2}{2}\right)^{d_n'^a-\frac{1}{2}}\cdot\Big\{\Big[\tfrac{1}{2}[\gamma^\sigma\gamma^\tau]\varkappa D'-4\gamma^\sigma D'(12)^\tau\Big]\sum_{K=0}^{n}\tilde{Y}^{\mu_1\cdots\mu_n}_{n,K,0}\,{}_K\tilde{F}_{\sigma\tau\mu_1\cdots\mu_n}(x_2)+$$

$$+2\Big[\gamma^\sigma\varkappa D'\varkappa-\gamma^\sigma D'x_{12}^2-2\gamma^\sigma(12)^\nu D'(12)_\nu+2(\varkappa D'-D'\varkappa)\Big]\sum_{K=0}^{n}\tilde{\tilde{Y}}^{\lambda\mu_1\cdots\mu_n}_{n,K,0}\tilde{\tilde{F}}_{\sigma\lambda\mu_1\cdots\mu_n}(x_2)\Big\}-$$

$$-i\sum_{n=1}^{\infty}b_n\left(\frac{x_{12}^2}{2}\right)^{d_n^a-\frac{1}{2}}\Big\{\Big[\gamma^\sigma\gamma^\mu\varkappa D'-2\gamma^\sigma D'(12)^\mu+2\gamma^\mu D'(12)^\sigma\Big]\sum_{K=0}^{n-1}\Big[(12)^\nu\tilde{Y}^{\mu_1\cdots\mu_{n-1}}_{n,K,0}\,{}_K\tilde{F}_{\sigma\nu\mu\mu_1\cdots\mu_{n-1}}(x)+$$

$$+\frac{x_{12}^2}{2}\tilde{\tilde{Y}}^{\lambda\mu_1\cdots\mu_{n-1}}_{n,K,0}\tilde{\tilde{F}}_{\sigma\lambda\mu\mu_1\cdots\mu_{n-1}}(x_2)\Big]+\Big[-\gamma^\sigma\gamma^\lambda\varkappa D'(12)_\lambda-D'x_{12}^2\gamma^\sigma+2(\varkappa D'-D'\varkappa)(12)^\sigma\Big]\cdot$$

$$\cdot\sum_{K=0}^{n-1}\Big[(12)^\tau\tilde{Y}^{\mu_1\cdots\mu_n}_{n,K,1}\,{}_{K+1}\tilde{F}_{\sigma\tau\mu_1\cdots\mu_n}(x_2)+\frac{x_{12}^2}{2}\tilde{\tilde{Y}}^{\mu_1\cdots\mu_n\lambda}_{n,K,1}\tilde{\tilde{F}}_{\sigma\lambda\mu_1\cdots\mu_n}(x_2)+(12)^\tau\tilde{\tilde{Y}}^{\mu_1\cdots\mu_{n-1}\lambda}_{n,K,0}\tilde{\tilde{F}}_{\tau\lambda\sigma\mu_1\cdots\mu_{n-1}}(x_2)\Big]\Big\}-$$

$$-i\sum_{n=0}^{\infty}b_n'\left(\frac{x_{12}^2}{2}\right)^{d_n^a-\frac{1}{2}}\Big\{\Big[\Big(h_2'^a\gamma^\sigma+\gamma_1\gamma^\sigma\varkappa\Big)D'-2\gamma_1D'(12)^\sigma\Big]\sum_{K=0}^{n}\Big[(12)^\tau\tilde{Y}^{\mu_1\cdots\mu_n}_{n,K,0}\,{}_K\tilde{F}_{\sigma\tau\mu_1\cdots\mu_n}(x_2)+$$

$$+\frac{x_{12}^2}{2}\tilde{\tilde{Y}}^{\mu_1\cdots\mu_n\lambda}_{n,K,0}\tilde{\tilde{F}}_{\sigma\lambda\mu_1\cdots\mu_n}(x_2)\Big]+\Big[\gamma_1\varkappa D'\varkappa+h_2'^a(\varkappa D'-D'\varkappa)-$$

$$-2\gamma_1(12)^\nu D'(12)_\nu-\gamma_1D'x_{12}^2\Big](12)^\tau\sum_{K=0}^{n}\tilde{\tilde{Y}}^{\mu_1\cdots\mu_n\lambda}_{n,K,0}\tilde{\tilde{F}}_{\lambda\tau\mu_1\cdots\mu_n}(x_2)\Big\}$$

here $D=D^{h^a_n+1}_{d_n^a}$, $D'=D^{h^a}_{d_n^a}$, $h_1'^a=4+n-d_n^a-d_1'+d_2'$, $h_2'^a=h_1'^a+2$.

Finally for $A^{(5)}$. P. we get from (2.8):

$$
\begin{aligned}
A^{(5)}_{:} P.\ \Psi(x_1)\widetilde{\Psi}(x_2) = & -i\sum_{n=1}^{\infty} a_n^{(5)}\left(\frac{x_{12}^2}{2}\right)^{d_n^{(5)}}\Big\{\Big[\Big(\frac{1}{2}\varkappa[\gamma^{\sigma},\gamma^{\tau}]\gamma^{\mu}D - [\gamma^{\sigma},\gamma^{\tau}]((12)^{\mu}D - D(12)^{\mu}\Big)+\\
& + 4\gamma^{\sigma}\gamma^{\mu}((12)^{\tau}D - D(12)^{\tau})\Big]\sum_{k=0}^{n-1}\widetilde{Y}^{\mu_1\ldots\mu_{n-1}}_{n,k,0}\,{}_{k}\widetilde{F}^{(5)}_{\sigma\tau\mu\mu_1\ldots\mu_{n-1}}(x_2) + \qquad (3.5)\\
& + \frac{1}{2}\Big[2\gamma^{\sigma}(\varkappa D - D\varkappa)(12)^{\tau} + [\gamma^{\sigma},\gamma^{\tau}]\big(Dx_{12}^2 - \varkappa D\varkappa + 2(12)^{\nu}D(12)_{\nu}\big)\Big]\sum_{k=0}^{n-1}\widetilde{Y}^{\mu_1\ldots\mu_n}_{n,k,1}\,{}_{k+1}\widetilde{F}^{(5)}_{\tau\sigma\mu_1\ldots\mu_n}(x_2) -\\
& - \Big[2\gamma^{\sigma}\gamma^{\mu}\big(Dx_{12}^2 - \varkappa D\varkappa + 2(12)^{\nu}D(12)_{\nu}\big) - 4\gamma^{\sigma}(\varkappa D - D\varkappa)(12)^{\mu} + 4\gamma^{\mu}(\varkappa D - D\varkappa)(12)^{\sigma}\Big]\cdot\\
& \cdot\sum_{k=0}^{n-1}\widetilde{\widetilde{Y}}^{\lambda\mu_1\ldots\mu_{n-1}}_{n,k,0}\widetilde{\widetilde{F}}^{(5)}_{\sigma\lambda\mu\mu_1\ldots\mu_{n-1}}(x_2) + \Big[\gamma^{\sigma}\varkappa Dx_{12}^2 - 4Dx_{12}^2(12)^{\sigma}\Big]\sum_{k=0}^{n-1}\widetilde{\widetilde{Y}}^{\lambda\mu_1\ldots\mu_n}_{n,k,1}\widetilde{\widetilde{F}}^{(5)}_{\sigma\lambda\mu_1\ldots\mu_n}(x_2)\Big\} -\\
& - i\sum_{n=0}^{\infty} a_n^{\prime(5)}\left(\frac{x_{12}^2}{2}\right)^{d_n^{\prime(5)}}\Big\{\Big[\frac{1}{2}\big(h^{\prime a} - \mathcal{Z}_1\varkappa\big)[\gamma^{\sigma},\gamma^{\tau}]D - 4\mathcal{Z}_1\gamma^{\tau}\big((12)^{\tau}D - D(12)^{\tau}\big)\Big]\cdot\\
& \cdot\sum_{k=0}^{n}\widetilde{Y}^{\mu_1\ldots\mu_n}_{n,k,0}\,{}_{k}\widetilde{F}^{(5)}_{\sigma\tau\mu_1\ldots\mu_n}(x_2) + 2\Big[\big(h^{\prime a} - \mathcal{Z}_1\varkappa\big)\gamma^{\sigma}(\varkappa D - D\varkappa) + \mathcal{Z}_1\gamma^{\sigma}\big(x_{12}^2D +\\
& + Dx_{12}^2 + 2(12)^{\nu}D(12)_{\nu}\big) + 2\mathcal{Z}_1\big((12)^{\sigma}(\varkappa D - D\varkappa) - (\varkappa D - D\varkappa)(12)^{\sigma}\big)\Big]\sum_{k=0}^{n}\widetilde{\widetilde{Y}}^{\lambda\mu_1\ldots\mu_n}_{n,k,0}\widetilde{\widetilde{F}}^{(5)}_{\sigma\lambda\mu_1\ldots\mu_n}(x_2)\Big\} -\\
& - \sum_{n=1}^{\infty} b_n^{(5)}\left(\frac{x_{12}^2}{2}\right)^{d_n^{\prime\prime(5)}}\Big\{\Big[\big(h^{\prime a} - \mathcal{Z}_1\varkappa\big)\gamma^{\sigma}\gamma^{\mu}D + 2\mathcal{Z}_1\gamma^{\mu}\big((12)^{\sigma}D - D(12)^{\sigma}\big) - 2\mathcal{Z}_1\gamma^{\sigma}\big((12)^{\mu}D -\\
& - D(12)^{\mu}\big)\Big]\sum_{k=0}^{n-1}\Big[(12)^{\tau}\widetilde{Y}^{\mu_1\ldots\mu_{n-1}}_{n,k,0}\,{}_{k}\widetilde{F}^{(5)}_{\sigma\tau\mu\mu_1\ldots\mu_{n-1}}(x_2) + \frac{x_{12}^2}{2}\widetilde{\widetilde{Y}}^{\prime\lambda\mu_1\ldots\mu_{n-1}}_{n,k,0}\widetilde{\widetilde{F}}^{(5)}_{\sigma\lambda\mu\mu_1\ldots\mu_{n-1}}(x_2)\Big] +\\
& + \Big[\big(h^{\prime a} - \mathcal{Z}_1\varkappa\big)\gamma^{\sigma}(\varkappa D - D\varkappa) + \mathcal{Z}_1\gamma^{\sigma}\big(x_{12}^2D + Dx_{12}^2 + 2(12)^{\nu}D(12)_{\nu}\big) + 2\mathcal{Z}_1\big((12)^{\sigma}(\varkappa D - D\varkappa) -
\end{aligned}
$$

$$-(\varkappa D-D\varkappa)(12)^{\sigma}\Big)\Big]\sum_{k=0}^{n-1}\Big[(12)^{\tau}\widetilde{Y}^{\mu_1\dots\mu_n}_{n,k,1}\widetilde{F}^{(s)}_{k\,\sigma\tau\mu_1\dots\mu_n}(x_2)+\frac{x_{12}^2}{2}\widetilde{\widetilde{Y}}^{\lambda\mu_1\dots\mu_n}_{n,k,1}\widetilde{\widetilde{F}}^{(s)}_{\sigma\lambda\mu_1\dots\mu_n}(x_2)\Big]+$$

$$+\Big[-\big(h^{1a}-\mathcal{Z}_1\varkappa\big)\gamma^{\mu}(\varkappa D-D\varkappa)+\mathcal{Z}_1\gamma^{\mu}\big(x_{12}^2D+Dx_{12}^2+2(12)^{\nu}D(12)_{\nu}\big)+2\mathcal{Z}_1\Big((12)^{\mu}(\varkappa D-D\varkappa)-$$

$$-(\varkappa D-D\varkappa)(12)^{\mu}\Big)\Big]\sum_{k=0}^{n-1}(12)^{\sigma}\,\widetilde{\widetilde{Y}}^{\mu_1\dots\mu_{n-1}\lambda}_{n,k,0}\,\widetilde{\widetilde{F}}^{(s)}_{\sigma\lambda\mu\mu_1\dots\mu_{n-1}}(x_2)\Big\}-$$

$$-i\sum_{n=0}^{\infty}\ell_n^{\prime(s)}\left(\frac{x_{12}^2}{2}\right)^{d_n^{a(s)}-1}\Big\{\Big(-D'x_{12}^2+\varkappa D'\varkappa-2(12)^{\nu}D'(12)_{\nu}\Big)(12)^{\sigma}\sum_{k=0}^{n}\widetilde{\widetilde{Y}}^{\lambda\mu_1\dots\mu_n}_{n,k,0}\widetilde{\widetilde{F}}^{(s)}_{\sigma\lambda\mu_1\dots\mu_n}(x_2)+$$

$$+\Big[\varkappa\gamma^{\sigma}D'+2D'(12)^{\sigma}-(12)^{\sigma}D'\Big]\sum_{k=0}^{n}\Big[(12)^{\tau}\widetilde{Y}^{\mu_1\dots\mu_n}_{n,k,0}\widetilde{F}^{(s)}_{k\,\sigma\tau\mu_1\dots\mu_n}(x_2)+\frac{x_{12}^2}{2}\widetilde{\widetilde{Y}}^{\lambda\mu_1\dots\mu_n}_{n,k,0}\widetilde{\widetilde{F}}^{(s)}_{\sigma\lambda\mu_1\dots\mu_n}(x_1)\Big]\Big\}$$

here $D=D^{h^{a(s)}+1/2}_{d_n^{a(s)}}$, $D'=D^{h^{a(s)}-1/2}_{d_n^{a(s)}}$, $h^{1a}=d_2'-d_1'-d_n+n+5$.

3.2. Canonical dimensions

In the case of canonical dimensions we cannot use (1.16) and (1.22), (1.23), (1.24) to obtain formulae analogous to the relations written above without "unphysical" components. However if the tensors are conserved it turns out that in the only possible case here $d_1'=d_2'=d'$ the terms with the "unphysical" components cansel out and the expansion reduces to:

$$S.P.\ \Psi(x_1)\widetilde{\Psi}(x_2)=-i\sum_{n=1}^{\infty}S_n\left(\frac{x_{12}^2}{2}\right)^{-d'+3/2}\Big[\big(2\gamma^{\mu}+\mathcal{Z}_1\gamma^{\mu}\varkappa\big)(12)^{\mu_1}\dots(12)^{\mu_{n-1}}\overset{(n-1)}{D}{}^{h+1}_{d_n}-$$

$$-2\mathcal{Z}_1(12)^{\mu_1}\dots(12)^{\mu_{n-1}}(12)^{\mu}\overset{(n)}{D}{}^{h+1}_{d_n}\Big]O_{\mu\mu_1\dots\mu_{n-1}}(x_2)-i\sum_{n=0}^{\infty}S_n'\left(\frac{x_{12}^2}{2}\right)^{-d'+1/2}\varkappa(12)^{\mu_1}\dots(12)^{\mu_n}\overset{(n)}{D}{}^{h}_{d_n}O_{\mu_1\dots\mu_n}(x_2)\qquad(3.6)$$

$$S^{(s)}_{\cdot} P\, \psi(x_1)\bar{\psi}(x_2) = -\sum_{n=0}^{\infty} S^{(s)}_n \left(\frac{x_{12}^2}{2}\right)^{-d'+1} \left[2 - d_1' 12\right](12)^{\mu_1} \cdots (12)^{\mu_n} \overset{(n)}{D}{}^{h_n}_{d_n} O_{\mu_1 \dots \mu_n}(x_2) -$$

$$-\sum_{n=1}^{\infty} S_n^{(n-1)} \left(\frac{x_{12}^2}{2}\right)^{-d'+1} \left[\gamma^{\mu} 12 (12)^{\mu_1} \cdots (12)^{\mu_{n-1}} \overset{(n-1)}{D}{}^{h_n}_{d_n} + 2(12)^{\mu_1} \cdots (12)^{\mu_{n-1}} (12)^{\mu} \overset{(n)}{D}{}^{h_n}_{d_n}\right] O_{\mu\mu_1 \dots \mu_{n-1}}(x_2)$$

here $\overset{(n)}{D}{}^{h}_{d} = \int_0^1 du\, u^{-h-1}(1-u)^{d+h+n-1} {}_0F_1\left(d-1, \frac{x_{12}^2}{4} u(1-u)\Box_2\right) \exp(u(12)\partial_2)$.

Analogous formulae can be written for the contribution of $F_{\alpha\beta\mu_1\dots\mu_n}$. (Let us note that in the case $d_1' \neq d_2'$ the three-point functions $\langle \psi \bar{\psi} A_{\mu_1 \dots \mu_n} \rangle_c$ vanish so the corresponding constants in the expansion must also be equal to zero).

The case of nonconserved tensors with canonical dimensions is more complicated+). Our method does not give any result with physical components only. But there exists a conjecture (which is verified in the simplest case n=1) that for $d_1' = d_2' = d'$ formulae (3.6) hold also for this case. (Let us note that this is not a consequence of our basic formulae; in (3.6) $(12)^{\mu_1} \dots (12)^{\mu_n}$ are written before the differential operator D^h_d which is justified for traceless and conserved tensors only.)

The above written expansions are checked in the usual manner. One multiplies both sides by one of the basic fields, takes the vacuum expectation value and verifies that the L.H.S. and R.H.S. (in which only one term is left because of orthogonality of the two-point functions) coincide. This is done in Appendix C for the case n=1 of the symmetric tensors.

+) There is a conjecture /5/ which excludes this case but no complete proof is given.

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APPENDIX A

Elimination of Unphysical Components in the Antisymmetric Case

From (1.19b) using (1.20) and (1.21) we get

$$2(d_n^a - 3 - n + K) F_{\alpha\beta\mu_1\ldots\mu_{n-K}\underbrace{5\ldots5}_{K}} = \partial^\lambda F_{\alpha\beta\lambda\mu_1\ldots\mu_{n-K}\underbrace{5\ldots5}_{K-1}} + \tag{A.1}$$

$$+ 2 F_{5\beta\alpha\mu_1\ldots\mu_{n-K}\underbrace{5\ldots5}_{K-1}} + 2 F_{\alpha5\beta\mu_1\ldots\mu_{n-K}\underbrace{5\ldots5}_{K-1}} ,$$

and

$$2(d_n^a - 3 - n + K) F_{\alpha5\mu_1\ldots\mu_{n-K}\underbrace{5\ldots5}_{K}} = \partial^\lambda F_{\alpha5\lambda\mu_1\ldots\mu_{n-K}\underbrace{5\ldots5}_{K-1}} . \tag{A.2}$$

From (A.2) we obtain inductively

$$F_{\alpha5\mu_1\ldots\mu_{n-K}\underbrace{5\ldots5}_{K}} = \frac{\Gamma(d_n^a - n - 2)}{2^K \Gamma(d_n^a - n - 2 + K)} \partial^{\lambda_1}\ldots\partial^{\lambda_K} F_{\alpha5\mu_1\ldots\mu_{n-K}\lambda_1\ldots\lambda_K} . \tag{A.3}$$

From (A.1) we obtain inductively using (A.3):

$$F_{\alpha\beta\mu_1\ldots\mu_{n-k}\underbrace{5\ldots5}_{k}} = \frac{\Gamma(d_n^a - n - 2)}{2^k\,\Gamma(d_n^a - n - 2 + k)}\,\partial^{\lambda_1}\ldots\partial^{\lambda_{k-1}}\Big[\partial^{\lambda_k} F_{\alpha\beta\mu_1\ldots\mu_{n-k}\lambda_1\ldots\lambda_k} + \tag{A.4}$$

$$+2k\left(F_{\alpha5\mu_1\ldots\mu_{n-k}\beta\lambda_1\ldots\lambda_{k-1}} + F_{5\beta\mu_1\ldots\mu_{n-k}\alpha\lambda_1\ldots\lambda_{k-1}}\right)\Big].$$

Now to get rid of the unphysical components we must express $F_{\alpha5\mu_1\ldots\mu_n}$ in terms of $F_{\alpha\beta\mu_1\ldots\mu_n}$. To do this we make use of (1.19a) getting:

$$(d_n^a - 2)F_{\alpha5\mu_1\ldots\mu_{n-k}\underbrace{5\ldots5}_{k}} = \frac{1}{2}\partial^\lambda F_{\alpha\lambda\mu_1\ldots\mu_{n-k}\underbrace{5\ldots5}_{k}} + \sum_{i=1}^{n-k} F_{\alpha\mu_i\mu_1\ldots\hat{\mu}_i\ldots\mu_{n-k}\underbrace{5\ldots5}_{k+1}}\,,\ n \neq 0. \tag{A.5}$$

We take (A.5) at k=0 and insert (A.4) in it at k=1. We write down n+1 equations of this type for $F_{\mu_i5\mu_1\ldots\hat{\mu}_i\ldots\mu_{n+1}}$ and solve the resulting system of linear equations (in this case $d_n^a \neq 2$ or $d_n^a \neq n+3$) with the result:

$$F_{\alpha5\mu_1\ldots\mu_n} = \frac{1}{2(d_n^a - n - 3)(d_n^a - 1)}\Big[\Big(d_n^a - n - 2 - \frac{1}{d_n^a - 2}\Big)\partial^\lambda F_{\alpha\lambda\mu_1\ldots\mu_n} + \tag{A.6}$$

$$+\sum_{i=1}^{n}\partial^\lambda F_{\alpha\mu_i\lambda\mu_1\ldots\hat{\mu}_i\ldots\mu_n} - \frac{1}{d_n^a - 2}\sum_{i=1}^{n}\partial^\lambda F_{\mu_i\lambda\alpha\mu_1\ldots\hat{\mu}_i\ldots\mu_n}.$$

Finally we insert (A.6) in (A.3) and (A.4) to get (1.22) and (1.23), respectively.

In the cases $d_n^a=2$ or $d_n^a=n+3$ we must require

$$\partial^\lambda F_{\alpha\lambda\mu_1\ldots\mu_n} = 0\,, \quad \partial^{\mu_1} F_{\alpha\beta\mu_1\ldots\mu_n} = 0$$

in order to ensure the compatibility of the above system of linear equations, but in these cases the system has infinitely many solutions and (A.6) is not valid. (The same considerations would be valid for $d^a_n=1$ and $d^a_n=n+2$, but these values are excluded by positivity).

For n=o we obtain the formula (1.24) directly from (1.19a).

APPENDIX B

Two- and Three-Point Functions

It is well known that the requirements of conformal invariance completely determine the two- and three-point functions [6].

The two-point functions are determined up to a constant. Besides, the following selection rule is true

$$\langle O_{\mu_1\ldots\mu_m}(x_1)\, O_{\nu_1\ldots\nu_n}(x_2)\rangle_0 = 0 \tag{B.1}$$

unless m=n and $d_m=d_n$ [x].

x) We take the generators (corresponding to special conformal transformations) of the little group K_μ to be equal to zero.

We write down some of the two- and three-point functions needed for checking of the expansion (some of them are well known from other papers)

$$\langle O(x_1)\,O(x_2)\rangle_c = c_o \frac{\Gamma(d_o)}{(4\pi)^2}\left(\frac{4}{x_{12}^2}\right)^{d_o}, \tag{B.2}$$

$$\langle O_\mu(x_1)\,O_\nu(x_2)\rangle_c = c_1 \frac{\Gamma(d_1)}{(4\pi)^2}\left(\frac{4}{x_{12}^2}\right)^{d_1} V_{\mu\nu}, \tag{B.3}$$

where

$$V_{\mu\nu} = g_{\mu\nu} + 2\,\frac{(12)_\mu\,(12)_\nu}{x_{12}^2} .$$

$$\langle O_{\mu\nu}(x_1)\,O_{\rho\sigma}(x_2)\rangle_c = c_2 \frac{\Gamma(d_2)}{(4\pi)^2}\left(\frac{4}{x_{12}^2}\right)^{d_2}\left(V_{\mu(\rho}V_{\sigma)\nu} - \frac{1}{2}g_{\mu\nu}g_{\rho\sigma}\right) \tag{B.4}$$

$$\langle F_{\mu\nu}(x_1)\,F_{\rho\sigma}(x_2)\rangle_c = C_o^a \frac{\Gamma(d_o^a)}{(4\pi)^2}\left(\frac{4}{x_{12}^2}\right)^{d_o^a} V_{\mu[\rho}V_{\sigma]\nu} \tag{B.5}$$

$$\langle F_{[\alpha\beta]\nu}(x_1)\,F_{[\alpha'\beta']\nu'}(x_2)\rangle_c = C_1^a \frac{\Gamma(d_1^a)}{(4\pi)^2}\left(\frac{4}{x_{12}^2}\right)^{d_1^a}\Big[V_{\alpha[\alpha'}V_{\beta']\beta}V_{\nu\nu'} - \tag{B.6}$$

$$-\frac{1}{2}V_{\nu'[\alpha}g_{\beta][\alpha'}V_{\beta']\nu} + \frac{1}{2}g_{[\nu[\alpha}V_{\beta][\alpha'}g_{\beta']\nu']} + \frac{1}{2}g_{\nu[\alpha}g_{\beta][\alpha'}g_{\beta']\nu}$$

$$\langle \Psi(x_1)\bar{\Psi}(x_2)O(x_3)\rangle_c = -i g_0\, \hat{12}\left(\frac{2}{x_{12}^2}\right)^{\frac{d_1'+d_2'-d_o+1}{2}}\left(\frac{2}{x_{13}^2}\right)^{\frac{d_o+d_1'-d_2'}{2}}\left(\frac{2}{x_{23}^2}\right)^{\frac{d_o+d_2'-d_1'}{2}} \tag{B.7}$$

$$\langle \Psi(x_1)\bar{\Psi}(x_2)O_\mu(x_3)\rangle_c = -i\left(\frac{2}{x_{12}^2}\right)^{\frac{d_1'+d_2'-d_1}{2}}\left(\frac{2}{x_{13}^2}\right)^{\frac{d_1+d_1'-d_2'-1}{2}}\left(\frac{2}{x_{23}^2}\right)^{\frac{d_1+d_2'-d_1'-1}{2}}. \tag{B.8}$$

$$\cdot\left\{g_1(x_{13}^2)^{-1}\hat{13}\,\gamma_\mu\,\hat{32}(x_{23}^2)^{-1} + g_1'(x_{12}^2)^{-1}\hat{12}\,n_\mu\right\}.$$

where $$n_\mu = \frac{(13)_\mu}{x_{13}^2} - \frac{(23)_\mu}{x_{23}^2} \quad . \tag{B.9}$$

$$< \Psi(x_1)\,\tilde{\Psi}(x_2)\,O_{\mu\nu}(x_3)>_c = -i\left(\frac{2}{x_{12}^2}\right)^{\frac{d_1'+d_2'-d_2+1}{2}} \left(\frac{2}{x_{13}^2}\right)^{\frac{d_2+d_1'-d_2'-2}{2}} \left(\frac{2}{x_{23}^2}\right)^{\frac{d_2+d_2'-d_1'-2}{2}} \cdot$$

$$\cdot\left\{ g_2\left[\frac{1}{2} g_{\mu\nu} \not{12}(x_{13}^2)^{-1}(x_{23}^2)^{-1} + \not{13}(x_{13}^2)^{-1}\gamma_\mu n_\nu \not{23}(x_{23}^2)^{-1}\right] + g_2' \not{12}\left[\tfrac{1}{4} g_{\mu\nu}(x_{13}^2 x_{23}^2)^{-1} + (x_{12}^2)^{-1} n_\mu n_\nu\right]\right\} \tag{B.10}$$

$$< \Psi(x_1)\,\tilde{\Psi}(x_2)\,F_{\alpha\beta}(x_3)>_0 = -i\left(\frac{2}{x_{12}^2}\right)^{\frac{d_1'+d_2'-d_0^a+1}{2}} \left(\frac{2}{x_{13}^2}\right)^{\frac{d_0^a+d_1'-d_2'}{2}} \left(\frac{2}{x_{23}^2}\right)^{\frac{d_0^a+d_2'-d_1'}{2}} \cdot \tag{B.11}$$

$$\cdot\left\{ f_0 (x_{13}^2)^{-1} \not{13}[\gamma_\alpha, \gamma_\beta] \not{13}\not{12} + f_0' (x_{23}^2)^{-1} \not{12}\not{23}[\gamma_\alpha, \gamma_\beta]\not{23}\right\} \cdot$$

APPENDIX C

Relations between the OPE and the Three-Point Functions

We consider the vector contribution to the expansion, i.e., the case m=1. Let us multiply both sides of our full expansion with $O_\alpha(x_3)$ and take the vacuum expectation values. Then as a consequence of the selection rule (B.1) it follows:

$$< \Psi(x_1)\,\tilde{\Psi}(x_2)\,O_\mu(x_3)>_0 = < S_1 . T.\left(\Psi(x_1)\,\tilde{\Psi}(x_2)\right) O_\mu(x_3)>_0 \, . \tag{C.1}$$

It is more convenient to make the calculations using $S_1 T$ not in the final form (3.2) but in some intermediate form:

$$S_1 T\; \Psi(x_1)\bar{\Psi}(x_2) = -\left(\frac{2}{x_{12}^2}\right)^{d'-d/2}\Big\{-S_1 T(x_1)(\beta X_1)\left[\tau_3 \partial_1 - 2i\tau_-(x_1\partial_1 - h)\right]\cdot \tag{C.2}$$

$$\cdot \beta^A D_d^{h+1}\Theta_A(X_2)(\beta X_2)T(x_2) + iS_1'\tau_+ \not{x}\frac{2}{x_{12}^2}X_1^A D_d^h \Theta_A(X_2)\Big\}$$

(here $X^A = \frac{1}{\mathscr{X}}\eta^A$, $T(x_1)$ is defined by (1.9), $d=d_1$ and fur simplicity $d_1'=d_2'=d'$, $h=-\frac{d+1}{2}$).

Then expressing

$$O_\mu(x_3) = \Theta_\nu(X_3) - x_{3\nu}\left[\Theta_5(X_3) + \Theta_6(X_3)\right]$$

(we have used (1.13))and using the relation for conformal invariant two-point function in η - space

$$\langle\Theta_A(X_2)\Theta_\mu(X_3)\rangle_0 = C_1\frac{2^d\Gamma(d)}{(4\pi)^2}\left[g_{A\mu}(X_2X_3) - X_{3A}X_{2\mu} + C_1' X_{2A}X_{3\mu}\right](X_2X_3)^{-d-1} \tag{C.3}$$

we obtain for the R.H.S. of (C.1):

$$R.H.S. = -\left(\frac{2}{x_{12}^2}\right)^{d'-d/2} C_1\frac{2^d\Gamma(d)}{(4\pi)^2}\Big\{S_1\left[-\tau_+\partial_1 + i\tau_+\tau_-(x_1\partial_1 - 2x_1\partial_1 + 2h)\right]\cdot$$

$$\cdot\left[\gamma_\mu\tau_3 - 2i\tau_- x_{3\mu} + (\beta X_3)\frac{\partial_{3\mu}}{d}\right]D_d^{h+1}\left(\frac{2}{x_{23}^2}\right)^d(\tau_+ - i\tau_-\tau_+\not{x}_2) + S_1'\frac{2}{x_{12}^2}i\tau_+\not{x}\left((13)_\mu + \frac{x_{13}^2}{2d}\partial_{3\mu}\right)D_d^h\left(\frac{2}{x_{13}^2}\right)^d\Big\} = \tag{C.4}$$

$$= -i\tau_+\left(\frac{2}{x_{12}^2}\right)^{d'-d/2}\left(\frac{2}{x_{13}^2}\frac{2}{x_{23}^2}\right)^{\frac{d-1}{2}}(-1)^{\frac{d-1}{2}}\frac{\Gamma(\frac{d}{2})C_1}{(4\pi)^2}\Big\{-\left(\frac{d-1}{2}\right)^2 S_1\frac{\not{x}_{13}}{x_{13}^2}\gamma_\mu\frac{\not{x}_{23}}{x_{23}^2} + S_1'\frac{\not{x}_{12}}{x_{12}^2}n_\mu\Big\},$$

where n_μ is given by (B.9).

Comparing (C.4) and (B.8) we obtain the following relations between the constants S_1, S_1' of the expansion, g_1, g_1' of the three-point function and the constant C_1 of the two-point function:

$$g_1 = -C_1 \frac{\Gamma\left(\frac{d}{2}\right)}{(4\pi)^2} \left(\frac{d-1}{2}\right)^2 S_1 , \tag{C.5}$$

$$g_1' = C_1 \frac{\Gamma\left(\frac{d}{2}\right)}{(4\pi)^2} S_1' .$$

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