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1. In paper [1] the spin structure of Feynman matrix elements of scattering amplitude which describe the interaction of two relativistic particles in one-meson exchange approximation has been studied. These matrix elements correspond to some quasi-potential by which the interaction is described in the quasi-potential method for relativistic two-particle problem proposed by Logunov and Tavkhelidze [2,3]. In what follows, like in [1], we shall use the equation for the wave function describing the relative motion of two relativistic particles with spin 1/2, obtained within the framework of Kadyshevsky quasi-potential approach [4-6].

$$E_p(E_p-E_q)\Psi_q(\vec{p})_{\sigma_1\sigma_2}=\frac{1}{(4\pi)^3}\sum_{\sigma_1',\sigma_2'=-\frac{1}{2}}^{\frac{1}{2}}\int\frac{d^3\vec{k}}{E_k}V^{\sigma_1'\sigma_2'}_{\sigma_1\sigma_2}(\vec{p},\vec{k};E_q)\Psi_q(\vec{k})_{\sigma_1'\sigma_2'} \quad (1)$$

or in the case of scattering

$$\Psi_q(\vec{p})_{\sigma_1\sigma_2}=\frac{(2\pi)^3}{m}\delta(\vec{p}-\vec{q})\sqrt{\vec{p}^2+m^2}\,\xi_{\sigma_1}\xi_{\sigma_2}+$$
$$+\frac{1}{E_p(E_p-E_q-i\varepsilon)}\cdot\frac{1}{(4\pi)^3}\cdot\sum_{\sigma_1',\sigma_2'=-\frac{1}{2}}^{\frac{1}{2}}\int\frac{d^3\vec{k}}{E_k}V^{\sigma_1'\sigma_2'}_{\sigma_1\sigma_2}(\vec{p},\vec{k};E_q)\Psi_q(\vec{k})_{\sigma_1'\sigma_2'}, \quad (2)$$

where

$$E_p=p_0=\sqrt{\vec{p}^2+m^2}\,; \qquad E_q=q_0=\sqrt{\vec{q}^2+m^2}\,.$$

In the second approximation in coupling constant the quasi-potential $V^{\sigma_1'\sigma_2'}_{\sigma_1\sigma_2}(\vec{p},\vec{k};E_q)$ coincides with the Feynman matrix element of the scattering amplitude, that corresponds to diagrams of one-boson exchange.

In equations (1) and (2) all the momenta of particles belong to the mass shell

$$p_0^2 - \vec{p}^{\,2} = m^2 \quad . \tag{3}$$

Equality (3) defines the three-dimensional manifold of hyperboloid on the upper sheet of which the Lobachevsky space is realized. The integration in (1) and (2) is performed with the volume element on hyperboloid $\frac{d^3\vec{k}}{E_k}$, which is the volume element in Lobachevsky space. So, it is possible to consider the geometry of the momentum space in equations (1) and (2) to be the Lobachevsky geometry /7/.

In /1/ equation (2) with a quasi-potential corresponding to one-meson exchange interaction has been transformed to a form in which an interaction is described by local in Lobachevsky space quasi-potential. The spin structure of such quasi-potential looks like a direct geometrical relativistic generalization of the spin structure of quantum-mechanical potentials in the sense of replacement of the Euclidean geometry of momentum space by Lobachevsky geometry in relativistic case.

The aim of the present work is to study the spin structure of electromagnetic interaction of two relativistic particles with the help of method developed in /1/. We shall restrict ourselves to consideration of the electron-positron interaction only, and for the quasi-potential in (1) and (2) we shall take the Feynman matrix elements, corresponding to one-photon exchange. As it will be shown later the spin structure of this interaction looks like the geometrical relativistic generalization of the spin structure of Breit interaction.

2. Let us consider at first an interaction of two electrons. In accordance with the general rules for constructing the quasi-potential from the matrix elements of relativistic scattering amplitude T /1-4,6/ in the second order in coupling constant,

to quasi-potential V there corresponds the matrix element of the scattering amplitude $\langle \vec{p}, \sigma_1 \sigma_2 | T | \vec{k}, \sigma_1' \sigma_2' \rangle$, conformable to the Feynman diagrams on Fig. 1.

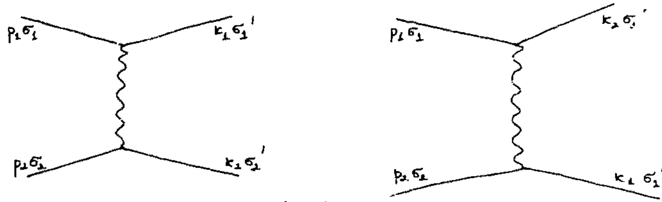

Fig. 1.

The first diagram describes the"direct" interaction and the second - the"exchange" one. The momenta of incoming particles p_1, p_2 and the momenta of outgoing particles k_1, k_2 belong to the centre of mass system (c.m.s.), i.e. $\vec{p_1} = -\vec{p_2} = \vec{p}$; $\vec{k_1} = -\vec{k_2} = \vec{k}$, the solid line corresponds to a spinor particle, the waved line- to photon. Thus, for the matrix element of the quasi-potential V there exists the following representation (the index (2) means the second approximation in the coupling constant).

$$\langle \vec{p}, \sigma_1 \sigma_2 | V^{(2)}(E_q) | \vec{k}, \sigma_1' \sigma_2' \rangle = \langle \vec{p}, \sigma_1 \sigma_2 | T^{(2)} | \vec{k}, \sigma_1' \sigma_2' \rangle =$$
$$= \langle \vec{p}, \sigma_1 \sigma_2 | V^{(2)}_{dir}(E_q) | \vec{k}, \sigma_1' \sigma_2' \rangle - \langle \vec{p}, \sigma_1 \sigma_2 | V^{(2)}_{exch}(E_q) | \vec{k}, \sigma_1' \sigma_2' \rangle , \tag{4}$$

where

$$\langle \vec{p}, \sigma_1 \sigma_2 | V^{(2)}_{dir}(E_q) | \vec{k}, \sigma_1' \sigma_2' \rangle =$$
$$= 4\pi e^2 \frac{\left[\bar{u}^{\sigma_1}(\vec{p}) \gamma^{\mu} u^{\sigma_1'}(\vec{k})\right] g^{\mu\nu} \left[\bar{u}^{\sigma_2}(-\vec{p}) \gamma^{\nu} u^{\sigma_2'}(-\vec{k})\right]}{(p_1 - k_1)^2} \tag{5}$$

and

$$\langle \vec{p}, \sigma_1 \sigma_2 | V^{(2)}_{exch}(E_q) | \vec{k}, \sigma_1' \sigma_2' \rangle = \\ = 4\pi e^2 \cdot \frac{[\bar{u}^{\sigma_1}(\vec{p}) \gamma^\mu u^{\sigma_2'}(\vec{k})]\, g^{\mu\nu} [\bar{u}^{\sigma_2}(-\vec{p}) \gamma^\nu u^{\sigma_1'}(\vec{k})]}{(p_1 - k_2)^2} \quad (6)$$

Let us consider the part of the quasi-potential, describing the direct interaction.

The Dirac bispinors $u^{\sigma}(\vec{k})$, normalized by condition

$$\bar{u}^{\sigma}(\vec{k})\, u^{\sigma'}(\vec{k}) = 2m\, \delta_{\sigma\sigma'} \quad (7)$$

can be represented as follows:

$$u^{\sigma}(\vec{k}) = S_{\vec{k}}\, u^{\sigma}(0) \; . \quad (8)$$

In spinor representation, where γ - matrices are of the form:

$$\gamma^\mu = \begin{pmatrix} 0 & g^{\mu\mu}\sigma_\mu \\ \sigma_\mu & 0 \end{pmatrix} \quad (9)$$

(σ_μ -Pauli matrices with $\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$), bispinor in the rest frame has the form

$$u^{\sigma}(0) = \sqrt{m} \begin{pmatrix} \xi_\sigma \\ \xi_\sigma \end{pmatrix} , \quad (10)$$

where ξ_σ - two-component spinor with the norm $\overset{+}{\xi}_\sigma \xi_{\sigma'} = \delta_{\sigma\sigma'}$.
In this representation the matrix $S_{\vec{k}}$ is diagonal [*)]

$$S_{\vec{k}} = \begin{pmatrix} H_{\vec{k}} & 0 \\ 0 & \overset{+}{H}{}^{-1}_{\vec{k}} \end{pmatrix} . \quad (11)$$

---

*) With the help of the matrix $\vec{\alpha} = \gamma^0 \vec{\gamma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & -\vec{\sigma} \end{pmatrix}$ $S_{\vec{k}}$ can be represented as

$$S_{\vec{k}} = \sqrt{\frac{k_0 + m}{2m}} \left(1 + \frac{\vec{\alpha}\vec{k}}{k_0 + m} \right)$$

In standard representation, where $\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $\vec{\alpha}$ matrix has the form $\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}$. The transition from bispinors in spinor representation to those in standard representation can be performed

The matrix $H_{\vec{k}} = \frac{k_0 + m + \vec{\sigma}\vec{k}}{\sqrt{2m(k_0+m)}}$ from $SL(2,C)$ group corresponds to the pure Lorentz transformation $\Lambda_{\vec{k}}$, which transforms the rest system of the particle into a system where the particle momentum is $k = (k_0, \vec{k})$, i.e. $\Lambda_{\vec{k}}(m, 0) = (k^0, \vec{k})$. Let us introduce for the 4-vector $\Lambda_p^{-1} k$ the following notation

$$
\begin{aligned}
(k(-)p)^0 &\equiv (\Lambda_p^{-1} k)^0 = \frac{k^0 p^0 - \vec{k}\vec{p}}{m} = \sqrt{m^2 + (\vec{k}(-)\vec{p})^2} \\
\vec{k}(-)\vec{p} &\equiv \overrightarrow{(\Lambda_p^{-1} k)} = \vec{k} - \frac{\vec{p}}{m}\left(k^0 - \frac{\vec{k}\vec{p}}{p^0+m}\right) .
\end{aligned}
\tag{12}
$$

Vector $\vec{\Delta} = \vec{k}(-)\vec{p}$ is the difference of two vectors in Lobachevsky space, realized on the upper sheet of hyperboloid (3). In the nonrelativistic limit it reduces to the usual difference of two vectors $\vec{\Delta}_3 = \vec{k} - \vec{p}$ in the Euclidean space. So it can be treated as a relativistic generalization of three-dimensional momentum transfer vector.

Two-component spinor ξ_σ describes the polarization state of a particle in the rest system of the particle the axes of which coincide with the axes of the system where particle momentum is equal to $\vec{k}$. Thus, σ here is the spin projection on the third axis $h_3(\vec{k})$ of the coordinate system connected with momentum $\vec{k}$.

The quasi-potential $V^{\sigma_1'\sigma_2'}_{\sigma_1\sigma_2}(\vec{p},\vec{k};E_q)$ in equations (1), (2) can be obtained from $\langle \vec{p},\sigma_1\sigma_2 | V(E_q) | \vec{k},\sigma_1'\sigma_2' \rangle$ by passing from bispinors to two- component spinors

$$
\begin{aligned}
\langle \vec{p},\sigma_1\sigma_2 | V(E_q) | \vec{k},\sigma_1'\sigma_2' \rangle &= \sum_{\alpha,\beta,\rho,\mu=0}^{3} \bar{u}^{\sigma_1}_{\alpha}(\vec{p})\,\bar{u}^{\sigma_2}_{\beta}(-\vec{p})\, V_{\alpha\beta,\rho\mu}(\vec{p},\vec{k};E_q)\, u^{\sigma_1'}_{\rho}(\vec{k})\, u^{\sigma_2'}_{\mu}(-\vec{k}) \\
&= \xi^{+}_{\sigma_1}\xi^{+}_{\sigma_2}\, V^{\sigma_1'\sigma_2'}_{\sigma_1\sigma_2}(\vec{p},\vec{k};E_q)\, \xi_{\sigma_1'}\xi_{\sigma_2'} .
\end{aligned}
\tag{13}
$$

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with the help of matrix $S_0 = S_0^{-1} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$.

Now, in expression for a current in (3) we shall make the transition to bispinors in the rest systems of particles :

$$j^{\mu}_{\sigma_1\sigma_1'}(\vec{p},\vec{k}) = \bar{u}^{\sigma_1}(\vec{p})\gamma^{\mu}u^{\sigma_1'}(\vec{k}) = \bar{u}^{\sigma_1}(0)\, S^{-1}_{\vec{p}}\gamma^{\mu}S_{\vec{k}}\; u^{\sigma_1'}(0) \; . \quad (14)$$

Keeping the definition of Wigner rotation, described by matrix $V(H_p,\kappa) \in SU(2)$*),

$$S^{-1}_{\vec{p}}\, S_{\vec{k}} = S_{\vec{k}\leftrightarrow\vec{p}} \cdot I \otimes D^{\frac{1}{2}}\{V^{-1}(H_p,\kappa)\} \quad (15)$$

(I - is the unity matrix $4\otimes 4$), (14) takes the form

$$j^{\mu}_{\sigma_1\sigma_1'}(\vec{p},\vec{k}) = \bar{u}^{\sigma_1}(0)\, S^{-1}_{\vec{p}}\gamma^{\mu}S_{\vec{p}} \cdot S_{\vec{k}\leftrightarrow\vec{p}} \cdot I\otimes D^{\frac{1}{2}}\{V^{-1}(H_p,\kappa)\}u^{\sigma_1'}(0) \quad (16)$$

An application of the well-known formula

$$S^{-1}_{p}\gamma^{\mu}S_{p} = (\Lambda_p)^{\mu}_{\nu}\,\gamma^{\nu} \quad (18)$$

allows us to represent (16) like

$$j^{\mu}_{\sigma_1\sigma_1'}(\vec{p},\vec{k}) = \frac{1}{m}\, \overset{+}{u}{}^{\sigma_1}(0)\left\{p^{\mu} + i\gamma^{5}W^{\mu}(\vec{p})\right\} S_{\vec{k}\leftrightarrow\vec{p}}\, I\otimes D^{\frac{1}{2}}\{V^{-1}(H_p,\kappa)\}\cdot u^{\sigma_1'}(0). \quad (19)$$

The 4-vector of the relativistic spin $W^{\mu}(\vec{p})$ /9/ has the following components

$$W^{0}(\vec{p}) = \frac{1}{2}\vec{\sigma}\vec{p}$$
$$\vec{W}(\vec{p}) = \frac{1}{2}\vec{\sigma}m + \frac{\vec{p}\,(\frac{1}{2}\vec{\sigma}\vec{p})}{p_0 + m} \quad (20)$$

and γ^5 -matrix in spinor representation is

$$\gamma^{5} = i\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (21)$$

From (19) there follows the formula for the current $j^{\mu}_{\sigma_1\sigma_1'}(\vec{p},\vec{k})$:

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*) The function $D^{\frac{1}{2}}\{V^{-1}(H_p,\kappa)\}$ can be written as

$$D^{\frac{1}{2}}\{V^{-1}(H_p,\kappa)\} = \sqrt{\frac{(p_0+m)(k_0+m)}{2m(\Delta_0+m)}}\left\{1 - \frac{\vec{p}\vec{k} + i\vec{\sigma}[\vec{p}\vec{k}]}{(p_0+m)(k_0+m)}\right\} \quad (17)$$

$$j^{\mu}_{\sigma_1\sigma_1'}(\vec{p},\vec{k}) = \sum_{\sigma_{1p}=-\frac{1}{2}}^{\frac{1}{2}} j^{\mu}_{\sigma_1\sigma_{1p}}(\vec{p},\vec{k})\, D^{\frac{1}{2}}_{\sigma_{1p}\sigma_1'}\left\{V^{-1}(\Lambda_p, k)\right\}, \tag{22}$$

where *)

$$j^{\mu}_{\sigma_1\sigma_{1p}}(\vec{p},\vec{k}) = \frac{2}{\sqrt{2m(\Delta_0+m)}}\, \xi^{+}_{\sigma_1}\left\{ p^{\mu}(\Delta_0+m) + W^{\mu}_1(\vec{p})(\vec{\sigma}_1\vec{\Delta})\right\}. \tag{23}$$

Let us mention that in nonrelativistic limit $\Delta_0 = \sqrt{m^2+\vec{\Delta}^2}$ takes the form

$$\Delta_0 \approx mc + \frac{(\vec{k}-\vec{p})^2}{2mc} \tag{24}$$

and

$$D^{\frac{1}{2}}\left\{V^{-1}(\Lambda_p, k)\right\} \approx 1 - i\,\frac{\vec{\sigma}\,[\vec{p}\,\vec{k}]}{4m^2c^2}. \tag{25}$$

Making use of (24), (25) and (23) it is easy to find the nonrelativistic limit of the current (14) **)

$$j^{0}_{\sigma_1\sigma_1'}(\vec{p},\vec{k}) \approx 2m\left\{1 + \frac{(\vec{k}-\vec{p})^2}{8m^2c^2} + i\,\frac{\vec{\sigma}\,[\vec{p}\,\vec{k}]}{4m^2c^2}\right\}_{\sigma_1\sigma_1'} \tag{26}$$

$$\vec{j}_{\sigma_1\sigma_1'}(\vec{p},\vec{k}) \approx \frac{1}{c}\left\{(\vec{p}-\vec{k}) + 2\vec{k} + i\left[\vec{\sigma}(\vec{p}-\vec{k})\right]\right\}_{\sigma_1\sigma_1'} \tag{27}$$

From (22), (23) and (17) the symmetry relations follow

$$j^{0}(-\vec{p},-\vec{k}) = j^{0}(\vec{p},\vec{k}) \quad ; \quad \vec{j}(-\vec{p},-\vec{k}) = -\vec{j}(\vec{p},\vec{k}). \tag{28}$$

Relations (22) and (28) allow us to represent the quasi-potential

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*) General parametrization for vector current with the help of relativistic spin vector is given in /10/.

**) The difference of the second term of (26) from the analogous expression in /13/ follows from the fact that together with the transformation to nonrelativistic limit there has been done an additional transformation from two-component spinors to "Schrödinger" amplitudes, i.e. the substitution $\xi = \left(1 - \frac{\vec{p}^2}{8m^2c^2}\right)\varphi_{Schr.}$

(5) like:

$$\langle \vec{p}, \sigma_1 \sigma_2 | V^{(2)}_{dir}(E_q) | \vec{\kappa}, \sigma'_1 \sigma'_2 \rangle = \tag{29}$$

$$= \sum_{\sigma_{1p}, \sigma_{2p} = -\frac{1}{2}}^{\frac{1}{2}} \langle \vec{p}, \sigma_1 \sigma_2 | V^{(2)}_{dir}(E_q) | \vec{\kappa}, \sigma_{1p} \sigma_{2p} \rangle D^{\frac{1}{2}}_{\sigma_{1p} \sigma'_1} \{ V^{-1}(H_p, \kappa) \} D^{\frac{1}{2}}_{\sigma_{2p} \sigma'_2} \{ V^{-1}(H_p, \kappa) \},$$

where, obviously,

$$\langle \vec{p}, \sigma_1 \sigma_2 | V^{(2)}_{dir}(E_q) | \vec{\kappa}, \sigma_{1p} \sigma_{2p} \rangle = \xi^{+}_{\sigma_1} \xi_{\sigma_2} V^{(2)\, \sigma_{1p} \sigma_{2p}}_{dir\ \sigma_1 \sigma_2} (\vec{\kappa} \leftrightarrow \vec{p}; \vec{p}; E_q) \xi_{\sigma_{1p}} \xi_{\sigma_{2p}} =$$

$$= \frac{ j^{\mu}_{\sigma_1 \sigma_{1p}}(\vec{p}, \vec{\kappa})\, g^{\mu\nu}\, j^{\nu}_{\sigma_2 \sigma_{2p}}(-\vec{p}, -\vec{\kappa}) }{ (p - \kappa)^2 } . \tag{30}$$

The role of Wigner rotation, that appears in expression for the quasi-potential, can be easily understood if one considers the transformation law for the state vector under the Lorentz transformation $\Lambda^{-1}_{\vec{p}}$:

$$U(\Lambda^{-1}_p) | \vec{\kappa}, \sigma \rangle = \sum_{\sigma' = -\frac{1}{2}}^{\frac{1}{2}} D^{\frac{1}{2}}_{\sigma \sigma'} \{ V^{-1}(H_p, \kappa) \} | \overrightarrow{\Lambda^{-1}_{p} \kappa}, \sigma' \rangle \tag{31}$$

So the matrix of polarization index transformation depends on the momentum of the state. Therefore, for the matrix element (5) the right indices σ'_1, σ'_2 and left ones σ_1, σ_2 transform under the Lorentz transformation in a different way. By the terminology of the authors of paper /10/, where this question has been studied, the spin indices are "sitting" on their own momenta. This is based on the fact that spin projection on the third axes is defined in the rest system of a particle. But, in general, to each momentum there corresponds its own system, so the axes of spin quantization

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has been performed.

z and z' are different for right and left indices. The function $D^{\frac{1}{2}}_{\sigma_{1p}\sigma_1}\{V^{-1}(H_p,k)\}$ matches the axes of quantization and by this operation it performs the "removing" of spin indices from the momentum $\vec{k}$ to the momentum $\vec{p}$. Thus, the indices σ_1, σ_2 and σ_{1p}, σ_{2p} of the quasi-potential (30) are "sitting" on the same momentum $\vec{p}$.

Let us mention that in the left hand side of the equations (1) and (2) the spin indices of the wave function $\Psi_q(\vec{p})_{\sigma_1\sigma_2}$ are "sitting" on the momentum $\vec{p}$, and in the right hand side the spin indices of the function $\Psi_q(\vec{k})_{\sigma_1'\sigma_2'}$ are "sitting" on the momentum $\vec{k}$. Let us perform now in equations (1) and (2) the transition to spin indices of the same nature, by defining under the sign of integration the wave function

$$\Psi_q(\vec{k})_{\sigma_{1p}\sigma_{2p}} = \sum_{\sigma_1',\sigma_2'=-\frac{1}{2}}^{\frac{1}{2}} D^{\frac{1}{2}}_{\sigma_{1p},\sigma_1'}\{V^{-1}(H_p,k)\} D^{\frac{1}{2}}_{\sigma_{2p},\sigma_2'}\{V^{-1}(H_p,k)\} \Psi_q(\vec{k})_{\sigma_1'\sigma_2'} \quad (32)$$

spin indices of which σ_{1p} and σ_{2p} are "sitting" on the momentum $\vec{p}$. As a result, the equations (1) and (2) take the form:

$$E_p(E_p-E_q)\Psi_q(\vec{p})_{\sigma_1\sigma_2} = \frac{1}{(4\pi)^3}\sum_{\sigma_{1p},\sigma_{2p}=-\frac{1}{2}}^{\frac{1}{2}} \int \frac{d^3\vec{k}}{E_k} V^{(2)\,\sigma_{1p}\sigma_{2p}}_{dir\;\sigma_1\sigma_2}(\vec{k}(-)\vec{p};\vec{p};E_q)\Psi_q(\vec{k})_{\sigma_{1p}\sigma_{2p}} \quad (33)$$

or

$$\Psi_q(\vec{p})_{\sigma_1\sigma_2} = \frac{(2\pi)^3}{m}\delta(\vec{p}-\vec{q})\sqrt{\vec{p}^2+m^2}\,\xi_{\sigma_1}\xi_{\sigma_2} + \frac{1}{E_p(E_p-E_q-i\epsilon)}\cdot\frac{1}{(4\pi)^3}\sum_{\sigma_{1p},\sigma_{2p}=-\frac{1}{2}}^{\frac{1}{2}}\int\frac{d^3\vec{k}}{E_k}\cdot V^{(2)\,\sigma_{1p}\sigma_{2p}}_{dir\;\sigma_1\sigma_2}(\vec{k}(-)\vec{p};\vec{p};E_q)\Psi_q(\vec{k})_{\sigma_{1p}\sigma_{2p}} \quad (34)$$

where all spin indices in (33) and (34) are "sitting" on the same momentum $\vec{p}$. Natural character of this transformation to the form (33-34) can be understood when solving the quasi-potential equation by the iteration method. Indeed, putting in (2) as the first approximation the expression $\frac{(2\pi)^3}{m}\delta(\vec{p}-\vec{q})\sqrt{\vec{p}^2+m^2}\,\xi_{\sigma_1}\xi_{\sigma_2}$, describing the free motion, we are to perform in addition the "removing" of spin indices σ_1 and σ_2 from the momentum $\vec{p}$ to the momentum $\vec{\kappa}$. At the same time, when iterating (34) we do not need this additional operation, because it is taken into account automatically by the transformation (32). It is easy to obtain an explicit form of the quasi-potential in (33) and (34) by substituting into (30) equality (23) and making use of the relations (28). The denominator in (30) can be represented as /7/:

$$\frac{1}{(p-\kappa)^2}=\frac{1}{2m^2-2p\kappa}=\frac{1}{2m(m-\Delta_0)}\ . \tag{35}$$

As a result, the operator of quasi-potential $V^{(2)}_{dir}(\vec{\kappa}(-)\vec{p};\vec{p};E_q)$, which corresponds to the matrix element of the scattering amplitude (5), can be written through its spin structures in the following way

$$\begin{aligned}
V^{(2)}_{dir}&(\vec{\kappa}(-)\vec{p};\vec{p};E_q)= \\
&=-4\pi e^2\cdot\frac{2m}{\Delta_0-m}- \\
&-4\pi e^2\cdot\frac{(\vec{\sigma}_1\vec{\Delta})(\vec{\sigma}_2\vec{\Delta})-(\vec{\sigma}_1\vec{\sigma}_2)\vec{\Delta}^2}{\vec{\Delta}^2}- \\
&-4\pi e^2\cdot\frac{i\vec{\sigma}_1[\vec{p}\vec{\Delta}]+i\vec{\sigma}_2[\vec{p}\vec{\Delta}]}{\Delta_0-m}\cdot\frac{2p_0}{m^2}- \\
&-4\pi e^2\cdot\frac{p_0^2(\Delta_0+m)+2p_0(\vec{p}\vec{\Delta})-2m^3}{\Delta_0-m}\cdot\frac{2}{m^2}- \\
&-4\pi e^2\cdot\frac{(\vec{\sigma}_1\vec{p})(\vec{\sigma}_1\vec{\Delta})\cdot(\vec{\sigma}_2\vec{p})(\vec{\sigma}_2\vec{\Delta})}{\vec{\Delta}^2}\cdot\frac{2}{m^2}\ ,
\end{aligned} \tag{36}$$

where

$$\vec{\Delta} = \vec{k} (-) \vec{p}.$$

Now let us clarify the meaning of different terms appearing in (36). The first term, as it is clear from (35), describes the Coulomb interaction of spinless particles, because it corresponds to the same Feynman diagram as drawn in Fig. (1), if one would consider the solid lines there as corresponding to scalar particles, but not to spinor ones.

In nonrelativistic limit, neglecting in (36) all the terms proportional to $\frac{1}{c}$, only the first term contributes, which according to (24), transforms into the nonrelativistic Coulomb potential $-4\pi e^2 \cdot \frac{4m^2}{(\vec{k}-\vec{p})^2}$. In the next $\frac{1}{c^2}$ approximation, a nonvanishing contribution comes from the second, third and fourth terms in (36). The last term does not contribute because of its proportionality to $\frac{1}{c^4}$. Thus, we find that in the nonrelativistic limit, with account of terms of the order of $\frac{1}{c^2}$ only, the operator of quasi-potential (36) is

$$V^{(2)}_{nonrel}\left(\vec{k}-\vec{p}; \vec{p}; E_q\right) = \tag{37}$$

$$= -4\pi e^2 \cdot \frac{4m^2}{\vec{\Delta}_3^{\,2}} -$$

$$-4\pi e^2 \cdot \frac{1}{c^2} \cdot \frac{(\vec{\sigma}_1\vec{\Delta}_3)(\vec{\sigma}_2\vec{\Delta}_3) - (\vec{\sigma}_1\vec{\sigma}_2)\vec{\Delta}_3^{\,2}}{\vec{\Delta}_3^{\,2}} -$$

$$-4\pi e^2 \cdot \frac{4}{c^2} \cdot \frac{i\vec{\sigma}_1[\vec{p}\vec{\Delta}_3] + i\vec{\sigma}_2[\vec{p}\vec{\Delta}_3]}{\vec{\Delta}_3^{\,2}} -$$

$$-4\pi e^2 \cdot \frac{2}{c^2} \cdot \frac{\vec{\Delta}_3^{\,2} + 4\vec{p}\vec{\Delta} + 4\vec{p}^{\,2}}{\vec{\Delta}_3^{\,2}},$$

where

$$\vec{\Delta}_{э} = \vec{k} - \vec{p}$$

is the difference of vectors in Euclidean space.

The terms in the second line in (36) and (37) describe the spin-spin interaction and have the same form in the relativistic case as in the nonrelativistic one. The transformation from one theory to another in these terms is performed by change of Lobachevsky geometry to Euclidean one for the nonrelativistic case. So it is possible to say that the spin-spin interaction in the second order in coupling constant has an "absolute" geometrical character according to replacement of nonrelativistic theory by relativistic one. The terms in the third line in (36) and (37) describe the spin-orbital interaction. The spin structure of this interaction also has an "absolute" geometrical meaning. The terms in the fourth line of (36) and (37) have the orbital origin.

Now the last term in (36) can be expanded in the following spin structures

$$4\pi e^2 \cdot \frac{2}{m^2} \cdot \frac{(\vec{\sigma}_1\vec{p})(\vec{\sigma}_2\vec{\Delta})\cdot(\vec{\sigma}_2\vec{p})(\vec{\sigma}_2\vec{\Delta})}{\vec{\Delta}^2} = \tag{38}$$

$$= 4\pi e^2 \cdot \frac{2}{m^2} \cdot \frac{(\vec{p}\vec{\Delta})^2 + [\vec{p}\vec{\Delta}]^2}{\vec{\Delta}^2} +$$

$$+ 4\pi e^2 \cdot \frac{1}{m^2} \cdot \frac{i\vec{\sigma}_1[\vec{p}\vec{\Delta}] + i\vec{\sigma}_2[\vec{p}\vec{\Delta}]}{\vec{\Delta}^2} \cdot (\vec{p}\vec{\Delta})^2 +$$

$$4\pi e^2 \cdot \frac{1}{m^2} \cdot \frac{\{i\vec{\sigma}_1[\vec{p}\vec{\Delta}] + i\vec{\sigma}_2[\vec{p}\vec{\Delta}]\}^2}{\vec{\Delta}^2} .$$

The equations (1), (2) and (33), (34) are written in c.m.s., so the wave function $\Psi_q(\vec{p})_{\sigma_1\sigma_2}$ describes the relative motion of two particles with equal masses and spin 1/2. Therefore, we can add the spins of particles using usual additional rules for two angular momenta in quantum mechanics. Let us define the wave function with the total spin $S = 0, 1$

$$\Psi_q(\vec{p})_{S,\sigma} = \sum_{\sigma_1,\sigma_2=-\frac{1}{2}}^{\frac{1}{2}} \langle \tfrac{1}{2}\tfrac{1}{2};\sigma_1\sigma_2 | S\sigma \rangle \, \Psi_q(\vec{p})_{\sigma_1\sigma_2} . \tag{39}$$

Using the relation for Clebsh-Gordan coefficients

$$\sum_{\sigma_1',\sigma_2'=-\frac{1}{2}}^{\frac{1}{2}} \langle \tfrac{1}{2}\tfrac{1}{2};\sigma_1'\sigma_2' | S\sigma \rangle \langle \tfrac{1}{2}\tfrac{1}{2};\sigma_1'\sigma_2' | S'\sigma' \rangle = \delta_{SS'}\,\delta_{\sigma\sigma'} \tag{40}$$

we represent an equation (33) in the form

$$E_p(E_p - E_q)\Psi_q(\vec{p})_{\sigma_1\sigma_2} =$$

$$= \frac{1}{(4\pi)^3} \sum_{S',\sigma_p'} \int \frac{d^3\vec{k}}{E_k} V_{dir\; S\,\sigma}^{(2)\; S'\sigma_p'}(\vec{k}(-)\vec{p};\vec{p};E_q)\,\Psi_q(\vec{k})_{S',\sigma_p'} , \tag{41}$$

where the quasi-potential $V^{(2)\,S'\sigma_p'}_{S\sigma}(\vec{k}(-)\vec{p};\vec{p};E_q)$ is defined by equality

$$\langle \vec{p}, S\sigma | V^{(2)}(E_q) | \vec{k}, S'\sigma_p \rangle =$$

$$= \sum_{\sigma_1,\sigma_2=-\frac{1}{2}}^{\frac{1}{2}} \langle \tfrac{1}{2}\tfrac{1}{2};\sigma_1\sigma_2 | S\sigma \rangle \langle \vec{p},\sigma_1\sigma_2 | V^{(2)}(E_q) | \vec{k},\sigma_{1p},\sigma_{2p} \rangle \langle \tfrac{1}{2}\tfrac{1}{2};\sigma_{1p}\sigma_{2p} | S'\sigma_p' \rangle . \tag{42}$$

On transforming the second line in (36) to the form

$$\frac{(\vec{\sigma}_1\vec{\Delta})(\vec{\sigma}_2\vec{\Delta}) - (\vec{\sigma}_1\vec{\sigma}_2)\vec{\Delta}^2}{\vec{\Delta}^2} = 2\,\frac{(\vec{S}\vec{\Delta})^2 - \vec{\Delta}^2\vec{S}^2 + \vec{\Delta}^2}{\vec{\Delta}^2} ,$$

where

$$\vec{S} = \frac{1}{2}(\vec{\sigma}_1 + \vec{\sigma}_2)$$

and taking the last term in the form (38) it becomes clear that spin operators of particles emerge in quasi-potential only in the form of the total spin $\vec{S}$. Thus, the quasi-potential (36) commutes with the operator $\vec{S}^2$, i.e. the total spin of the system is conserved. In general case, the singlet-triplet transitions are forbidden because of the Pauli principle. Therefore the quasi-potential in (33) can be taken being diagonal in the total spin

$$\langle \vec{p}, S\sigma | V^{(2)}(E_q) | \vec{k}, S'\sigma'_p \rangle = \langle \vec{p}, S\sigma | V^{(2)}(E_q) | \vec{k}, S\sigma'_p \rangle \delta_{SS'} .$$

That leads to a separation of equation (33) into two equations, one-for the triplet state and the second - for the singlet one.

Now let us study another part of quasi-potential, that corresponds to a diagram of "exchange" interaction given by expression (6). The denominator in (6) can be represented in the following way

$$(p_1 - k_2)^2 = 2m^2 - 2p_1k_2 = 2m\left(m - \Delta^0_+\right), \tag{43}$$

where

$$\Delta^0_+ \equiv (\Lambda_p k)^0 = \frac{p_0 k_0 + \vec{p}\vec{k}}{m} = \sqrt{m^2 + \vec{\Delta}^2_+} \tag{44}$$

and

$$\vec{\Delta}_+ \equiv \overrightarrow{(\Lambda_p k)} = \vec{k}(+)\vec{p} = \vec{k} + \frac{\vec{p}}{m}\left(k^0 + \frac{\vec{k}\vec{p}}{p^0 + m}\right). \tag{45}$$

Obviously, applying to (6) the same transformations (14-19) , allowing to represent the "direct" quasi-potential (5) in the form (29), we come to analogous representation for "exchange" quasi-potential

$$\langle \vec{p}, \sigma_1\sigma_2 | V^{(2)}_{exch}(E_q) | \vec{k}, \sigma'_1\sigma'_2 \rangle = \tag{46}$$

$$= \sum_{\sigma_{1p},\sigma_{2p}=-\frac{1}{2}}^{\frac{1}{2}} \langle \vec{p}, \sigma_1\sigma_2 | V^{(2)}_{exch}(E_q) | \vec{k}, \sigma_{1p}, \sigma_{2p} \rangle D^{\frac{1}{2}}_{\sigma_{1p}\sigma'_1}\{V^{-1}(\Lambda_p, -k)\} D^{\frac{1}{2}}_{\sigma_{2p}\sigma'_2}\{V^{-1}(\Lambda_p, k)\}.$$

It is easy to be convinced of that the explicit expression for "exchange" quasi-potential can be obtained from (36) by changing the vector $\vec{\Delta} = \vec{k}(-)\vec{p}$ by the vector $\vec{\Delta}_+ = \vec{k}(+)\vec{p}$ and making use of equality

$$-\vec{k}(-)\vec{p} = -\vec{k}(+)\vec{p}.$$

The $\mathcal{D}^{\frac{1}{2}}\left\{V^{-1}(H_p;-k)\right\}$ functions in (46) describe Wigner rotation that performs a "removing" of spin indices from momentum $-\vec{k}$ onto momentum $\vec{p}$.

3. Now let us consider an important case of electron-positron interaction. In papers /15-17/ the quasi-potential method was applied for description of this system. In /6,17/ it was shown that the wave function of this system satisfies equations (1) and (2). Our aim here is to show that the spin structure of the quasi-potential in this case can be built analogously to electron -electron interactions with the help of elements of Lobachevsky space. As before, our consideration will be confined to the quasi-potential in the second approximation in coupling constant. Quite analogously to (4) the quasi-potential is constructed with the help of matrix elements of scattering amplitude, consisting of two parts:

$$\langle \vec{p}, \sigma_1\sigma_2 | V^{(2)}(E_q) | \vec{k}, \sigma_1'\sigma_2' \rangle = \langle \vec{p}, \sigma_1\sigma_2 | T^{(2)} | \vec{k}, \sigma_1'\sigma_2' \rangle = \tag{47}$$

$$= \langle \vec{p}, \sigma_1\sigma_2 | V^{(2)}_{scatt.}(E_q) | \vec{k}, \sigma_1'\sigma_2' \rangle - \langle \vec{p}, \sigma_1\sigma_2 | V^{(2)}_{ann}(E_q) | \vec{k}, \sigma_1'\sigma_2' \rangle .$$

The first term in (47)

$$\langle \vec{p}, \sigma_1 \sigma_2 | V^{(2)}_{scatt} (E_q) | \vec{k}, \sigma_1' \sigma_2' \rangle =$$
$$= -4\pi e^2 \cdot \frac{\left[\bar{u}^{\sigma_1}(\vec{p}) \gamma^{\mu} u^{\sigma_1'}(\vec{k})\right] g^{\mu\nu} \left[\bar{v}^{\sigma_2'}(-\vec{k}) \gamma^{\nu} v^{\sigma_2}(-\vec{p})\right]}{(p_1 - k_1)^2} \qquad (48)$$

corresponds to "scattering" diagram (see Fig. 2).

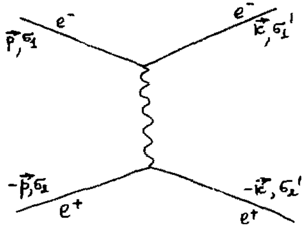

Fig. 2.

and to the expression *)

$$\langle \vec{p}, \sigma_1 \sigma_2 | V^{(2)}_{ann.} (E_q) | \vec{k}, \sigma_1' \sigma_2' \rangle = -4\pi e^2 \frac{\left[\bar{u}^{\sigma_1}(\vec{p}) \gamma^{\mu} v^{\sigma_2}(-\vec{p})\right] g^{\mu\nu} \left[\bar{v}^{\sigma_2'}(-\vec{k}) \gamma^{\nu} u^{\sigma_1'}(\vec{k})\right]}{(p_1 + p_2)^2} \qquad (49)$$

there corresponds " annihilation " diagram, drawn in Fig. 3.

---

*) Bispinor $u^{\sigma}(\vec{k})$, corresponding to positive energy solution of Dirac equation, and negative energy bispinor $v^{\sigma}(\vec{k})$ are connected with the expansion of spinor field in creation and annihilation operators as follows:[6]

$$\Psi(x) = \frac{1}{(2\pi)^{3/2}} \sum_{\sigma=-\frac{1}{2}}^{\frac{1}{2}} \int \frac{d^3\vec{k}}{k_0} \left\{ e^{ikx} b^+_{\sigma}(\vec{k})\, v^{\sigma}(\vec{k}) + e^{-ikx} a_{\sigma}(\vec{k})\, u^{\sigma}(\vec{k}) \right\} \qquad (50)$$

$$\bar{\Psi}(x) = \Psi^+(x)\gamma^0 = \frac{1}{(2\pi)^{3/2}} \sum_{\sigma=-\frac{1}{2}}^{\frac{1}{2}} \int \frac{d^3\vec{k}}{k_0} \left\{ e^{ikx} a^+_{\sigma}(\vec{k})\, \bar{u}^{\sigma}(\vec{k}) + e^{-ikx} b_{\sigma}(\vec{k})\, \bar{v}^{\sigma}(\vec{k}) \right\} \qquad (51)$$

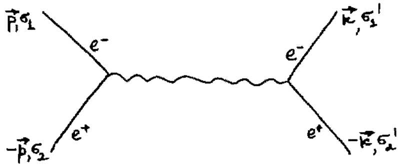

Fig. 3.

Let us perform a transition in (48) to the charge-conjugated bispinors that describe positrons (cf. with paper /6/) with the help of matrix $C = \gamma^0\gamma^5$.

$$v = C\,\bar{u}^{T c} \quad ; \quad \bar{v} = u^{T c} \cdot C^{T -1} \; . \tag{52}$$

A charge-conjugated bispinor $u^c(\vec{k})$ has the same form as bispinor (8):

$$u^{c\sigma}(\vec{k}) = \sqrt{m}\begin{pmatrix} \dfrac{k_0 + m + \vec{\sigma}\vec{k}}{\sqrt{2m(k_0+m)}} & \varphi^\sigma \\ \dfrac{k_0 + m - \vec{\sigma}\vec{k}}{\sqrt{2m(k_0+m)}} & \varphi^\sigma \end{pmatrix} , \tag{53}$$

where

$$\overset{+}{\varphi}{}^\sigma \varphi^{\sigma'} = \delta_{\sigma\sigma'} \; .$$

As a result the operator of the quasi-potential (48), rewritten as

$$\langle \vec{p},\sigma_1\sigma_2 | V^{(2)}_{scatt}(E_q) | \vec{k},\sigma_1'\sigma_2' \rangle = -4\pi e^2 \cdot \frac{\left[\bar{u}^{\sigma_1}(\vec{p})\gamma^\mu u^{\sigma_1'}(\vec{k})\right] g^{\mu\nu} \left[\bar{u}^{c\sigma_2}(-\vec{p})\gamma^\nu u^{c\sigma_2'}(-\vec{k})\right]}{(p-k)^2} \tag{54}$$

after the transformations, analogous to those used to obtain from (5) the expression (36), will be of the same form as (36).

Now we shall study the expression (49). In order to get a possibility to use in (49) the relation (15) for separation of Wigner rotation let us change the order of bispinors in (49). An application of Firtz theorem allows to represent the numerator of (49) in the form:

$$\left[\bar{u}^{\sigma_1}(\vec{p})\gamma^{\mu}v^{\sigma_2}(-\vec{p})\right] g^{\mu\nu}\left[\bar{v}^{\sigma_2'}(-\vec{k})\gamma^{\nu}u^{\sigma_1'}(\vec{k})\right] = \tag{55}$$

$$= \left[\bar{u}^{\sigma_1}(\vec{p})\cdot u^{\sigma_1'}(\vec{k})\right]\cdot\left[\bar{v}^{\sigma_2'}(-\vec{k})\cdot v^{\sigma_2}(-\vec{p})\right] -$$

$$-\frac{1}{2}\left[\bar{u}^{\sigma_1}(\vec{p})\gamma^{\mu}u^{\sigma_1'}(\vec{k})\right]\cdot g^{\mu\nu}\left[\bar{v}^{\sigma_2'}(-\vec{k})\gamma^{\nu}v^{\sigma_2}(-\vec{p})\right] +$$

$$+\frac{1}{2}\left[\bar{u}^{\sigma_1}(\vec{p})\gamma^{\mu}\gamma^{5}u^{\sigma_1'}(\vec{k})\right] g^{\mu\nu}\left[\bar{v}^{\sigma_2'}(-\vec{k})\gamma^{\nu}\gamma^{5}v^{\sigma_2}(-\vec{p})\right] +$$

$$+\left[\bar{u}^{\sigma_1}(\vec{p})\gamma^{5}u^{\sigma_1'}(\vec{k})\right]\cdot\left[\bar{v}^{\sigma_2'}(-\vec{k})\gamma^{5}v^{\sigma_2}(-\vec{p})\right] = ,$$

Let us perform the transition in (55) to charge-conjugated bispinors with the help of (52). Taking into account the relations

$$\overset{T}{C}{}^{-1}\gamma^{\nu}C = \gamma^{\nu} \quad ; \quad \overset{T}{C}{}^{-1}\gamma^{5}C = -\overset{T}{\gamma}{}^{5}$$

we obtain for (49) the following representation

$$\langle \vec{p},\sigma_1\sigma_2 \mid V^{(a)}_{ann}(E_q)\mid \vec{k},\sigma_1'\sigma_2'\rangle = \tag{56}$$

$$= 4\pi e^2\cdot\frac{\left[\bar{u}^{\sigma_1}(\vec{p})\cdot u^{\sigma_1'}(\vec{k})\right]\cdot\left[\bar{u}^{c\,\sigma_2}(-\vec{p})\cdot u^{c\,\sigma_2'}(-\vec{k})\right]}{(p_1+p_2)^2} +$$

$$+4\pi e^2\cdot\frac{\left[\bar{u}^{\sigma_1}(\vec{p})\gamma^{\mu}u^{\sigma_1'}(\vec{k})\right]\cdot g^{\mu\nu}\left[\bar{u}^{c\,\sigma_2}(-\vec{p})\gamma^{\nu}u^{c\,\sigma_2'}(-\vec{k})\right]}{(p_1+p_2)^2} +$$

$$+4\pi e^2\cdot\frac{\left[\bar{u}^{\sigma_1}(\vec{p})\gamma^{\mu}\gamma^{5}u^{\sigma_1'}(\vec{k})\right]\cdot g^{\mu\nu}\left[\bar{u}^{c\,\sigma_2}(-\vec{p})\gamma^{\nu}\gamma^{5}u^{c\,\sigma_2'}(-\vec{k})\right]}{(p_1+p_2)^2} +$$

$$+ 4\pi e^2 \cdot \frac{\left[\bar{u}^{\sigma_1}(\vec{p})\gamma^5 u^{\sigma_1'}(\vec{k})\right]\left[\bar{u}^{c\sigma_2}(-\vec{p})\gamma^5 u^{c\sigma_2'}(-\vec{k})\right]}{(p_1+p_2)^2} .$$

In c.m.s. the denominator of (49) can be written as

$$\frac{1}{(p_1+p_2)^2} = \frac{1}{(p_1^0+p_2^0)^2} = \frac{1}{4(m^2+\vec{p}^{\,2})} . \tag{57}$$

Thus, after transformation of quasi-potential (49) to the form (56) we can use the relation (15). As a result analogously to (29) the quasi-potential (56) can be represented in the form

$$\langle \vec{p},\sigma_1\sigma_2 | V^{(2)}_{ann}(E_q)|\vec{k},\sigma_1'\sigma_2'\rangle =$$

$$= \sum_{\sigma_{1p},\sigma_{2p}=-\frac{1}{2}}^{\frac{1}{2}} \langle \vec{p},\sigma_1\sigma_2 | V^{(2)}_{ann}(E_q)|\vec{k},\sigma_{1p}\sigma_{2p}\rangle D^{\frac{1}{2}}_{\sigma_{1p}\sigma_1'}\{V^{-1}(H_p,k)\} D^{\frac{1}{2}}_{\sigma_{2p}\sigma_2'}\{V^{-1}(H_p,k)\},$$

where

$$\langle \vec{p},\sigma_1\sigma_2 | V^{(2)}_{ann}(E_q)|\vec{k},\sigma_1'\sigma_2'\rangle = \tag{58}$$

$$= \xi^{+\sigma_1}\varphi^{+\sigma_2}\, V^{(2)\,\sigma_{1p}\sigma_{2p}}_{ann\;\sigma_1\sigma_2}(\vec{k}\leftrightarrow\vec{p};\vec{p};E_q)\,\xi^{\sigma_{1p}}\xi^{\sigma_{2p}} .$$

So , in the case of electron-positron interaction, described by the quasi-potential (47), after the transition in equations (1) and (2) to the wave function (32), which spin indices are "sitting" on the same momentum $\vec{p}$, equations (1) and (2) take the form (33)-(34).

Let us study now the spin structure of the quasi-potential $V^{(2)\,\sigma_{1p}\sigma_{2p}}_{ann\;\sigma_1\sigma_2}(\vec{k}\leftrightarrow\vec{p};\vec{p};E_p)$. The spin structure, given by the second term in (56) is defined by (22). The structures, corresponding to the first and last line in (56), have been found in paper/1/;

$$\left[\bar{u}^{\sigma_1}(\vec{p})\cdot u^{\sigma_{1p}}(\vec{k})\right]\left[\bar{u}^{\sigma_2}(-\vec{p})\, u^{\sigma_{2p}}(-\vec{k})\right]\equiv \mathcal{J}_S(\vec{p},\vec{k}) = \tag{59}$$
$$= 2m(\Delta_0+m)\left(\xi^{+\sigma_1}\xi^{\sigma_{1p}}\right)\left(\varphi^{+\sigma_2}\varphi^{\sigma_{2p}}\right),$$

$$\left[\bar{u}^{\sigma_1}(\vec{p})\gamma^5 u^{\sigma_{1p}}(\vec{k})\right]\cdot\left[\bar{u}^{\sigma_2}(-\vec{p})\gamma^5 u^{\sigma_{2p}}(-\vec{k})\right]\equiv \mathcal{J}_{PS}(\vec{p},\vec{k}) = \tag{60}$$
$$= -\frac{2m}{\Delta_0+m}\,\xi^{+\sigma_1}\varphi^{+\sigma_2}\left\{(\vec{\sigma}_1\vec{\Delta})(\vec{\sigma}_2\vec{\Delta})\right\}\xi^{\sigma_{1p}}\varphi^{\sigma_{2p}}.$$

The spin structure of the quasi-potential (58) corresponding to the third line in (56) is defined by the spin structure of the axial current

$$j_A^{\mu}(\vec{p},\vec{k}) = \bar{u}(\vec{p})\,\gamma^{\mu}\gamma^5\, u(\vec{k}). \tag{61}$$

The matrix $S_{\vec{p}}$, due to its dependence on matrix $\vec{\alpha}$, commutes with γ^5. It makes it possible to use relations (15) and (18). So, after separation of Wigner rotation, we find the expression for axial current components:

$$j^{\mu}_{A\,\sigma_{1p}\sigma_{1p}}(\vec{p},\vec{k}) = -\frac{2i}{\sqrt{2m(\Delta_0+m)}}\,\xi^{+\sigma_1}\left\{p^{\mu}(\vec{\sigma}_1\vec{\Delta}) + W_1^{\mu}(\vec{p})(\Delta_0+m)\right\}\varphi^{\sigma_{1p}} \tag{62}$$

Taking account of symmetry relations

$$j_A^0(-\vec{p},-\vec{k}) = -j_A^0(\vec{p},\vec{k})\ ;\ \vec{j}_A(-\vec{p},-\vec{k}) = \vec{j}_A(\vec{p},\vec{k}) \tag{63}$$

we come to the equality

$$\left[\bar{u}^{\sigma_1}(\vec{p})\gamma^{\mu}\gamma^5 u^{\sigma_{1p}}(\vec{k})\right]\cdot g^{\mu\nu}\cdot\left[\bar{u}^{c\sigma_2}(-\vec{p})\gamma^{\nu}\gamma^5 u^{c\sigma_{2p}}(-\vec{k})\right]\equiv \mathcal{J}_A(\vec{p},\vec{k}) =$$
$$= \xi^{+\sigma_1}\varphi^{+\sigma_2}\left\{\frac{2(p_0^2+\vec{p}^2)}{m(\Delta_0+m)}\cdot(\vec{\sigma}_1\vec{\Delta})(\vec{\sigma}_2\vec{\Delta}) + \right. \tag{64}$$
$$+ 4p_0\frac{(\vec{\sigma}_1\vec{p})(\vec{\sigma}_2\vec{\Delta})+(\vec{\sigma}_1\vec{\Delta})(\vec{\sigma}_2\vec{p})}{m} +$$
$$\left. + 2(\Delta_0+m)\frac{2(\vec{\sigma}_1\vec{p})(\vec{\sigma}_2\vec{p})+(\vec{\sigma}_1\vec{\sigma}_2)m^2}{m}\right\}\xi^{\sigma_{1p}}\varphi^{\sigma_{2p}}$$

Thus, finally the quasi-potential which describes the interaction of electron and positron and stands in equations (40) and (64) is

$$V^{(2)\sigma_{1p}\sigma_{2p}}_{\sigma_1\ \sigma_2}\left(\vec{k}(-)\vec{p};\ \vec{p};E_q\right) =$$

$$= V^{(2)\ \sigma_{1p}\sigma_{2p}}_{scatt\ \sigma_2\ \sigma_1}\left(\vec{k}(-)\vec{p};\vec{p};E_q\right) - V^{(2)\ \sigma_{1p}\sigma_{2p}}_{ann\ \sigma_2\ \sigma_1}\left(\vec{k}(-)\vec{p};\vec{p},E_q\right),$$

where the quasi-potential operator $V^{(2)}_{Scatt}\left(\vec{k}(-)\vec{p};\vec{p};E_q\right)$, as it was shown before, is given by (36) with the opposite sign, and

$$V^{(2)\ \sigma_{1p}\sigma_{2p}}_{ann.\ \sigma_2\ \sigma_1}\left(\vec{k}(-)\vec{p};\vec{p};E_q\right) = \tag{65}$$

$$= 4\pi e^2\cdot\frac{\mathcal{J}_S(\vec{p},\vec{k}) + \frac{1}{2}\mathcal{J}_V(\vec{p},\vec{k}) + \frac{1}{2}\mathcal{J}_A(\vec{p},\vec{k}) + \mathcal{J}_{PS}(\vec{p},\vec{k})}{4\left(m^2+\vec{p}^{\,2}\right)}$$

The form of $\mathcal{J}_S(\vec{p},\vec{k})$, $\mathcal{J}_P(\vec{p},\vec{k})$ and $\mathcal{J}_A(\vec{p},\vec{k})$ is defined by relations (59), (60) and (64), respectively. With the help of (22) and (36), it is easy also to define $\mathcal{J}_A(\vec{p},\vec{k})$

$$\mathcal{J}_V(\vec{p},\vec{k}) \equiv \left[\bar{u}^{\sigma_1}(\vec{p})\gamma^{\mu}u^{\sigma_{1p}}(\vec{k})\right]\cdot g^{\mu\nu}\left[\bar{u}^{c\sigma_2}(-\vec{p})\gamma^{\nu}u^{c\,\sigma_{2p}}(-\vec{k})\right] =$$

$$= \xi^{+\sigma_1}\psi^{+\sigma_2}\Bigg\{4m^2 + 2m\,\frac{(\vec{\sigma}_1\vec{\Delta})(\vec{\sigma}_2\vec{\Delta}) - (\vec{\sigma}_1\vec{\sigma}_2)\,\vec{\Delta}^2}{\Delta_0+m} +$$

$$+ 4p_0\,\frac{i\vec{\sigma}_1\left[\vec{p}\vec{\Delta}\right] + i\vec{\sigma}_2\left[\vec{p}\vec{\Delta}\right]}{m} +$$

$$+ 4\,\frac{p_0^2(\Delta_0+m) + 2p_0(\vec{p}\vec{\Delta}) - 2m^3}{m} +$$

$$+ 4\cdot\frac{(\vec{\sigma}_1\vec{p})(\vec{\sigma}_1\vec{\Delta})\cdot(\vec{\sigma}_2\vec{p})(\vec{\sigma}_2\vec{\Delta})}{m\left(\Delta_0+m\right)}\Bigg\}\,\xi^{\sigma_2}\psi^{\sigma_{2p}}$$

Now, let us look for the nonrelativistic limit of the quasi-potential (65). Taking into account that the denominator in (65) is proportional to $\frac{1}{4m^2c^2}$, the contribution to $V^{(2)}_{ann}(\vec{k}(-)\vec{p};\vec{p};E_q)$ will give the first term in (66), $\mathcal{J}_{\mathcal{S}}$ and the term $\frac{2(\Delta_0+m)}{m}$. $\cdot\, m^2(\vec{\sigma}_1\vec{\sigma}_2)$ in the third line of (64). As a result in the nonrelativistic limit only the expression

$$-4\pi e^2\cdot m^2\cdot \overset{+}{\xi}{}^{\sigma_1}\overset{+}{\varphi}{}^{\sigma_2}\left\{\frac{1}{2m^2c^2}\cdot\left(3+\vec{\sigma}_1\vec{\sigma}_2\right)\right\}\xi^{\sigma_{1p}}\varphi^{\sigma_{2p}} \quad (67)$$

contributes to "annihilation" part of the quasi-potential (47). This expression is used in Breit equation for description of positronium.

4. In conclusion we are going to summarize the results obtained here. We have taken Feynman matrix elements of the scattering amplitude, corresponding to one-photon exchange, as the quasi-potential. It turns out, that after separation from them of the Wigner rotation, that has the kinematical nature, they are parametrized through the quantities defined on the hyperboloid(3), i.e. belonging to Lobachevsky space. An expansion of the quasi-potential in spin structures was carried out without using the expansion in $\frac{1}{c}$. It is remarkable that in this consideration some terms in the expansion of the quasi-potential in spin structures and the spin structures themselves look like the direct geometrical (in the sense of change of Euclidean geometry by Lobachevsky one) relativistic generalization of spin structures and potentials in quantum mechanics. The natural continuation of formalism developed here will consist in its formulation in the relativistic configuration representation /7/ because in this representation the relativistic Coulomb problem has been explicitly solved for the spinless case.

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References

1. N.B.Skachkov. JINR Communication E2-7159, Dubna, (1973).
2. A.A.Logunov, A.N.Tavkhelidze. Nuovo Cim., 29, 380 (1963).
3. V.G.Kadyshevsky, A.N.Tavkhelidze.Problems of Theoretical Physics, Nauka, Moscow(1969) (In Russian).(In /3/ one can find a more complete list of the articles on the quasi-potential approach).
4. V.G.Kadyshevsky. Nucl.Phys., B6, 125 (1968).
5. V.G.Kadyshevsky. JETP, 46, 654 (1964); JETP, 46, 872 (1964); Dokl. Akad. Nauk SSSR, 760, 573 (1965).
6. V.G.Kadyshevsky, M.D.Mateev. Nuovo Cim., 55A, 275 (1967).
7. V.G.Kadyshevsky, R.M.Mir-Kasimov, N.B.Skachkov. Nuovo Cim., 55A, 233 (1968).
8. V.G.Kadyshevsky, R.M.Mir-Kasimov, N.B.Skachkov. " Particles and Nucleus", vol.2, N.3, (translation of "Elementary Particles and Atomic Nucleus", Dubna (1970)).
9. Yu.M.Shirokov. JETP 21, 748 (1951).
10. A.A.Cheshkov, Yu.M.Shirokov. JETP, 44, 1982 (1963) (In Russian),
11. A.A.Cheshkov. JETP, 50 144, (1966) (In Russian).
12. V.P.Kozhevnikov, V.E.Troitski, S.V.Trubnikov, Yu.M.Shirokov. TMP, 10, 47 (1972). (In Russian).
13. P.Moussa, R.Stora. Analysis of Scattering and Decay , Gordon and Breach , New-York-London-Paris (1968).
14. V.B.Berestetsky, E.M.Lifshitsh, A.P.Pitaevsky. Relativistic quantum theory , part I, Nauka, Moscow (1968) (In Russian).
15. R.N.Faustov. Nucl.Phys., 75, 669 (1966); Nuovo Cim., 69A, 37, (1970).
16. R.N.Faustov. Paticles and Nucleus , vol.3, N.1, 238 (1972) (In Russian).
17. A.S.Ilchev, M.D.Mateev. JINR Communication E2-5225, Dubna (1970).

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